PROFESSOR: OK. Now, before we start with the prepared questions, I have another question for you. It's sort of taking a straw poll before and after discussing something.

And the question for you, I want to ask you is, what is the nature of shot noise? The whole set of questions will build up to teach you something about what is shot noise.

In other words, when you measure in photons, you start with coherent beams, you always have sort of square root n. And the question I want you to ask is, when we measure square root n fluctuations, when we detect n photons, is this caused by the nature of the measurement process-- sort of how we detect photons-- or is it an intrinsic quantity of the quantum fields?

So I just want to get your intuition, your gut feeling, with the shot noise-- it's maybe not black and white, but if you think it's more related to the measurement-- Well, let me write it down.

Shot noise. OK, so the question is, shot noise, is it caused by the quantum measurement process, projection of whatever you know about it?

Second part, is it a property of the quantum fields? For instance, if you don't make a measurement, if you run the quantum fields through a beam splitter, we can split it. We can split the shot noise even before we measure it. That would mean quantum fields.

And maybe then, third is both. And the fourth is none of the above. So let's just take an opinion. And I'm not discussing the answer at this point. I will come back to that at the end of those 10 or 15 minutes and ask you the question again.

So I just want to see what your intuitive understanding is when you measure the

number of photons in a laser beam in an optical field and you see fluctuations. Quantum measurement versus properties of a quantum field.

Quantum measurement means mainly it caused the moment you detected. And quantum fields means it's there already before, and it can be, whatever, modified. Quantum fields or both. All right. Good.

Fine. Let's now go through the question. We first have to make sure that we use the same definition and measure things in the same unit. We want to talk about coherent states. So the first is just little, quick checks.

Coherent states are eigenstates of the Hamiltonians of a or a dagger? For the easy question, I will just give you 10 seconds. All right. Stop. Display. Yes. OK. That's good.

Next question. Which states realize the minimum quantum uncertainty with delta p delta q equals h-bar over 2? I give you five different states. And in order to use the clicker, you should just answer how many of them are minimum uncertainty states. Ready, set, go.

So three or two. Let's just go through them. The vacuum, definitely. The coherent state, definitely. The thermal state, definitely not.

Now, number states, they are eigenstates. But let's just take a big number state. It's a big lean in the quasi-probability. It has a huge delta x and a huge delta p. It's definitely-- it's a very quantum like state, very nonclassical. But it doesn't have minimum uncertainty.

Final question. x and p, are those minimum uncertainty states? x and p are a little bit cryptic, because one is infinite, the other one is 0. And infinite times 0 is h-bar over 2, yes. So it's a minimum uncertainty state, because we discussed it. It is the limit of squeezing, when we perform the limit that the squeezing parameter goes to infinity.

You can send me your lawyer, and I may have to step back from it. But the way I

present it, this is one correct answer. It's a limit of an infinitely squeezed state. So if the squeezed vacuum-- which I forgot actually to ask-- if the squeezed vacuum is a minimum uncertainty state, then the x and p state are the limiting case.

By the way, your clicks are getting recorded, and the clicks are responsible for half of your semester grade. This was a joke.

You can actually use clickers and record them, but then each student would have a clicker with an already set number. But as you see, you just take a random clicker out of the box and put it back in. So I have no idea what your clicker number is. And to use the clicker is solely for our joint entertainment here.

OK, next question. What features are unique to nonclassical light? And nonclassical light means-- well, we have discussed nonclassical lights but their different aspects.

One is that the roots of the two quadrature components-- the uncertainty of the two quadrature components-- the cosine and sine quadrature components are unequal; negative cosine probabilities; a correlation function, which is smaller than 1; and sub-Poissonian statistics. So I've given you four criteria. How many of those four are features which we will not associate with classical light, only with nonclassical light?

OK. Stop. Display. So there's a wide distribution. Let's quickly discuss the answer.

Negative cosine probabilities, well, if you use w distribution and p distribution, negative probabilities are always associated with nonclassical light. You can also say it's the definition of nonclassical light.

g 2 function of-- maybe, I should not say smaller or equal -- smaller than 1 is only possible for nonclassical light. Sub-Poissonian statistics is also only possible for nonclassical light. Now, the fact that the two quadrature components are different, you can just use some, and you can choose classical states and prepare them, that you have more certainty in the sine quadrature than in the cosine quadrature, so definitely not.

OK, now, we really need prominently to discuss quantum noise and shot noise, the

coherent state. So just to make sure that we relate-- I will relate later everything to photon number. Because photon number is intuitive. You can count, you can feel the photons fluctuation of square root n and immediately tell you Poissonian, if it's less, sub-Poisson, all that. So I want to make sure we are correctly associating photon number with coherent states.

So, therefore, the first question is if you have a coherent state, alpha, and we detect all the photons with a photodiode of 100% efficiency, is the number of detected photons alpha or alpha squared? Yes. Alpha is, so to speak, the field. And alpha squared is the intensity.

So, therefore, form now one, I will always use units for the photocurrent in such a way that the photocurrent, or the integrated photocurrent, is the number of photons. So when I say, what is the current associated with a coherent state, it's just alpha squared.

Just one side remark. If you use a laser beam, you sort of have a stream of photons, but you have a time constant in your detector, and every millisecond you do a measurement. And then, what I mean by alpha or alpha squared is the number of photons which arrive during the time constant. So that's sort of the mapping.

