Physics 8.321, Fall 2002
Homework #5

Due **Wednesday, October 16** by 4:30 PM in the 8.321 homework box in 4-339B.

1. For each part of this problem you are asked to find an approximation to the energies of one or more of the lowest-lying quantum states for a particular potential. You may use any approximation technique you wish to determine the energy eigenvalues. You may use a computer if you wish, or you can work by hand. Please include a sketch or graph of the eigenfunctions in each case. In all parts you may use units with $\hbar = m = 1$.

(a) Find the ground state and first excited state energies for a particle in the 1D potential

$$V(x) = \frac{1}{4}x^4 .$$

(b) Find the ground state and first excited state energies for a particle in the 1D potential

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{24}x^4 .$$

(c) Find the ground state energy for a pair of particles in the harmonic oscillator potential $V(x) = x^2/2$. The interaction energy between the particles is given by $-\sqrt{2}|x-y|$, where x, y are the positions of the two particles. You may assume that these particles are fermions, so that $\psi(x,y) = -\psi(y,x)$. Note that the Hamiltonian for this system is equivalent to that of a single particle moving in two dimensions x, y in the potential

$$W(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 - \sqrt{2}|x-y| .$$

(d) Find the ground state energy for a particle in the 2D potential

$$V(x,y) = \frac{1}{4}x^4 + \frac{1}{6}y^6 + 2xy .$$

2. This problem not included for copyright reasons.