

Massachusetts Institute of Technology
Physics Department

Physics 8.20
Introduction to Special Relativity

IAP 2005

Problem Set 4

1. Non-relativistic limit

The correct relativistic formula for the energy of a particle of rest mass m and speed v is $E(v) = m\gamma(v)c^2$.

- (a) Expand $E(v)$ to order v^4 . Identify the first correction to the usual non-relativistic formula for the kinetic energy, $T = \frac{1}{2}mv^2$.
- (b) How large is the correction of part a) for the Earth relative to the Sun? In other words, approximately how much heavier is the Earth on account of its motion around the Sun? You will need the Earth's mass and its orbital speed to complete this estimate.
- (c) How large a percentage error did you make by using the approximation of part a) instead of the exact result in the case of the Earth's motion around the Sun?

2. Hard work

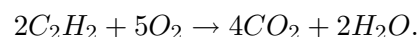
A proton initially moves at $v = 0.9c$ in some reference frame.

- (a) How much work must be done on the proton to increase its speed from $0.9c$ to $0.99c$?
- (b) How much work must be done to increase its speed from $0.99c$ to $0.999c$?

[Hint: it is simplest to use energy units throughout this problem. Express the proton's rest mass in terms of its rest energy, $m_p c^2 = 939$ MeV, and use MeV as your units of work.]

3. Acetylene

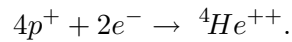
Acetylene (C_2H_2) is a particularly good fuel. When it burns completely to water and carbon dioxide the chemical reaction is



Go to your favorite source of chemistry information (the Web, perhaps, or the CRC Handbook) and determine how much energy is released per gram molecular weight of acetylene (the usual units are KCal/mole). Suppose 1000 kg of acetylene is combined with just enough oxygen to burn completely. What is the mass equivalent of the energy released in the reaction? [You will probably wish to convert KCal/mole to the MKS units of joules/kg.]

4. Solar Power

The net result of the fusion reaction that fuels the sun is to turn four protons and two electrons into one helium nucleus,



Other particles are given off (neutrinos and photons), but you can assume they eventually show up as energy. The masses of the relevant nuclei are as follows:

$$m_p = 1.6726 \times 10^{-27} \text{ kg.}$$

$$m_e = 9.1094 \times 10^{-31} \text{ kg.}$$

$$m_{{}^4\text{He}} = 6.6419 \times 10^{-27} \text{ kg.}$$

- (a) How much energy is released when a kilogram of protons combines with just enough electrons to fuse completely to form helium?
- (b) How many kilograms of methane would you have to burn to produce the same amount of energy?

5. $E = mc^2$

Assume the heat capacity of water is equal to 4.2 joules/g-K, and constant as a function of temperature. How much does the mass of a kilogram of water increase when it is heated from freezing (0°C) to boiling (100°C)?

6. Enormous energies

Quasars are the nuclei of active galaxies in the early stages of their formation. A typical quasar radiates energy at the rate of 10^{41} watts. At what rate is the mass of this quasar being reduced to supply this energy? Express your answer in solar mass units per year, where one solar mass unit, $1 \text{ smu} = 2 \times 10^{30} \text{ kg}$, is the mass of our sun.

7. Classical physics and the speed of light

- (a) How much energy would it take to accelerate an electron to the speed of light according to “classical” (before special relativity) physics?
- (b) With this energy what would its actual velocity be?

8. A useful approximation

- (a) Show that for an extremely relativistic particle, the particle speed u differs from the speed of light c by

$$\Delta u = c - u = \frac{c}{2} \left(\frac{m_0 c^2}{E} \right)^2$$

where m_0 is the rest mass and E is the energy.

- (b) Find Δu for electrons produced by
- i. MIT's Bates Accelerator Center, where $E = 800$ MeV.
 - ii. The Jefferson Lab (in Newport News, Virginia), where $E = 6$ GeV.
 - iii. The Stanford Linear Accelerator Center (in Palo Alto, California), where $E = 50$ GeV.

9. Pressure of a Gamma Ray Burster

Gamma ray "bursters" are among the most energetic and mysterious events in the Universe. They were discovered by the first satellites capable of detecting very high energy light (gamma rays), which are usually absorbed by our atmosphere. Initially it was thought that GRB's (as they are called) sent radiation out in all directions "isotropically". Based on the amount of radiation seen at the Earth and the estimated distance, astrophysicists calculated that the GRB emits an energy equal to the energy equivalent of about 1/10 smu (see problem 6) in about 100 seconds.

Assume the radiation is emitted isotropically and compute the pressure the Earth would experience if a GRB took place nearby in our galaxy, for example at a distance of 1,000 lyr.

More recent observations suggest that a GRB really emits radiation in a narrow cone. Thus the total radiation emitted is less than supposed in this problem. However, if you happen to be in the line of the cone of radiation, the effects of a nearby GRB would be very unpleasant.

10. Pressure of light II.

French §6, Problem 6-8, page 201.