We all usually talk about coherent state in an isolated system. But if you have a beam, you just chop the beam into pieces and associate that with your coherent state.

OK. So now, we are talking about the measurement of coherent beams. We will have now lots of coherent beams, running through beam splitters, being mixed at beam splitters and such. And we will always characterize the strength of the coherent beam by alpha squared. And that means in photons.

So now, the next question is-- again, just to make sure we are all on the same-- we have the same basis, what is now the variance? If I measure the fluctuating current, I measure the current, I have a coherent state, and I repeatedly measure the current, it will show fluctuations. And the question is, what is not the standard

deviation of the current, what is the variance, which is the standard deviation squared?

Yes, the standard deviation is square root n. The variance is n. OK. Stop. Hide.

OK, now finally, now we can have fun with beam splitters. So, so far, we have really just repeated a little bit of definitions of classical and nonclassical light at coherent states. We measured current. We know now in what units we need to measure the current. And we characterized the fluctuations in variance not in standard deviation for the rest of this unit.

But now, we run the coherent state through a beam splitter. And we have two photodiodes, one i1 and one i2. And we measure the current, i1. What is the variance now in the current you measure?

OK. It is A. It's n/2.

The way how you think about what it is, when you split a laser beam with a beam splitter, you get two laser beams. Each is half the intensity. So therefore, each creates a photocurrent, which is n/2.

And since one laser beam, which is a coherent state, gives two laser beams, which are two coherent states, each of them has a variance which is equal to the photon number. And this means, we have Poissonian statistics.

So we have n/2 photons in each beam. The standard deviation is the square root of n/2. And the variance is n/2.

Ask me, if there are any questions. I mean, this is still the preliminaries. We are not yet bringing in squeezed light. Then, it gets really interesting.

OK, we will always refer to that situation. We have a laser beam. We split it. We create-- and that's actually your homework assignment. We'll actually show wonderful operator algebra.

If you split a coherent beam, you get two coherent beams. And they are simply

characterized that the alpha value is now down by square root 2. The number is down by 2. And if the number is down by 2 and you have a coherent beam, the variance is down by 2. Poissonian statistics.

OK, next question is we have a beam splitter. We have two photodiodes, i1 and i2, and now we measure-- it's important for balanced homodyne detection-- we can now take the two currents, and we can add them or we can subtract them. So the question is, if we add the two currents, what is now the variance in the two currents?

Remember, each current had a variance of n/2. So now, you are asked, when you sum the currents of the two photodetectors, what happens to the two variances? Do they sum up? Do they--?

I don't know. Do people want to-- does somebody want to defend his or her choice? What do you think is the variance?

AUDIENCE: It should be the answer, you just measure before the beam splitter.

PROFESSOR: You said it should be as if I measure before the beam splitter, and then it would be n, because we have n photons.

Well, you can also see if you put in a mirror here, and you shine the two beams on the same photodiode. And then you would say you have n photons. And if there are Poissonian fluctuations, you would get n.

But there is one thing you should keep in mind. Since we have the beam splitter here-- this is the correct answer. But I want to point out one subtlety for you, which will become important later.

The beam splitter has an open part with the vacuum. And so when you said we can measure before or by sort of just combining the two beams with a big photodiode, we can measure after, that's now a question for you-- which, maybe hold the thought and ask me later, if it doesn't become clear with the following question.

Hasn't the vacuum entered here, and hasn't the vacuum, which has entered here added fluctuations? So if you put in a beam splitter, and we mix in the vacuum with

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our coherent states, shouldn't that possibly mean that if you use a big photodiode and measure everything together, that we measure the variance we had before. But isn't that possibly a contribution of the vacuum, because we were opening the door to the vacuum? Please hold the thought.

OK. The next thing is now, you can't have the solution that we take sort of a big photodiode and combine the two beams, because I'm asking you now is, what is the variance in i1 minus i2? So you subtract the two currents.

So the question is, if you have technical noise, the two beams would be fluctuating, and you would form i1 minus i2. All the technical noise would cancel away. But what is shot noise?

Is shot noise technical noise? Is shot noise that every time you prepare an ensemble, there is a hiccup, and you can split it and subtract it? So now, it's getting really interesting.

To what extent does a beam splitter-- does a split coherent state, which has shot noise, and you now perform the difference of the measurement, can you get rid of some of the shot noise or not? You vote, please.

Yes. So it doesn't matter whether we measure the sum or the difference. So it seems, we have two beams. And it seems that the noise in i1 and i2 is completely uncorrelated. And whether we perform the sum or the difference, it doesn't matter.

As long as you think about classical states and coherent states, you can get away with this notion. But now I'm going to add some squeezed light. And now, it's getting more subtle.

So we have exactly the same situation. We have our beam splitter. We split our coherent beam into two beams, exactly as before. But now, I'm playing with a vacuum. I'm playing with the open part of the beam splitter, and I put in squeezed vacuum.

And just to be specific, I've squeezed the vacuum in such a way that what is narrow

now is the quadrature component of the coherent state. In other words, the coherent state is cosine omega t. In sine omega t, we haven't been put in any photons. Our coherent state has really a classical oscillation in cosine omega t.

