8.13-14 Experimental Physics I & II "Junior Lab"
Fall 2007 - Spring 2008

Introductory Optics

MIT Department of Physics
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The purpose of this experiment is to give you practice in optical measurements in a study of three fundamental phenomena of geometrical and physical optics: image formation, interference and diffraction, and polarization. You are expected to dig most of the relevant theory out of the references, perform data analysis to learn about errors, and write a paper about it.

1. PREPATORY QUESTIONS

1. **Image formation:** (a) Predict the relation between the measured image distance t and object distance s for a lens with focal length 10 cm in the form of a plot of t against s. (b) If the object is 12.0 ± 0.5 cm away, where would you find the image with 68% probability?
2. **Interference:** A narrow beam (diameter ≈ 1 mm) of plane waves of wavelength 632.8 nm from a laser is incident on a reflection grating (see Figure 2 below) at a grazing angle of $\alpha = 1.0°$. The grating has periodic grooves separated by 0.06 cm. The reflected beams strike a screen oriented perpendicular to the plane of the grating and located 200 cm from the area of reflection. Predict the pattern of bright spots on the screen. Give the intensity profile of a double slit of 50μm slit width separated by 100μm on a screen 1m away for an incident laser operating at 632.8 nm.
3. **Polarization:** A parallel beam of light of intensity I_0 passes through two ideal linear polarizers with their transmission axes rotated through an angle θ with respect to one another. Predict the intensity $I(\theta)$ of the emergent beam as a function of the angle θ in the form of a plot of I against θ.
4. Read section 13.1 of Reference [1] (available from the Junior Lab E-Library) and describe in general terms how a laser works. Describe, quantitatively, the difference in output between a 1 mW HeNe laser and a 100 W incandescent lightbulb.
5. Having two lenses of 50 and 5 cm focal length, describe how you would build a telescope. Give a quantitative sketch and calculate the magnification.

CAUTION While optical equipment is generally not hazardous except when it involves high-power or high-intensity light sources, it is frequently fragile. Take special care not to drop optical components, set them down so their surfaces can be marred, or touch their surfaces with your fingers. Also, treat the HeNe lasers with respect; remember that the beam can travel to all parts of the room so be cognizant of who might be affected. Be particularly aware of potential reflections from objects located at some distance away from the experiment. Be sure to make use of beamstops.

2. IMAGE FORMATION

The lens equation for an ideal thin lens is

$$\frac{1}{s} + \frac{1}{t} = \frac{1}{f} \tag{1}$$

where s and t are the distances of the object and image from the center plane of the lens, respectively, and f is the focal length. The focal length is a property of the lens that depends on its shape and composition. The sign of s is positive if the direction of propagation of the light is from the object to the lens; the sign of t is positive if the light propagates from the lens to the image. This relation holds for convex, or "positive" ($f > 0$), lenses and for concave, or "negative", lenses ($f < 0$). The same equation and sign conventions also describe the object-image relations of concave ($f > 0$) and convex ($f < 0$) spherical mirrors. An image is called real if $t > 0$, virtual if $t < 0$. Note that if an image is virtual, the light does not pass through it. Other designations you might come across are "plano-convex" or "bi-convex" referring to whether or not both lens surfaces possess curvature. Typically, plano-convex lenses are used when imaging an object located an infinity and bi-convex lenses are used when $s/t < 5$.

2.1. Observing Real Images

Set up a 40-watt frosted-bulb light source illuminating the object (an arrow-cross works well), a 'positive' (convex or bi-convex) lens, and a frosted glass viewing screen. From the optics rack, select a lens of focal length $\approx 5-40$ cm.[1] Project an image of the object onto the frosted glass and vary the positions of the elements until it is "sharp". Examine this image with another more powerful lens (i.e. shorter focal length lens) used as a magnifying glass. Remove the screen and examine the image plane in empty space directly both with your naked eye and with a magnifying glass. Describe your observations

[1] This is easily checked to first order by holding up the lens in front of a window and noting the distance at which an object at infinity (e.g. a neighboring building) is focused upon a piece of white paper.

in your lab notebook with carefully drawn ray diagrams that show the critical rays from the source through the lens to the image on your eye's retina (your eye is a camera). Use a straight edge and a schematic "object" in the form of an arrow with its foot on the optical axis, as illustrated in Figure 1.