11. Pentaquarks

Particle physicists are currently debating the evidence for a new and quite unusual particle made of quarks. It is unusual because it cannot be made of three quarks (the proton, neutron, and all their friends are made of qqq); instead it has to be a bound state of four quarks and an antiquark, $qqqq\bar{q}$. The reason we know this is that its charges (which include ordinary electric charge, but also another called “strangeness” or “hypercharge”) cannot be made out of the charges of three quarks. It is called the Θ particle, and it is the first example of a new class of particles called “pentaquarks”. The Θ can be created in the collision of a neutron (n) and another particle called a “kaon” (K). The rest masses of these particles are $m_n c^2 = 939$ MeV, $m_K c^2 = 492$ MeV, and $m_\Theta = 1540$ MeV.

(a) If an experimenter wants to make a Θ by colliding a K with a neutron target at rest, what is the minimum energy the kaon must have? Be sure to indicate whether you are giving its total energy or just its kinetic energy.

(b) What is the kaon’s velocity as measured in the neutron target rest frame?

(c) Once the Θ is made, it decays quickly back into a neutron and a kaon. In the Θ rest frame, what is the momentum of the emitted neutron? What is the velocity of the neutron and of the kaon?

12. Impossible processes

French §6, Problem 6-14, page 202. This problem is easiest if you use invariants.

13. Compton scattering

Derive the relationship between scattering angle and wavelength change for Compton scattering.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

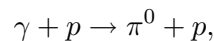
Explain what the notation means. What is $\Delta\lambda$? What is $\cos\theta$? [You can derive this using energy and momentum conservation or you can use invariants to make this problem easier.]

14. Limits of production

French §7, Problem 7-3, page 225.

15. An ultimate energy

Magnetic fields in the universe accelerate charged particles in intergalactic space up to enormous energies. These energetic particles rain down on the Earth as “cosmic rays”. Particle physicists conjecture that there is an ultimate limit to the energy of cosmic rays due to their collisions with the “cosmic background radiation” (CBR) that suffuses the Universe. Suppose the cosmic rays are protons (with rest mass $m_p = 1.67 \times 10^{-27}$ kg.). Suppose the CBR consists of photons of energy 2.1×10^{-4} eV. [This is the energy corresponding to the temperature ($E = kT$) of $2.3^\circ K$ that characterizes the background radiation.] The reaction that degrades the energy of the cosmic rays is “pion production”



where γ is a CBR photon and the π^0 is a “meson” with rest mass $m_{\pi^0} = 0.240 \times 10^{-27}$ kg.

- (a) What is the threshold energy of the proton, *ie.* what is the minimum proton energy necessary for this reaction to go in the frame where the photons have energy 2.1×10^{-4} eV?

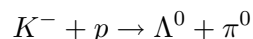
Hint 1: As always, the use of invariants will make this problem much simpler.

Hint 2: Consider the threshold condition in the center of mass of the final proton and pion. What is their configuration at threshold?

- (b) Suppose the proton has just enough energy to make the reaction possible. What is its energy after the collision?

16. A magic energy

The K^- meson and the Λ^0 hyperon are two commonly encountered unstable particles. For example, they are commonly produced in air showers by cosmic rays and several of each have whizzed through you during the time you have been working on this problem set. The reaction



can be used to make Λ^0 's at rest in the laboratory by scattering K^- mesons off a stationary proton (hydrogen) target.

- (a) Find the energy of the incident K^- beam required to just produce Λ^0 hyperons at rest in the lab.
- (b) What is the π^0 energy for this “magic” K^- energy?
- (c) Check momentum conservation.

- (d) Could the process be run the other way? That is, could a π^0 beam (assuming one was available) be used to make a K^+ at rest by the reaction $\pi^0 + p \rightarrow \Lambda^0 + K^+$?

The rest energies of all the particles involved are: $m_p c^2 = 939$ MeV, $m_{K^\pm} c^2 = 494$ MeV, $m_{\pi^0} c^2 = 135$ MeV, $m_{\Lambda^0} c^2 = 1116$ MeV.

17. s, t, and u

When particles scatter off one another, the fundamental physics effects are the same in all reference frames even though the energies of the particles and the angles at which they scatter differ from frame to frame. That means that the interesting information can depend only on Lorentz invariant combinations of the four momenta p_i^μ .

Consider the general “two-to-two” scattering process,

$$a + b \rightarrow c + d$$

where the four momentum and rest mass of particle i are p_i^μ and m_i , respectively, where $i = a, b, c, d$. Of course $p_i^2 = m_i^2 c^4$.

- (a) Show that all the products of the form $p_i \cdot p_j$ can be expressed in terms of the three variables,

$$\begin{aligned} s &\equiv (p_a + p_b)^2 \\ t &\equiv (p_a - p_c)^2 \\ u &\equiv (p_a - p_d)^2 \end{aligned}$$

and the rest masses m_i .

- (b) Show that s , t , and u are related by

$$s + t + u = \sum_{i=a,b,c,d} m_i^2 c^4$$

- (c) Show that when the masses are all equal $s \geq 4m^2$ and $t, u \leq 0$.