And what I have squeezed here with the vacuum is the noise in the cosine omega t quadrature component. So in other words, the coherent state and the short axis of the ellipse are the same quadrature component, which means, they can now interfere.

So the question is now the following. We split the beam. We have n/2 photons here. We have n/2 photons there. And what is now the result for the variance?

I formulated a condition up there, which is the strong local oscillator condition. You should assume, if you get confused, that the local oscillator is very, very strong. And if you have any doubts about some smaller terms, just crank up the power and make sure that it's really the local oscillator which dominates, which is the biggest number in all equations.

OK. So the reference is without squeezing, the noise, or the variance, in i1 was n/2. With n/2 photons, Poissonian statistics was n/2.

The question is now, when we use squeezed light, does the variance stay the same? Does it go up? Does it go down? Or does it go to 0?

By the way, at the end of the clicker question, if you come to me after class and honestly declare you got 100% on all questions, I buy you a lunch. OK, you're done?

What have you learned? It's almost random. I will give you a little bit formalism later, but I mean, let me say one thing.

I had a few discussion with students after class, and I came to the conclusion after last class that I could teach today about the Deutsch-Jozsa algorithms' wonderful applications of all that, but I came to the conclusion, you can go to the Wikipedia and read about yourself. Because this is simply using beam splitters, homodyne

detection. It's just using kind of all the tools.

But what I said instead is, I'd rather spend half the class today in trying to give you, with very simple examples, an intuitive understanding. I mean, everything I'm saying you can do with operator algebra. But to some extent, I feel when we learn physics, we use operators, we use equations, but in the end, we want to shape our intuition. And what I try to sort of challenge you today is, what happens to noise? So what I have to tell you now is-- OK, got to hide this.

The way you should look at it very soon, I hope, is the following, that when you split the coherent beam, you also split the noise. In other words, your little uncertainty circle goes down in the amplitude by square root of 2. It is split-- the reflection coefficient of the beam spit and transmission coefficient is 1 over square root 2. But what happens is, you let in the vacuum here. And so, the vacuum comes into the transmission coefficient of 1 over square root 2.

So what I want to teach you is that when we think back to this example and we had this situation with a vacuum, yes, it looked to us that we had n/2-- so this was i1 plus i2-- we had n/2 photons and the variance was n/2, it looked completely Poissonian as if nothing has happened.

But what you should understand is, that the beam splitter has taken out-- in amplitude, 1 over square root 2, when I square it-- half of the noise out of the coherent state. But half of the vacuum state was added. And half of the vacuum state and half of the intrinsic noise of the coherent state give you back this circle of unity. And you can pretty much neglect all subtleties of the quantum field.

But now, by messing up with a vacuum, by squeezing the vacuum, half of the shot noise comes from the beam splitter. It's a 50-50 beam splitter. But the other half comes from the open part. But if you squeeze it strongly, it's on the order epsilon. That means, it doesn't contribute.

And the correct answer is now, we have-- just a second. It is linear or quadratic? Variance is quadratic. The correct answer is here, because this is epsilon; this is

small. We only get half of the noise we had before. We had n/2, and now, we have n/4.

OK. Now, we take exactly the same situation. So the thought is that you measure the fluctuation here and you have only half of them, because this squeezed vacuum, you've squeezed out the noise from the vacuum part.

And since you have a strong coherent state, everything which is sine omega t doesn't matter. Cosine omega t is homodyned, everything fluctuation and cosine omega t. When we measure, the intensity is the square of phi, or it's a dagger a. And so if you measure a quadratic quantity and we have a very strong local oscillator, everything is projected on cosine omega t.

This is why only the cosine omega t fluctuations of discrete statement are here. It's a strong local oscillator limit. OK. Now, I hope you've learned something.

Next question is now, we do exactly the same thing, but now we have the two currents, and the question is, what is the variance in i-plus? I add the two currents. What do I get? And I give you the same choices as before.

Just as a reminder, when we did not squeeze, the variance was n. So the question is now, has this squeezing done nothing, or has it reduced the variance? What is happening?

The correct answer is C. It's n. And I will show you-- I owe you a little bit of math now.

But the way how you should feel is the following. The noise of the coherent oscillator is equally split. But the amplitude-- I mean, you can say 50% go through, 50% go through here. And if you add the two currents, you get back the full noise of the original coherent state.

You would say what about the squeezing? Well, the issue is the following. The squeezed light, the squeezed vacuum, is reflected with a reflection coefficient, which is minus 1 over square root of 2.

You know when you have two beams reflected, by unitarity, it's necessary that one has a negative, the other one has a positive reflection coefficient. So therefore, you have this squeezed vacuum appears in this arm with a minus sign and this arm with a plus sign. And if you sum up the two currents, you lose everything which is related to the vacuum part.

So when you measure i1 plus i2, what came from the vacuum part entered here with a minus sign, here with a plus sign. And therefore, you [INAUDIBLE] what you've done at the vacuum part.

This is when I told you earlier, you measure i1 plus i2, and we all agreed, the variance was n. And I said, well, this is exactly the same if you put a photodiode-- big photodiode-- behind the beam splitter or before the beam splitter. The vacuum has not contributed. That's what it seems. But even if you squeeze the vacuum, it cancels out.

OK. Now, the obvious next question-- but I think I've given you the answer-- is what happens when we now measure the difference, i1 minus i2? Again, without squeezing, the variance was n. So now, we measure i1 minus i2, and what happens?

