Lecture 23 - Topics

- Closed Strings
- Heterotic String Theory

$\Psi^I(\tau - \sigma)$, anticomm (Grassmann odd), 2D fermion (in worldsheet)

Creation operations: NS sector, b^I_{-r}, R sector: d^I_{-n}

Fermionic Oscillators

$$|NS\rangle \otimes \left|p^+, p_\tau\right\rangle$$

d^I_0: 8 O-Modes. 4 creation operators, 4 annihilation operators. With 4 creation operators, ζ_1,ζ_2,ζ_3,ζ_4 can create full set of vacua. Start with R vacuum $|0\rangle$. Act on it with the creation operators.

$$\begin{aligned} |R^a_1\rangle &\to |R_a\rangle \qquad a = 1, 2, \ldots, 8 \\ |R^a_2\rangle &\to |R_{\overline{a}}\rangle \qquad \overline{a} = \overline{1}, \overline{2}, \ldots, \overline{8} \end{aligned}$$

Introduce Fermion Number = F, operator:

$$(-1)^F |NS\rangle \otimes \left|p^+, p_T\right\rangle = - |NS\rangle \otimes \left|p^+, p_T\right\rangle$$

If F odd, then state fermionic (-). If F even, then state bosonic (+).

$$(-1)^F, b^I_{-r} = 0$$

$$\begin{aligned} (-1)^F b^I_{-3/2} |NS\rangle &= -b^I_{-3/2}(-1)^F |NS\rangle \\ &= +b^I_{-3/2} |NS\rangle \end{aligned}$$

$$\begin{array}{ll} (-1)^F = -1 & |NS\rangle \\ (-1)^F = +1 & b^I_{-1/2} |NS\rangle \\ (-1)^F = -1 & \alpha^J_{-1}, b^I_{-1/2} b^J_{-1/2} |NS\rangle \end{array}$$

Truncated NS sector: $(NS+)$. Just keep the states with $(-1)^F = +1$. Massless states and states with $M^2 \in Z$.

Closed String

Tensor product of state spaces of open of left and open of right.

How to build a closed string field?

$\overline{\alpha}^I_{-n}$	α^I_{-n}
Open bosonic string	Open bosonic string
Open superstring	Open superstring
$\begin{pmatrix} NS \\ R \end{pmatrix}$	$\begin{pmatrix} NS \\ R \end{pmatrix}$

Ways to combine:
1. Could take all states of left NS and combine with all states of right NS: (NS,NS) sector of closed superstrings
2. (NS,R) sector of closed superstrings
3. (R,NS) sector of closed superstrings
4. (R,R) sector of closed superstrings

(1) gives us spacetime bosons.
(2) and (3) gives us spacetime fermions.
(4) gives us spacetime bosons (very complicated).

Actually, 2 different ways of forming this string theory. Choose R's with same sign or choose R's of different signs.

Type II A	Type II B
(NS+,NS+)	(NS+,NS+)
(NS+,R+)	(NS+,R+)
(R-,NS+)	(R+,NS+)
(R-,R+)	(R+,R+)

10-dimensional superstring theories. We believe are really two manifestations of the same theory.

(NS+,NS+): Throw away $-$'s of NS sector on both left and right. Reasonable projection of NS sector: (NS+,NS+) eg: $b^I_{-1/2}\overline{b}^J_{-1/2}\left|NS\right\rangle_L \otimes \left|NS\right\rangle_R \otimes \left|p^+, p_T\right\rangle$

(lowest allowable state)

$$\frac{1}{2}\alpha' M^2_{\text{closed}} = \alpha' M^2_{\text{open}} + \alpha' M_{\text{open}'}$$

Universal bosons of string theory.

64 states, traceless. Gives massless states. 35 states for a 10-dim graviton. 28 states for Kalb Raman. 1 dilitant.

Type IIB:

$(NS+, R+) \to \bar{b}^I_{-1/2}|NS\rangle_L \otimes |R_{\bar{a}}R$ gives 64 fermions, plus an additional 64 from $(R+, NS+)$ giving 128 fermions. $(R+, R+) \to |R_{\bar{a}}\rangle \otimes \left|R_{\bar{b}}\right\rangle_R$ gives 64 bosons.

(R, R):
IIA: A_μ (D0-brane), $A_{\mu\nu\rho}$ (D2-brane) $\Rightarrow$ 64
IIB: A (D7-brane, magnetically charged object), $A_{\mu\nu}$ (D1-brane), $\mathrm{A}_{\mu\nu\rho\sigma}(D3 - brane, self - dual, spacefilling)$

So we often use the D3-brane of type IIB since it is space filling.

Heterotic String Theory

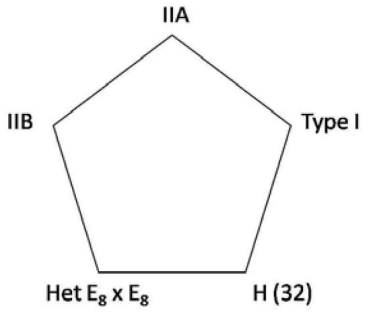
Open Bosonic String:
8, $X^I = \alpha^I_0$, $X^{A=1,\ldots,6}$. Don't have α^I_0. Sort of enough to make a string theory with 10 dimensions.

Open Superstring:
"$E_8 \times E_8$", 16, 1984: Know this has something to do with string theory since $16 + 10 = 26$

5 types of 10-dim. supersymm. string theories:
1. Type IIA
2. Type IIB
3. $E_8 \times E_8$ heterotic

4. $S_0(32)$ heterotic
5. Type I (open string, $32 = 2^{D/2}$ coincident, D-branes and orientation projections)

Interesting relations between them. Maybe these 5 are corners of one theory.



1. In early 1980s, bothersome. 5 theories of everything? Now we think it's really all 1.
2. Also have another theory, M-theory, with 2- and 5-branes proportional to parallel dimensions but no strings.
3. Want: lovely formulation like Einstein's equations. Currently don't have them.