Physics 8.322, Spring 2003
Homework #12

Due **Wednesday, May 14** by 4:00 PM in the 8.322 homework box in 4-339B.

1. Consider the one-dimensional potential step

$$V = \begin{cases} V = 0, & x < 0 \\ V = V_0 > 0, & x > 0 \end{cases}$$

A plane wave solution to the Klein-Gordon equation is incident from the left. From the Klein-Gordon equation find the reflection and transmission coefficients.

2. Transform the γ matrices found in class (the Pauli-Dirac representation) into the chiral (Weyl) representation using the transformation

$$\gamma^\mu_W = U\gamma^\mu U^{-1}, \qquad U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

Write the Dirac equation in this representation using $\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$. What happens to this equation as $m \to 0$?

3. Show that the 16 matrices Γ^n given by all 2^4 products of gamma matrices γ^μ are linearly independent as follows. First, prove the following properties:

(a) $(\Gamma^n)^2 = \pm 1, \forall n$

(b) $\mathrm{Tr}\,\Gamma^n = 0$ except for $\Gamma^1 = 1$.

(c) $\Gamma^n\Gamma^m$ is equal up to a sign to some other Γ^p, with $p = 1$ only when $n = m$.

Then assume $\sum_n a_n \Gamma^n = 0$. Multiply by Γ^m and take the trace, then show that each coefficient a_n vanishes.

4. Consider the negative energy solution to the Dirac equation for a particle at rest with $S_z = +1/2$

$$\psi^{(3)}(x) = e^{imc^2 t/\hbar}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Transform to a frame moving with velocity v along the x-axis and calculate $\psi^{(3)\prime}(x')$. Calculate the energy and momentum in the moving frame. What is the probability that a measurement of the spin in the z' direction yields $-1/2$?