OK. The correct answer is this. Well, when I say 0, it's always epsilon squared terms which I neglect. I just assume strong squeezing. I mean, I just want to give you the most conceptional problem.

What happens is the following. And I will show you the equation. The noise of the coherent state is split equally. So we have a delta a-- a is the field operator, or the quadrature operator here-- and a delta a here.

And therefore, the original shot noise is common mode, because the reflection and transmission coefficients are the same. And if you subtract the current, you completely get rid of all noise, which was originally associated with your coherent state.

Now, what happens is the small noise we get, but now, we have to consider the vacuum part. But what comes in on the same quadrature component point on the vacuum tube is strongly split. And because the beam splitter has different signs for reflection and transmission, the noise is anti-correlated.

And if you take the difference, you will actually measure the noise of the input of the open part of the beam splitter, which would mean, in terms of order epsilon-- I didn't work it out. I think it's epsilon squared. But it's a power of epsilon.

So anyway, it seems your intuition fails here. So let me try to give you a few equations which explain that. In your homework and in most of the course, we really want to work with operators. We do transformation of operators and such.

But I've found-- for shaping my intuition-- I found a simplified approach very helpful. And I've posted two references for you. One is the article by Schumacher, and one are some older lecture notes, where I summarize some results of this article. And this goes as follows.

When you have a coherent input, you can say you have a deterministic classical field. And then, you have some noise in one quadrature component, which is cosine omega t. And the i is a reminder that there's also noise in sine omega t. So you can really say the electric field has a sharp value, but then, there is some fluctuation in the coefficient of cosine omega t and a little bit fluctuation in the coefficient of sine omega t.

And what is important now is, if you shine that on a photodiode, this is sort of the field. The current is proportional to the square of it. You have to be a little bit careful where you put complex conjugation, but it is the square of it.

And now, if we use the strong local oscillator assumption, we only take alpha squared, which gives us a number of photons. But then, those fluctuations are homodyned. We only take-- we don't consider that the square of the fluctuation. We only homodyne it. We use, at the cross term, where we multiply with alpha.

So if I now put in photon numbers for alpha, we get n photons. Yes, OK. And now,

we get some noise.

Well, the noise is the current squared. The noise is-- we take the average of the current squared and subtract the average of the current squared. So if you do that-- this is the current squared-- the current squared average, and the average current squared takes that away. So we get this here as the variance.

And now, I have to tell you that in the units I'm using here, this delta a1 squared for coherent state-- so if I show the standard circle-- the standard Heisenberg uncertainty circle-- this quantity is 1/4. I've just kept track of my constant. So therefore, we obtain in this picture that the variance is n. And this is Poissonian light with a coherent state.

And the fluctuation squared of the current, n, this is Poissonian statistics, square root in fluctuations of the current. The variance is n.

But what you should realize is one thing. The moment I have a strong coherent state, the sign quadrature component doesn't matter at all. Because if you multiply cosine with sine and average, you get 0. So we only consider the quadrature component, which is given by the coherent state.

This is a principle of homodyne. You set your quadrature component with your local oscillator. OK.

But now, I have a beam splitter. And the way you can think about it is, you can get everything in operator algebra, but just take my word or follow the procedure now that we are saying, we can always think about a quantum field that it has an average value and some fluctuations.

And if you pass it through an optical element or a beam splitter, we have a transmission coefficient, but it multiplies the mean value of the field. But it also multiplies the fluctuations of the field. So that's why I'm saying the beam splitter is actually splitting the fluctuations of the field. And therefore, you get reduced fluctuations in either beam. But we have to consider that there may be vacuum, and the vacuum has also some fluctuations and will reflect in or transmit through parts of

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the vacuum.

So now, we want to understand photocurrents. For photocurrents, we take those quantities and square them. So which one should I take? Well, let me take the first line.

So this is i1. So if I calculate now i1, I obtain t squared alpha squared, plus the cross product between the first two terms is 2 times t alpha times t delta a1. And then, I get a second term from the vacuum, which is 2tr alpha delta b2.

And let me assume that we have a balanced beam splitter, 50-50. So t squared is simply 1/2. Alpha squared is the number of photons. So this term becomes n/2.

t squared is 1/2. It cancels with the 2. We are left with alpha. And this is shot noise.

Here, appears our square root n. So you can say the shot noise comes because we have homodyned the quantum noise. The quantum noise, which is the same for all coherent states, has now been multiplied with square root n through the homodyne process. OK.

So I lost my line. That's delta a2. And then-- and this is important-- because of the mixing at the beam splitter, the quantum noise off the vacuum gets also homodyned by a factor of square root n.

So now, I think you see what I told you before. The transmitted beam has half of its noise coming from the coherent state, and half of the noise comes from the vacuum. So if I would now take the current, square it, calculate the variance, I would find that I have n/2 photons with a variance of n/2. But an identical contribution of the noise will come from the original coherent state and from the vacuum.

So therefore, when I replace the vacuum by a strongly squeezed vacuum, the beam, the current i1, lost half of its shot noise. We had n/2 photons with a noise-- with a variance, which was n/4. Any questions? Yes.

AUDIENCE: Why are the subscripts on the delta operator of [INAUDIBLE]?

