8.286 Lecture 18
November 14, 2013

COSMIC MICROWAVE BACKGROUND SPECTRUM AND THE COSMOLOGICAL CONSTANT

Summary of Lecture 17: Black-Body Radiation

Energy Density: $$u = \rho c^2 = g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3}\ ,$$

Pressure: $$p = \frac{1}{3}u\ .$$

Number density: $$n = g^*\frac{\zeta(3)}{\pi^2}\frac{(kT)^3}{(\hbar c)^3}\ ,$$

Entropy density: $$s = g\frac{2\pi^2}{45}\frac{k^4T^3}{(\hbar c)^3}\ .$$

Meaning of g and g^*

For photons, $g = g^* = 2$.

But neutrinos also contribute, as do e^+e^- pairs when $kT \gg m_ec^2$, and other particles at higher temperatures.

In general,

$$g = \begin{Bmatrix} 1 & \text{(bosons)} \\ \frac{7}{8} & \text{(fermions)} \end{Bmatrix} \times \text{number of particle “types”}$$

$$g^* = \begin{Bmatrix} 1 & \text{(bosons)} \\ \frac{3}{4} & \text{(fermions)} \end{Bmatrix} \times \text{number of particle “types”}$$

By “type,” we mean a complete specification of species, particle vs. antiparticle, and spin state.

---

g and g^* for Neutrinos

The correct values are given by pretending that neutrinos are massless, and have only one spin state: all ν's are left-handed ($\vec{J}\cdot\hat{p} = -\frac{1}{2}\hbar$) and all $\bar{\nu}$'s are right-handed.

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species}\\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4}\ .$$

$$g^*_\nu = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species}\\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2}\ .$$

Summary of Lecture 17

g and g^* for e^+e^- Pairs

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2}\,.$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3\,.$$

Summary of Lecture 17:

Radiation Density of the Present Universe

When e^+e^- pairs disappear from the thermal equilibrium mix, as kT falls below $m_e c^2$, they give all their entropy to the photons, and none to the neutrinos. Consequently (as you will show on Problem Set 7),

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma\,.}$$

Then

$$\boxed{u_{\text{rad},0} = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30}\frac{(kT_\gamma)^4}{(\hbar c)^3} = 7.01 \times 10^{-14}\ \text{J/m}^3\,.}$$

Summary of Lecture 17:

The Real Story of Neutrino Masses

Neutrinos have been observed to “oscillate” from one species to another, which is not allowed unless neutrinos have a nonzero mass:

$$\boxed{\begin{aligned} \Delta m^2_{21}\, c^4 &= (7.50 \pm 0.20) \times 10^{-5}\ \text{eV}^2\,, \\ \Delta m^2_{23}\, c^4 &= \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3}\,\text{eV}^2\,. \end{aligned}}$$

For a massive particle with spin J, all spin states

$$J_z/\hbar = -J, -J+1, \ldots, J$$

must exist. In particular, there must be right-handed neutrinos and left-handed antineutrinos.

Summary of Lecture 17

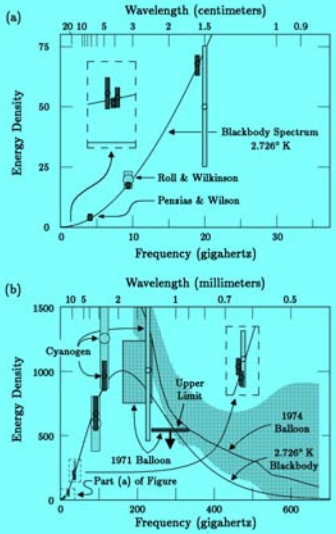
There are two possibilities:

Dirac Mass: Right-handed neutrino would be a new as-yet unseen type of particle. But it would interact so weakly that it would not have been produced in significant numbers during the big bang.

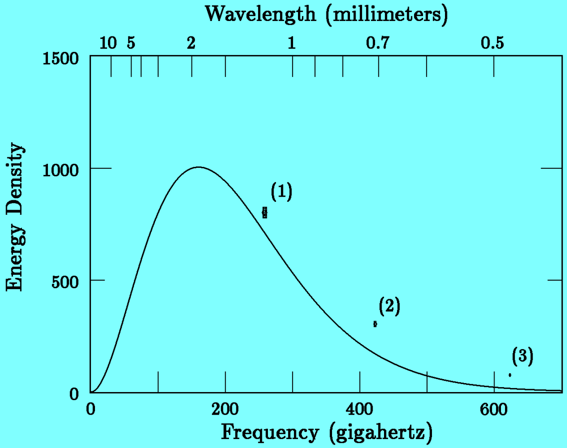
Majorana Mass: If *lepton number* is not conserved (which seems likely), so the neutrino is absolutely neutral, then the right-handed neutrino could be the particle that we call the anti-neutrino.

Summary of Lecture 17:
Thermal History of the Universe

- ☆ For 0.511 MeV $\ll kT \ll$ 106 MeV, $\boxed{kT = \dfrac{0.860\text{ MeV}}{\sqrt{t}\text{ (in sec)}}}$.
- ☆ Conservation of entropy implies that $\boxed{s \propto 1/a^3}$. When g is constant, this implies $\boxed{T \propto 1/a}$.
- ☆ At the densities found in the early universe, the hydrogen plasma becomes neutral atoms (hydrogen "recombines") at 4,000 K, and becomes transparent to photons ("photon decoupling") at 3,000 K. We estimated $T_{\text{decoupling}} \approx 380,000$ yr.



CMB Data in 1975

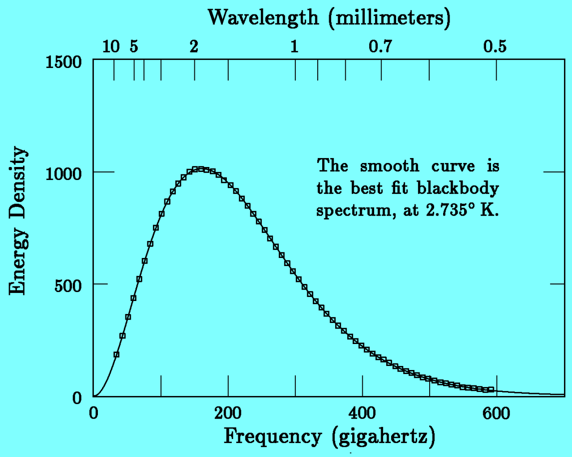


Data from Berkeley-Nagoya Rocket Flight, 1987

A Preliminary Measurement of the Cosmic
Microwave Background Spectrum by the
Cosmic Background Explorer (COBE) Satellite

J. C. Mather, E. S. Cheng, et al.

Cover Page of Original Preprint of the COBE Measurement of the CMB Spectrum, 1990



Original COBE Measurement of the CMB Spectrum, Jan 1990. Energy density is in units of electron volts per cubic meter per gigahertz.

8.286 The Early Universe
Fall 2013