8.512 Theory of Solids II
Spring 2009

LECTURE V

`Continued discussion on Kubo formula:`

Sanity Check with a *random potential*:

$$\overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2\delta(\mathbf{r}-\mathbf{r}')$$

$$\sigma_{\mu\mu}(q,\omega) = \frac{1}{\omega}\int d(\mathbf{r}-\mathbf{r}'\int dte^{-i\omega t}e^{i\mathbf{q}.(\mathbf{r}-\mathbf{r}')}\overline{<0|[j_{p\mu}(\mathbf{r},t),j_{p\mu}(\mathbf{r}',t)]|0>}$$

$$= \frac{1}{\omega}\frac{1}{\Omega}\int d\mathbf{r}\int d\mathbf{r}'\int dte^{-i\omega t}e^{i\mathbf{q}.(\mathbf{r}-\mathbf{r}')}\overline{<0|[j_{p\mu}(\mathbf{r},t),j_{p\mu}(\mathbf{r}',t)]|0>}$$

Where the bar in the above equation is the *impurity average*.

DC $q=0\ \ \omega\to 0$

$$\sigma(\omega) = \frac{1}{\omega}\frac{1}{\Omega}\sum_n <0|\int d\mathbf{r}j_{p\mu}(\mathbf{r})|n><n|\int d\mathbf{r}'j_{p\mu}(\mathbf{r}')|0>\delta(\omega-(E_n-E_0))$$

Where $|n>$ is an exact eigenvalue of the full *one-body* hamiltonian : $(H+V)|n>=E_n|n>$ In principle one can find the spectrum of $(H+V)$ so $|n>$ is the particlehole pair:

$$|n>=|\beta\overline{\alpha}>,\ E_\beta>E_F, E_\alpha<E_F$$

$$\int d\mathbf{r}<n|j_{p\mu}(\mathbf{r}|0>=\int d\mathbf{r}(\frac{e}{m})<n|\nabla_{\mathbf{r}}\sum_i\delta(\mathbf{r}-\mathbf{r}_i)|0>$$

$$=\int d\mathbf{r}(\frac{e}{m})\int d\mathbf{r}_1\varphi_\beta^*(\mathbf{r_1})\nabla_{\mathbf{r}}\delta(\mathbf{r}-\mathbf{r}_1)\varphi_\alpha(\mathbf{r_1})$$

$$=\frac{e}{m}\int d\mathbf{r}_1\varphi_\beta^*(\mathbf{r_1})\nabla_{\mathbf{r}_1}\varphi_\alpha(\mathbf{r_1}) = (e/m)<\beta|\nabla|\alpha>$$

$$\sigma(\omega) = \frac{\pi e^2}{\omega\Omega}\sum_{\alpha\beta}\overline{|<\beta|\frac{\nabla}{m}|\alpha>|^2}\,\overline{\delta(\omega-(E_\beta-E_\alpha))f(E_\alpha)(1-f(E_\beta))}$$

Where the "*broken*" average comes from our assumption that the random potential causes *uncorrelated* φ and E.

$$\sigma(\omega) = \frac{\pi e^2}{m^2\omega\Omega}\int_{-\omega}^{0} d\omega_1 \overline{\sum_\alpha \delta(\omega_1+E_\alpha)\sum_\beta \delta(\omega_1-E_\beta-\omega_1)}\,\overline{|<\beta|\frac{\nabla}{m}|\alpha>|^2}$$

`MORE APPROXIMATIONS:`

$\overline{|<\beta|\frac{\nabla}{m}|\alpha>|^2} = \overline{v^2}$
$\omega \gg$ level spacing $\Rightarrow$ $N(\omega) = \overline{\sum_\alpha \delta(\omega_1+E_\alpha)} \simeq N(0)$

Note that $N(0)$ is the density of state per (just!) unit energy, so it should diverge as $\Omega \to \infty$ or $\triangle \to 0:\ N(0) \approx \frac{1}{\Delta}$.

$$\boxed{\sigma(\omega\to 0) = \frac{e^2\pi N_0^2}{\Omega}\overline{v^2}}$$

Next we estimate $\overline{|<\beta|\frac{\nabla}{m}|\alpha>|^2:}$

Define l as the distance that wavefunction loses information about its phase so for a perfect plane wave (without scattering) $l \to \infty$ and for a very strong scattering impurity $l \to k_F^{-1}$. At this point we intend to find approximation for the $\overline{|<\beta|\frac{\nabla}{m}|\alpha>|^2}$ in the former regime or

$$k_F l \ll 1$$

Let's make a grid out of our sample where each section has the volume of

$$\nu = \frac{4\pi}{3}l^3$$

Definition of l suggests that within each box we can in principle associate a wave vector to our wavefunction:
So one an define

$$\delta_i \equiv \int^{\nu_i} d\mathbf{r} \psi^*_{\mathbf{k}'} \frac{1}{m} \frac{\partial}{\partial x} \psi_{\mathbf{k}}$$

associate $\mathbf{k}$ with α and $\mathbf{k}'$ with β. By this partitioning we have:

$$\overline{v^2} = \overline{v^2_{\alpha\beta}} = \frac{\Omega}{\nu} \overline{|\delta|^2}$$

$$\delta_i \approx \left(\int_0^{\nu} d\mathbf{r} \frac{e^{i(\mathbf{k}-\mathbf{k}').\mathbf{r}}}{\Omega} \frac{k}{m} \right) e^{i\phi_i}$$

Where ϕ_i is the random phase at the site i.

$$|\mathbf{k} - \mathbf{k}'| = 2k_F \sin \frac{\theta}{2} \simeq k_F \theta$$

for $k_F \theta l > 1$ we will encounter rapid oscillations and $\delta_i = 0$ and for

$$k_F \theta l < 1 \Rightarrow \delta_i = \frac{k\nu}{m\Omega} e^{i\phi_i}$$

In order to average over different boxes, we average over $\mathbf{k}$ and $\mathbf{k}'$ which amounts to average over θ.

Now the θ integration:

$$\overline{v^2} = \frac{\Omega}{\nu} (\frac{k\nu}{m\Omega})^2 \int_0^{1/kl} d\theta \frac{2\pi \sin\theta}{\pi}$$

$$\int_0^{1/kl} d\theta \frac{2\pi \sin\theta}{\pi} = \frac{1}{4(kl)^2}$$

$$\therefore \overline{v^2} = \frac{\pi l}{3m^2 \Omega}$$

$$\sigma(\omega \to 0) = \frac{e^2\pi^2}{3m^2} \left(\frac{N(0)}{\Omega} \right) l$$

Put $l = \tau v_f$; $n = \frac{k_F^3}{3\pi^2}$; $\frac{N(0)}{\Omega} = \frac{mk_F}{2\hbar^2\pi^2}$ in the

$$\sigma_{Boltzman} = \frac{ne^2\tau}{m}$$

you'll get the same result (Of course with different coefficient) $\sim$

$$\frac{e^2}{\hbar}\, k_F^2 l$$