PROFESSOR: Because I should have dropped them all together, saying we are only using the quadrature component of the local oscillator now. Everything else is orthogonal to it and cancels out. But if you want me to put them back, they should all be equal. I compiled this writeup from different lecture notes, which one had the cosine, one had the sine component. I just made a [? stitching error. ?]

OK. So this was i1. If I would now look at-- let me change the color-- at the current, i2, the only anything which changes is that because the quantum noise is transmitted with a positive coefficient, that I have to put a plus sign here. And now, you realize the magic.

When you form i1 plus i2, whatever came through the vacuum part, or through the open part, cancels out. Therefore, when we calculated-- or when I asked you for the variance in i1 plus i2, it was independent, whether there was squeezing, whether whatever was at the vacuum part. And therefore, it doesn't matter whether I put my power meter, measuring both currents-- both beams-- before the beam splitter or after the beam splitter. But-- yes, a question?

AUDIENCE: This last really depends on the convention of where we put the negative sign in reflection? Because if I had put the negative sign here and a positive there, then I would have been canceling my delta a's instead of b's? [INAUDIBLE] the process?

PROFESSOR: Yes. Well, so you should [INAUDIBLE] conventions here. It's clear that you have to be careful. We're talking about [? face-sensitive ?] detection.

So what happens is-- number one is-- there is really physics associated. When you have a beam splitter, one, let's assume you have a piece of glass, which has some coating on it. One beam is reflected when it goes from the vacuum to material interface. The other one is reflected from the material to vacuum interface.

AUDIENCE: So then, it depends on which face of the beam that will be put which--?

PROFESSOR: But one thing is absolutely clear. The two reflected beams, one of them has a minus sign. So therefore, when I do i1 minus i2, the contribution of the--

AUDIENCE: --If a's will cancel.

PROFESSOR: Oh. It is clear that in i1-- OK, yes. But the question is whether we have to perform i1 minus i2 or i1 plus i2. Well, I don't know if it helps you, but what happens is we have a propagating laser beam. If you would simply move your mirror by half a wavelength, you get a minus sign.

So eventually, all of the phases-- the beam splitter phase, the propagating phase-- all the phases have to be controlled. And in essence, what you do is you put an experiment together, you measure some noise, and then you move your mirror or you change your phase. And then, you really figure out which noise is now subtracting and which noise is constructively interfering. Other questions?

OK. But the other thing is now obvious. When we measure i1 minus i2, everything related to the coherent input, to the original noise of the beam, cancels out. And what we are measuring is only the noise which came in through the other part.

So this is the reason why, if you have interesting light you want to measure it, you use a local oscillator. But then, you do i1 minus i2. And this is what I presented to you at the end of last class, is the way how people have measured the signature of squeezing. Any questions?

And finally, I think this is one of the coolest things. Now, we want to talk about the displaced squeezed vacuum. Now, we put everything together.

Remember, we have learned by applying the squeezing operator, how we can squeeze the vacuum. And then, with the displacement operator, I can move the displaced vacuum, that there is an expectation value, alpha, for the field. So it now looks like a squeezed coherent state. So this is now the state we have generated.

And I explained to you that such a state can be generated by using a beam splitter in the limit where the transmission coefficient goes to 1. So the squeezed vacuum is pretty much transmitted without loss. But now, we have a very strong local oscillator.

Also, the reflection coefficient goes to 0. We can just crank up the power in alpha in

such a way that r alpha squared-- the number of photons which get reflected-- is n. So the limit we are looking at it is small n and large alpha. But what we keep constant-- throughout actually all those questions-- is that we have n photons in that beam. And I explained to you in class-- and I showed it to you with operators-- that this setup is really creating the displeased squeezed vacuum.

OK. Now, we have one laser beam, which has an average value of the field, alpha. It has certain properties. And now, I want to ask you, what is the variance of this beam? We measure the photocurrent of this beam. We measure-- of course, the photocurrent is n photons. But what is this variance?

So the choices are we have n photons. Is the variance n? We learned before, with the 50-50 beam splitter, that we were able, by squeezing the vacuum, to eliminate half of the shot noise. Maybe what I've done is wrong, and we get 2 and we increase the noise? Or does the noise really go to 0, which means it's on the order of epsilon, whatever I define for the squeezing for the short axis of this squeezed vacuum.

So now, we are not subtracting currents. No i1 minus i2. We have one beam hitting straight the photodiode. There are n photons per unit time. And the question is, what are the fluctuations of this beam? What is the variance?

Well, the correct answer is this one. What happens is the following. In terms of fields-- you should go through that again a little bit more slowly-- but what happens is the coherent state is sort of this disk. But when we make alpha larger and larger, the disk of the coherent state is the same.

But now, we have of a reflection coefficient, which goes to 0, such that the product of r alpha stays the same. But that means the reflected beam, we have divided the disk by the reflection coefficient-- multiplied with the reflection coefficient-- so therefore, the original noise of the coherent beam has been completely eliminated because of the small reflection coefficient.

So the only noise which is there, is from the other part. But we have strongly

squeezed the noise in this quadrature component. And therefore, the noise goes really down to 0 or to I think it's epsilon squared when epsilon is the normalized widths of the narrow part of the ellipse.

I've written down the math for you here, which is the following. We have a reflection coefficient for the coherent beam and the reflection coefficient for the fluctuating fields. Whereas, the squeezed fluctuations, they are fully transmitted.