2.2. The Focal Length of a Lens

Tabulate a series of measurements of the object-lens distance, s_i, and the lens-image distance, t_i, at six different positions. For each pair of measured values compute and tabulate as you go along the corresponding values of f_i according to the lens equation. Estimate and record the error (measurement uncertainty) of each s_i and t_i. Keep the random error (from at least 10 repeated measurements) separate from the systematic errors (uncertainties in the exact location of lens center, etc.) and give an estimate of the total error expressed as $f = x \pm r_{ran} \pm r_{sys}$. As you go along, plot $1/s_i$ against $1/t_i$ in your lab book next to the accumulating table of measurements. In the data analysis apply the techniques of error propagation (consult Reference [2]) through the lens equation to find the errors in the calculated values of f_i. Finally combine your individual measurements of f_i to obtain a best estimate of the lens's focal length and the error. The sample mean μ and error of the mean σ of a set of measurements x_i each with its error σ_i are given by the formulas in [2]

$$\mu = \frac{\sum x_i/\sigma^2}{\sum 1/\sigma^2}, \tag{2}$$

$$\sigma^2 = \frac{1}{\sum 1/\sigma_i^2}. \tag{3}$$

In carrying out a set of measurements like those above it is generally wise to explore as wide a range of the variables as you can, even though the fractional measurement uncertainties may be relatively large at the extremes. Remember σ_s and σ_t are obtained from repeated, independent (your partner) measurements. If you keep s fixed, you get σ_t only! Make sure you have ≥ 10 independent measurements at one or more set of (s_i, t_i).

2.3. Measuring Magnification

The rays that go through the center of an ideal (i.e. negligible thickness) thin lens, from whatever part of the object they arise, are not bent. From this fact it is obvious that the magnification of a lens is $M = -t_i/s_i$. Make a measurement to test this. Given a magnifying glass of 3× power, observe your image and evaluate the total magnification of this "microscope".

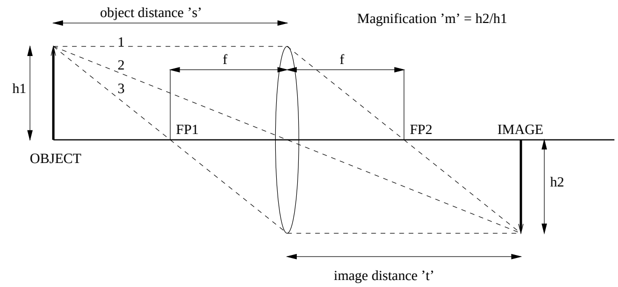

FIG. 1: Example of a ray trace diagram for the formation of a magnified real image by a positive lens. Three critical rays from object to image are shown: 1) the ray leaving the object parallel to the axis is deflected through FP2; 2) the ray through the center of the lens is undeflected; 3) the ray through FP1, though it does not actually pass through the lens, is still a valid ray for the analysis; it is deflected at the lens plane in the direction parallel to the axis. The intersection of any two of these rays defines the position of the image.

3. YOUNGS'S INTERFERENCE EXPERIMENT

The original experiment by Young in 1800, "disproved" Newton's particle theory of light by demonstrating wave interference. Note today's "wave-particle dualism", where photons and electrons (de Broglie) all can behave like waves or particles. In 1800 the source of light to illuminate the two pinholes was itself a pinhole illuminated by sunlight (why was the first pinhole essential?). To get interference all three pinholes have to be quite small, so the resulting fringe pattern is very faint. With a laser as the light source the experiment does not require a first pinhole (why?) and is easy to do.

For your interference experiments, use metal slides manufactured by Leybold-Didactic company (which can be identified by the "LD" in the upper left and the "46992" in the upper right). The measurements of the slit widths and spacings should be specified on the back of the slides. Mount a laser to the optical rail and direct the laser beam parallel to the optical rail and onto the small screen at a distance of $\sim$ 1m. **BE SURE THAT YOUR BEAM DOESN'T ENTER ANYONE ELSES WORK SPACE! USE BEAM-STOPS!** Attach the laser (or the slits) to the 1-axis translating stage (± 0.2 inches) mounted on the optical rail. Mount the diffraction grating "transparency" in the "plate holder" and position it into the beam close to the laser so the beam illuminates both slits. Do you see two interference patterns? If so, make sure you understand which is which. Observe and measure the interference fringes on the screen. The interference pattern is small, so you will probably need to measure it using a gauge micrometer over a few periods. You should also stick a small piece of white paper on the screen and mark the positions of the fringes on the sheet with a pencil. Be sure to tape the paper into your notebook as data! Measure the distance from the slits to the projection screen or detector. Determine the wavelength of the laser light from your data, and assess the error.

