8.286 Lecture 9
October 8, 2013

DYNAMICS OF HOMOGENEOUS EXPANSION, PART V (LAST!)

Summary of Lecture 8

Age of a Flat Matter-Dominated Universe:

$$a(t) \propto t^{2/3} \quad \Longrightarrow \quad t = \frac{2}{3} H^{-1}$$

For $H = 67.3 \pm 1.2$ km-s^{-1}-Mpc^{-1}, age $= 9.5 - 9.9$ billion years — but stars are older. Conclusion: our universe is nearly flat, but not matter-dominated.

The Big Bang Singularity: $a(0) = 0$, with infinite density, is a feature of our model, but not necessarily the real universe.

Horizon Distance: the present distance of the furthest particles from which light has had time to reach us.

$$a(t) \propto t^{2/3} \quad \Longrightarrow \quad \ell_{p,\text{horizon}} = 3ct = 2cH^{-1} \ .$$

Evolution of a Closed Universe:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} \ , \qquad \rho(t) a^3(t) = \text{constant} \ .$$

New variables:

$$\tilde a(t) \equiv \frac{a(t)}{\sqrt{k}} \ , \qquad \tilde t \equiv ct \qquad \text{(both with units of distance)}$$

$$\left(\frac{d\tilde a}{d\tilde t}\right)^2 = \frac{2\alpha}{\tilde a} - 1 \text{ where } \quad \alpha = \frac{4\pi}{3}\frac{G\rho\tilde a^3}{c^2} = \text{constant} \ .$$

$$\left(\frac{d\tilde a}{d\tilde t}\right)^2 = \frac{2\alpha}{\tilde a} - 1 \quad \Longrightarrow \quad d\tilde t = \frac{\tilde a \, d\tilde a}{\sqrt{2\alpha\tilde a - \tilde a^2}} \ .$$

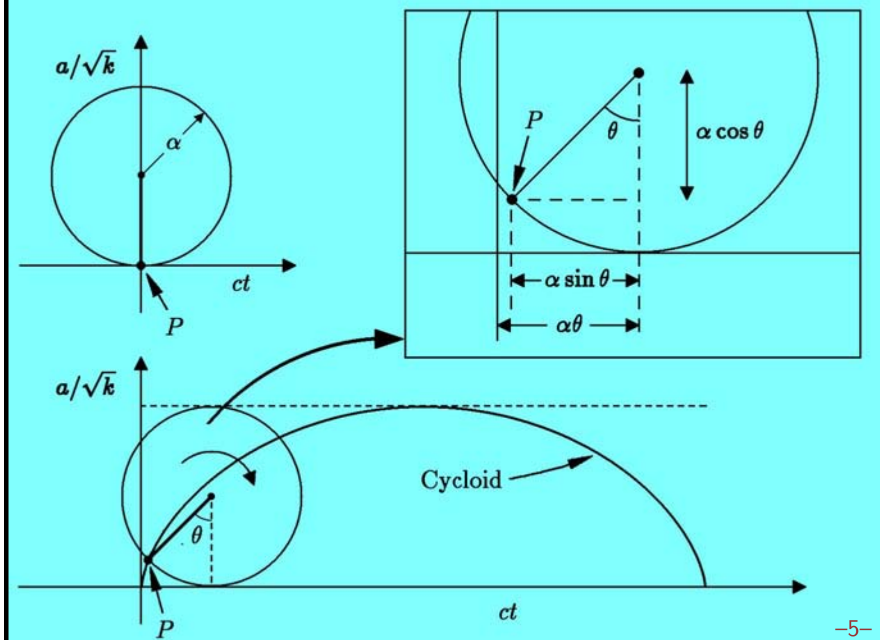
Then

$$\tilde t_f = \int_0^{\tilde t_f} d\tilde t = \int_0^{\tilde a_f} \frac{\tilde a \, d\tilde a}{\sqrt{2\alpha\tilde a - \tilde a^2}} \ ,$$

where $\tilde t_f$ is an arbitrary choice for a "final time" for the calculation, and $\tilde a_f$ is the value of $\tilde a$ at time $\tilde t_f$.

Evolution of a Closed Universe

$$ct = \alpha(\theta - \sin\theta) \; ,$$
$$\frac{a}{\sqrt{k}} = \alpha(1 - \cos\theta) \; .$$



Evolution of a Closed Universe

$$ct = \alpha(\theta - \sin\theta) \; ,$$
$$\frac{a}{\sqrt{k}} = \alpha(1 - \cos\theta) \; .$$

$$t = \frac{\Omega}{2|H|(\Omega-1)^{3/2}}\left\{\arcsin\left(\pm\frac{2\sqrt{\Omega-1}}{\Omega}\right) \mp \frac{2\sqrt{\Omega-1}}{\Omega}\right\} \; .$$

$$t = \frac{\Omega}{2|H|(\Omega-1)^{3/2}}\left\{\arcsin\left(\pm\frac{2\sqrt{\Omega-1}}{\Omega}\right) \mp \frac{2\sqrt{\Omega-1}}{\Omega}\right\} \; .$$

Quadrant	Phase	Ω	Sign Choice	$\sin^{-1}()$
1	Expanding	1 to 2	Upper	0 to $\frac{\pi}{2}$
2	Expanding	2 to ∞	Upper	$\frac{\pi}{2}$ to π
3	Contracting	∞ to 2	Lower	π to $\frac{3\pi}{2}$
4	Contracting	2 to 1	Lower	$\frac{3\pi}{2}$ to 2π