And what I'm completely neglecting is-- and you'll realize it already-- the other quadrature component. Because the moment I square it to obtain the current, I'm only considering parts, which are in the right quadrature component, that they get enhanced by alpha. So if I square that and neglect all terms which do not get multiplied with alpha, I get this line.

And now, you see that this term here-- the one which had the fluctuations of the coherent beam-- is what I just said. r times alpha is n. So this is n. It would be the normal variance. But now, we have another power of r. So therefore, delta a1 really got diminished. I mean, you see that actually already better in the previous line.

So therefore, in the limit of r, in the limit considered, there is no contribution from the fluctuations of the coherent beam. The only fluctuations come from the squeezing. And therefore, the variance is, well, 0 or epsilon squared.

So you can create a beam, which has no fluctuations-- or almost no fluctuation-- in the photon number by mixing a strong local oscillator with a squeeze vacuum. And that would mean now, if you take this beam and send it through a cesium cell and do spectroscopy of cesium, you have reduced noise, and you are more sensitive to obtain, to observe, a small quantity of cesium.

I've posted it actually-- this is exactly what I described here, how the first spectroscopy with squeezed light was done. And I've posted the paper of Jeff Kimble's group on our website.

So I'm wondering, by going through these examples, if I would ask you that the shot noise, which is square root n in the fields and n in the variance, after looking at the

beam splitter and the modes which are involved, how would you now answer this question? Is the shot noise related to quantum measurement, is it related to quantum fields, both, or none?

That's interpretation of physics; I think there is no right and wrong. I would actually very strongly-- my opinion is, it's B. It's really the quantum fields. The quantum fields which are split.

If the noise were done in the measurement, you cannot have constructive and destructive interference due to the positive and negative reflection coefficient. So for me, the noise, which is responsible for shot noise, is a field which can be split where you have constructive and destructive interference even before the measurement process takes place. But this is sort of my interpretation.

And let me maybe point out why I think that for those cases, the quantum measurement process is not relevant. Here, we've always talked about square root n fluctuations, where n is big. So the fluctuations of the coherent field-- all of the fluctuations we were discussing here-- had many, many photons.

So to some extent that we have maybe just a single photon, and we can only get one click and not two clicks, doesn't measure it. That doesn't matter here. All the fluctuations we considered here were fluctuations in the field, where many, many photons were involved. So, therefore, I think the quantum character of the measurement process does not really show up.

If I would construct something-- other examples-- where we wouldn't always have this strong local oscillator, this strong homodyning. Then I think we would find quantum noise of a single photon, and then I would say, we're getting closer, that's really quantum measurement. But here, I would say the short noise related to a strong local oscillator-- at least, I find it easier for go through those questions if I think about it as a quantum field, which can be split, added up. Constructive and destructive interference. Good. Questions? Yes?

AUDIENCE: So, when we're talking about [INAUDIBLE], can we actually talk about it without

referring to i, because that makes me think of measurements.

PROFESSOR: You mean i, as the current?

AUDIENCE: Yeah.

PROFESSOR: But we have to do something. OK, what happens is i was nothing else than the expectation value of a dagger a. In other words, whenever we had a photodiode-- I mean, we learn how to transform modes, and i was synonymous to performing a measurement, and the measurement is a dagger a. And when we talked about i squared, in order to get the noise, the operator involved was a a dagger, a a dagger. I think I will ask you something about the displaced.

Yeah, it's too nice a problem. I should ask about the displaced squeezed vacuum in the next problem set. I really want you to calculate, maybe with a higher order term-- it's a three-liner-- in operators, what is the noise? And in your calculation, photodiode will not appear. What will appear is "Calculate the expectation value of a dagger a. If you call it a measurement process, it's [? inobservable ?] and it's an expectation value of an emission operator. Other questions?

AUDIENCE: [INAUDIBLE] understand the [INAUDIBLE] for a stream of particles, which [INAUDIBLE] with a constant probability per unit time? [INAUDIBLE] The detector is also [INAUDIBLE]. I think you also get the square root.

PROFESSOR: Well, a lot of classical counting statistics needs to short noise. If you have independent particles with a certain probability, then you get short noise. But I would sort of press you hard to make a classical explanation for the last example I gave you, where we had a strong local oscillator. We reflected it and superimposed it with squeezed vacuum.

At that point, all the short noise has disappeared. So therefore, I would be hard-pressed to connect it with the classical probability of detecting a photon. Of course, classically speaking, the empty part of a beam splitter is empty, and squeezed vacuum doesn't exist classically. Yes, [INAUDIBLE].

AUDIENCE: So it might be kind of a silly question, but we say we have a minimum uncertainty state when delta x delta p is 0. But a lot of times, we talk about the number-phase uncertainty. So, I'm just curious, because the coherent state has the same circle of-- the area of the circle is the same, right?

I know this is wrong, but can't you always make the delta m delta phi uncertainty very small by having a very large n? Delta n stays the same, but the angular spread always goes down as you make the circle farther away from the origin?

PROFESSOR: So you're talking about an uncertainty relation we have not discussed. In the last homework, you looked at it. So what happens is if you have a coherent state, you would say the phase angle phi has this uncertainty. And then there's an uncertainty relation.