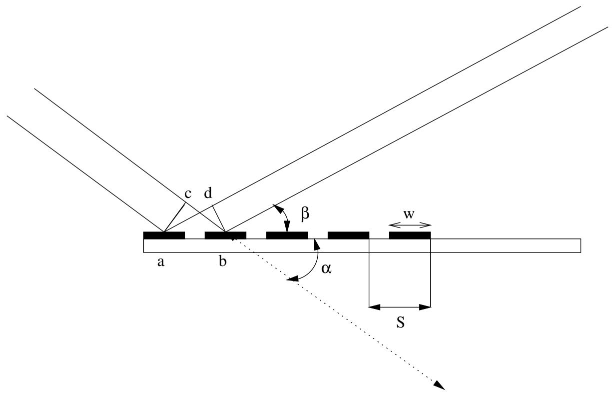

FIG. 2: Schematic diagram of a reflection grating and representative incident and reflected rays.

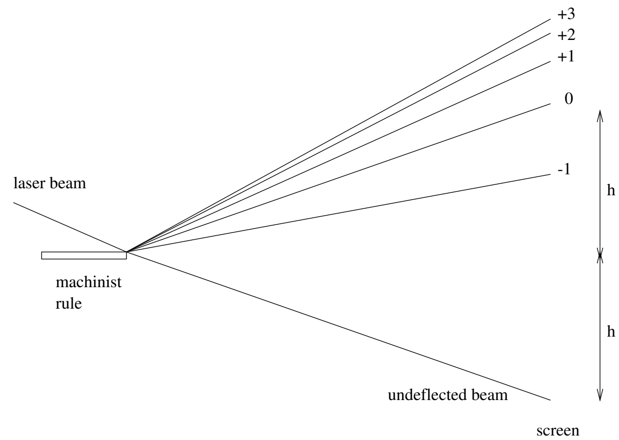

FIG. 3: Arrangement for measuring the interference pattern of parallel light reflected from a ruled grating.

3.1. Multiwave Interference

Reflection and transmission gratings produce the phenomenon of "multi-wave" interference which is employed in grating spectrometers for the analysis of atomic spectra. Optical gratings generally have many grooves per mm, typically 600 to 3,600 mm^{-1}, and are commonly used near normal incidence. However, a reflection grating with only a few grooves per cm can be used at a small grazing angle of incidence to produce the same phenomenon of multiwave interference.

This technique is specially useful in the X-ray region of the spectrum, where wavelengths are of the order of 1 Å, reflectivity is vanishingly small except for very small grazing angles of incidence, and transmission gratings for X-ray spectroscopy must have $\approx 30{,}000$ groves cm^{-1} with no X-ray absorbing material in the openings.[2]

A machinist's steel rule with a periodic engraved scale makes an effective grazing incidence reflection grating for visible laser light. Following a procedure described by [3], you can determine the wavelength of a laser beam by making a few simple measurements with such a rule. Figure 2 depicts a grating that consists of alternate strips of high and low reflectivity material running perpendicular to the page. Also shown are two parallel rays of an incident plane wave and two reflected, nearly parallel, rays of the Huygens wavelets spreading from reflection points separated by the distance S.

