MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J   May 12, 2006

Problem Set 13
Not To Be Turned In

Final Exam: Friday, May 26, 9:00 am - noon, Closed book exam. Bring a
calculator and up to two pages of notes. Last year's exam has been posted for practice and
to give you a good feel for the scope and style of the questions to be asked. There will be 8
problems to work; there may or may not be a choice of problems as there was last year.

Reading: Zeilik & Gregory: Chapters 24, 25, and 26.

Problem 1
 "Relativistic Doppler Effect"

Zeilik & Gregory, Chapter 24, Problem #1, page 486.
Refer to equation 8-14a in the text.

Problem 2

"Cygnus A Radio Galaxy"

Zeilik & Gregory, Chapter 24, Problem #3, page 486.
The units for *flux density* are $W\,m^{-2}\,Hz^{-1}$.
In this problem it is adequate to use the non-relativistic form $\Delta\lambda/\lambda \equiv z \simeq v/c$.

Problem 3

"Feeding a Black Hole"

Zeilik & Gregory, Chapter 24, Problem #8, page 486.

a. If a star of mass m_s and radius R passes a black hole of mass $M_{\mathrm{BH}}(\gg m_s)$ at a distance D,
the magnitude of its acceleration is GM_{BH}/D^2. The force per unit mass felt by an observer
at rest on the surface of the star at a point nearest the black hole is:

$$\frac{GM_{\mathrm{BH}}}{(D-R)^2} - \frac{Gm_s}{R^2} - \frac{GM_{\mathrm{BH}}}{D^2},$$

where the first and second terms are the opposing forces of gravity from the black hole and
the star, respectively, while the third term is the fictitious force associated with being in the
frame of reference that is accelerating with the center of mass of the star. The star will be
pulled apart near the point where the sum of these forces goes to zero. For the case where

$D \gg R$ this occurs when $D \lesssim (2M_{\mathrm{BH}}/m_s)^{1/3}R$. You can derive this condition by using a Taylor's series expansion for $1/(D-R)^2$. For a related discussion see Z&G, chapter 3-4-D, and equation 3-10.

b. Use an energy conversion factor of $\sim 10^{20}$ ergs/gram for matter falling into a black hole since at least 7% of its rest energy is released as radiation.

Problem 4

"Distance–Redshift Relation"

Zeilik & Gregory, Chapter 24, Problem #13, page 487.
(Note that 1 nm $= 10\mathring{\mathrm{A}} = 10^{-7}$ cm.) Compare the answer you get using $d = v/H_0$ (where the special relativistic relation between λ/λ_0 and v at high speeds is defined by equation 8-14a) with the answer obtained via equation (22-3d), where $d = cz(1+z/2)/[H_0(1+z)^2]$.

Problem 5

"H_0, τ_0, and ρ_{crit}"

Zeilik & Gregory, Chapter 25, Problem #3, page 501.

Problem 6

"Expanding Balloon Analogy"

Zeilik & Gregory, Chapter 25, Problem #4, page 501. In part (b) it's easier if you leave s/R as θ.

Problem 7

"Scale Factor and Age vs. Redshift"

Use the formalism that was developed in lecture for the evolution of the scale factor, a, as a function of time, t (after the formation of the Universe), to show that:

$$a = \left(\frac{3}{2}H_0 t\right)^{2/3},$$

for the special case where $\Omega_0 = 1$ ($\Omega_m = 1$; $\Omega_\Lambda = 0$). Use the "Lemaitre equation" that connects z and the scale factor:

$$1 + z = \frac{1}{a},$$

to find the relation between the z and the "age", t:

$$t = \frac{2}{3}H_0^{-1}\left(\frac{1}{1+z}\right)^{3/2}.$$

Find the age, t, of (i) our Galaxy, with $z = 0$, (ii) a quasar with $z = 6.4$, and (iii) the cosmic microwave background, with $z \simeq 1000$.

<u>Problem 8</u>

"Dust in the Universe"

Zeilik & Gregory, Chapter 26, Problem #1, page 515.
Take $\sigma = \pi r_{\text{grain}}^2$ as the scattering cross section. Assume a mass density of 1 g cm^{-3} (or 10^3 kg m^{-3}) for the *material* that makes up a dust grain.