Physics 8.321, Fall 2002

Homework #10

Due **Monday, November 25** by 4:30 PM in the 8.321 homework box in 4-339B.

1. Sakurai: Problem 12, Chapter 3 (page 243)

2. Sakurai: Problem 15, Chapter 3 (page 244)

3. Sakurai: Problem 17, Chapter 3 (page 244)

4. Consider a magnetic monopole with magnetic charge g located at the point $(0, 0, d)$, and a second monopole with magnetic charge $-g$ located at the point $(0, 0, -d)$. Find an expression for the vector potential associated with this configuration which is nonsingular for $r > d$. Describe the "Dirac string" of this configuration.

5. Compute the spherical harmonics $Y_{2m}(\theta, \phi)$ explicitly. Express these both as functions of θ, ϕ and of $z = \cos\theta, x = \sin\theta\cos\phi, y = \sin\theta\sin\phi$. Compute $\sum_m |Y_{2m}|^2$.

6. An $l = 1$ state has $m = 1$. The state is rotated by an angle θ about the y axis. What is the probability that a measurement of L_z will yield $m = 1$? Repeat with a rotation by θ about the z direction.

7. Using separation of variables, show that the eigenstates of the Hamiltonian for a spherically symmetric potential $V(\mathbf{r})$ may be written in the form

$$\Psi_{E,l,m} = R_{El}(r)Y_{lm}(\theta, \phi)$$

where $R_{El}(r) = \frac{1}{r}u_{El}(r)$ and u_{El} satisfies

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r)\right] u_{El}(r) = Eu_{El}(r)\,.$$

8. Consider a particle in the 2D potential $(m = 1)$

$$V(x, y) = \frac{1}{2}\omega^2(x^2 + y^2)\,.$$

Use raising and lowering operators $a_x^\dagger, a_y^\dagger, a_x, a_y$ to compute the spectrum and degeneracies of the Hamiltonian. For each value of the energy, what eigenvalues of J_z are possible? Are H, J_z a complete set of commuting observables?