AUDIENCE: Maybe you should have talked about this problem yesterday. I think the uncertainty and the size of the disc is constant, [INAUDIBLE] in alpha. But alpha is proportional not to n, but the square root of n. [INAUDIBLE] an arrow propagation [INAUDIBLE] square root of n. If we move the circle further away, the uncertainty alpha stays the same, but the uncertainty of n goes actually up. But we don't see this [INAUDIBLE], because we only have alpha, not [INAUDIBLE].

PROFESSOR: Yes, you have to be careful. That's another comment I wanted to make, but thanks for reminding of that. When we have coherent states, it looks that the uncertainty is always the same, because it's a disk. But we learned today, and in the last class that the uncertainty of the coherent state when you measure the photocurrent is square root n. What happens is the following: the uncertainty-- and this is what I was referring to-- the uncertainty, let's say delta a in one [? quadrature ?] component, is constant for the coherent state, independent of n.

But that would mean what you really calculate, when you calculate the photocurrent, is alpha plus delta a which is constant. But then you square it. In the limit of a strong coherent state, you get alpha squared, which is the number of photons, plus 2 alpha times delta a. So the delta a, which is constant, gets multiplied with square root n here. So therefore when you go to launch a coherent state, you increase the

number uncertainty, but by the same factor, you reduce the phase uncertainty. Well, I thought this would take half an hour. It took an hour, but I hope it was time well spent.

So let's get started on the next unit, which is fully dedicated to single photons. So after learning about different aspects of the electromagnetic field, we now want to focus on the purest quantum states, which are single photons. And this unit it is definitely motivated by quantum information processing. We want to use control over single photos to realize interesting devices. We talk about beam splitters and phase shifters, but then we put beam splitters and phase shifters together to make Mach-Zehnder interferometers. And then eventually I can show you how Mach-Zehnder interferometers with a non-linearity can be used to realize quantum gates.

But let me make maybe one general comment here. This unit here on single photons has actually two purposes. It tries to give you some fundamentals, which are really necessary to follow recently literature in quantum information science, so we introduce some concepts of gates and truth tables and such.

But for those of you who are maybe less interested in quantum information science, I have to say that as a more traditional atomic physicist, I've never worked in quantum information science myself. I started with atomic molecular spectroscopy, and advanced from laser cooling to quantum gases. I feel that using the quantum logic formalism is a very elegant way of organizing your thoughts. So I do feel even for the rest of the world, who is not doing kind quantum information process, which is a pretty big fraction of AMO science these days, even for the rest of us, to use concepts of quantum logic and quantum information is very, very powerful.

So in other words, I would say right now every atomic physicist should have some knowledge in what happens to beam splitters, what happens to singular photons when they're manipulating, and the way, the approach, how we formulate it using concepts of quantum information processing is extremely powerful. Like in quantum mechanics, you need to know the Schrodinger representation and the Heisenberg interpretation. You can't get away with one.

Similarly, for some aspects of the propagation of quantum states, you should just use the language of quantum information processing. Just sort of to give you already an outlook where I think everything comes together, I've taught for 15 years, 20 years, about decoherence and master equation. And I know how to derive it from general quantum physics, and I will show you that later in this course. But then I learned from Professor Ike Chuang, when I co-taught the course with him, that he could actually make a model for decoherence by just passing light through many, many beams splitters. So there is a beam splitter model for decoherence, which is just beautiful. You can give derive a master equation by just sending light through many, many beams splitters.

So that taught me something, that actually those was elementary devices for quantum information science can really act as conceptual models for more complicated, more general things. So I've decided now, over the last few years, to make quantum information language an essential ingredient of this course. But I should say over the last couple of years, I've de-emphasized some of the more formal algorithmic things, but I'm focusing on the concepts. And I think some concepts are really highlighted by using quantum information aspects.

So with that introduction, we want to talk about single photons. And I want to introduce for you the single photon qubit. Now, [INAUDIBLE] would think, if you talk about single photons, you automatically have one qubit. 0 is no photon. One is the single photon state. We make in addition later on, this is not the most robust qubit, because if, due to unavoidable losses or limited quantum efficiency, you have one photon, and you lose it, you think the bit has been flipped. So you don't want to really use that, but we will use, in a moment, a single photon in two different quantum states. And then, if you detect zero, you know you've lost it. We will talk about one photon, which is either here or there. And this is zero of our qubit, and this would be one of our qubit. But we will come to that later.

I first need the Hilbert space of zero one to introduce a few things, and then we'll use what we've learned to define the dual-rail single photon state. But for now we don't have to make this distinction. The question we want to answer is what

happens when those states pass through optical components. I want to show you how beam splitters and phase shifters can be used-- very simple optical components-- they can be used to realize an arbitrary single qubit operation. And then we introduce non-linear interferometers, and we get two qubit operations.

So let's start out very simple and talk about phase shifters and beam splitters. And this is pretty much how far we will get today in this chapter. Well, if you have a single photon and time passes on, the single photon gets phase shifted. If you pass it through a dispersive medium, it will have another phase shift. So this is how phase shifts can be realized. But there is one caveat, and that immediately tells us we can't get away with just a single photon in one fiber and phase shifting it, because everything phase shifts a photon. Time and everything which is in the beam path. So whenever we talk about the phase, we need a reference.

So therefore, we're immediately drawn that one mode and one photon is not enough, even if you're interested in one-photon gates, we need two modes. Just think about two wave guides or two optical fibers. We call them mode A and B. If you put one photon in-- and this is now the symbol for a phase shifter, a little box with the letter phi-- then we get a phase shift of the state one. And the second mode A acts as a reference mode to detect the shift of phase, let's say by interfering the two photons. So in general, we have an input state, and we obtain an output state, which has a phase shift. We'll come back to phase shifters later.

So we'll have two modes. The next thing we can do is we can combine modes, and this is done with a beam splitter. So we've talked about beam splitters in the context of homodyne detection. We've talked about beams splitters most of the first hour with a clicker question, and I was mainly referring to the beam splitter before in a rather non-formal way. If you've a 50:50 beam splitter, and you have two inputs, you get a symmetric and antisymmetric combination of the two, if it's a 50:50 beam splitter. What else can you get? Or, I used the concept of just reflection and transmission coefficients. I think that's all important to get an intuitive feel for the beam splitter. But now I use a more rigorous approach.

I have a beam splitter, which is characterized by an angle theta. We will later find out what theta is. And we have an input mode A and a second input mode B, and those modes get mixed now. We have output modes B prime and output modes A prime. And we can use the Schrodinger picture and talk about the photon states in those modes, but I prefer to use the Heisenberg picture, so when I put A, A prime, B and B prime in, just think about annihilation operators which annihilate photons in those modes. So what I postulate is that this system is described by the following Hamiltonian. And I'm sort of telling you, this Hamiltonian describes this beam splitter.

You will say, why and how? Well, I will tell you what this Hamiltonian does to the modes, and then we have the quantum description of the beams splitter, and you will then see in your homework that this quantum description of the beam splitter does exactly what it is expected to do to coherent states. It just splits a laser beam into two, with a reflection and transmission coefficient which are cosine squared theta and sine squared theta. And I will also show you that this beam splitter is doing the same, splitting with cosine squared and sine squared theta probabilities, a single photon. So in essence, I give you the Hamiltonian, I discuss that the transfer matrix, which is built upon this Hamiltonian, does everything you ever wanted a beam splitter to do, and therefore you can say, this defines the beam splitter.

OK, if you have a Hamiltonian, that means after propagating through the Hamiltonian, we have the following transfer. You may be familiar with putting the unitary time evolution here, and the time which has elapsed, but we are always using a fixed time after the light has transformed the beam splitter. So you can see the time t is absorbed in the definition of h, the Hamiltonian, and b is really the unitary time evolution from [? being ?] before and [? being ?] after the beam splitter.

So the transformation, which transforms the modes at this beam splitter, has now the following unitary operator. And we have modes a, b, mode one and two, before the beam splitter. After the beam splitter, they have been transformed to a prime and b prime. And we obtain that by taking the input modes and doing unitary time evolution. You do simply the operator transformation for unitary transformation. So

we can now transform the operators a and b.

I'm not going with you through the exercise how you transform an operator. I think in undergraduate quantum mechanics classes, you do the Baker-Campbell-Hausdorff formula, how to use with operators in the exponent. And if you do with that, you find-- not surprisingly, a beam splitter is a linear element-- that what you get is a linear combination of a and b. So if you don't like me postulating an operator and saying, "Believe me, this is a beam splitter," you can reverse engineer. You can say, this really looks like the modes, the operators a b get linearly superposed with cosine and sine. This is what a beam splitter does, is it mixes two modes. And then you can reverse engineer and figure out that it's exactly the Hamiltonian above which does it.

So your homework will show that this happens exactly for coherent states. So if you've a coherent state with alpha, you get a coherent state with alpha cosine theta in one arm, and alpha sine theta in the other arm, and you will see that everything works beautifully. I have to use one convention now for a 50:50 beam splitter. And I use a convention that I chose theta to be pi over 2 b and 90 degrees. And we want to use symbolic language, because we want to draw diagrams of multiple beam splitters and such.

So this tilted square is the symbol for beam splitter. We have input ports a and b on the left side. We have output ports, which are now b prime and a prime. And for the 50:50 beam splitter, the modes are symmetric and antisymmetric superpositions. So this is the symbol for the unitary transformation b. And since one output port has a minus sign, it's antisymmetric, and one is positive. We usually put a dot there which distinguishes the two different ports. So this is the operator B.

We have already used, for the unitary transformation, the operator b dagger. So for the operator b dagger, using the equations above, we have input modes a b, but what happens now is for the operator b dagger, the output modes are interchanged. So we have a plus b over square root a, a minus b over square root 2, and therefore the symbol for b dagger has the dot here. I just want to warn you that

there are other phase conventions in the literature which are equivalent. But sometimes if you read a paper and all of the minus signs appear somewhere else, well, people have a different phase convention.

You can postulate that you have a beam splitter without having the i here. And everything works similarly. It's just that, for instance, in this equation, you get an i here. So imaginary units and plus-minus signs and that busyness are phase conversations. If you'd say, hey, but if you another definition, don't you have another beam splitter? Yes, it is another beam splitter, but I can always get it out of my beam splitter-- which now is our beam splitter-- by putting a phase shifter afterwards. So it doesn't limit the generality of the discussion by defining our beam splitter in this way.

OK, time is over. Reminder, we have another class this week on Friday, and finally, your graded homework is here. Have a good afternoon.