At a given spot toward which the rays are converging on a distant screen, the amplitudes of the two Huygens wavelets will interfere constructively provided the difference in the path lengths along the rays from the incident plane wave front represented by the line ac is an integer number of wavelengths, i.e.

$$S(\cos\alpha - \cos\beta) = n\lambda. \tag{4}$$

For small angle this equation can be approximated by

$$S\frac{\beta^2 - \alpha^2}{2} = n\lambda. \tag{5}$$

Since the amplitude of every wavelet that contributes to the optical disturbance at the spot on the screen can be paired up with another emanating from a point a distance S away, the condition for constructive interference must hold for all the light that reaches the spot. From Equation 5 one can expect to observe a series of spots on the screen at positions corresponding to the reflection angles.

$$\beta_n = \sqrt{\frac{2n\lambda}{S} + \alpha^2} \tag{6}$$

The experiment consists of measuring α and β_n for $n = \ldots -1, 0, 1, 2, 3, \ldots$. and checking the constancy of the quantity $S\left(\frac{\beta^2-\alpha^2}{2n}\right)$ which is a measure of the wavelength of the light. (A more complete theory of the multiwave interference is given in Appendix A.)

3.1.1. *Multiwave Interference - Procedure*

Mount the laser and a steel ruler on the optical rail separated by ~ 0.5m. Tilt the ruler slightly so that the beam strikes the 1/32" (or finer) scale at grazing incidence near the tip of the ruler, allowing a portion of the beam to escape reflection. Place a screen at a distance of

[2] Such gratings are now being manufactured for X-ray astronomy at the MIT Microstructures Laboratory. Diffraction of X-rays from ruled gratings at grazing incidence is of fundamental importance in the absolute determination of the wavelengths of X-ray emission lines which can then be used in Bragg reflection measurements to determine the sizes of the unit cells of crystals and the value of Avagadro's number!

~ 2 m away from the ruler to show the locations of the unreflected portion of the beam and the multiple reflected beams. Measure the positions of the spots and the distance from the reflection point to the screen. These are the data you need to check Equation 6 and compute the wavelength of the laser light and give an error estimate.

4. POLARIZATION THEORY

Light is a transverse wave in the electromagnetic field. The intensity (ergs $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}$) of a light wave is proportional to the square of the amplitude of the oscillating electric field. An ideal linear polarizer transmits only the component of the electric field oscillating in the direction of the "transmission axis". Thus two ideal polarizers oriented so their transmission axes are perpendicular to one another will completely absorb an incident beam of light. The filters used in this experiment are good but not ideal polarizers. In this experiment you will measure the transmission coefficient of two commercially available (Edmund Scientific) filters as a function of the angle between their transmission axes.

4.1. Polarization - Procedure

On an optical rail arrange an incandescent lamp, a rotatable Polaroid filter and a photodiode detector so that the intensity of light passing through the filter is registered by the photodiode. Record and plot the signal voltage as a function of the angle of orientation of the polarizer. Next, insert a second polarizer in front of the first, and again record and plot the detector voltage as a function of the orientation angle. Explain your data in light of the answer to Preparatory Question 3. Figure out a way to determine the direction of the transmission axis of a polarizing filter, i.e. the orientation in the plane of the filter of the electric field of the transmitted wave. Recall from 8.03 how the reflectivity of a dielectric surface depends on the polarization of the incident light.

5. STATISTICAL EXERCISE

1. For two fixed values of s, make 16 indepedent measurements of t. Plot the two distributions and evaluate the mean and variance of t.

2. Show explicitly by error propagation, how this translates to a value of uncertainty for the focal length $\pm$ error. Are the two determinations consistent?

3. Make a fit to ALL (s, t) measurements to determine the final focal length $\pm$ uncertainty.

6. POSSIBLE TOPICS FOR ORAL EXAM

1. Derivation from Snell's law of the lens equation for a thin lens of glass with index of refraction n and radii of curvature R_1 and R_2.

2. Theory of the ruler diffraction experiment.

3. Derivation and solution of the partial differential equations governing electromagnetic waves in space.

4. Theoretical explanation of the results you obtained in the polarization experiment.

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[1] E. Hecht, *Optics* (Addison Wesley, 2002), 4th ed.
[2] P. R. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003), 3rd ed.
[3] A. Schalow, Am. J. Phys. **30**, 922 (1965).
[4] M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, 1999), 7th ed.
[5] B. Rossi, *Optics* (Addison Wesley, 1957).

APPENDIX A: MULTIPLE WAVE INTERFERENCE FROM A GRAZING INCIDENCE REFLECTION GRATING

For a general discussion of the principles of multiple wave interference see References [1, 4, 5] or other texts on physical optics. Here we apply those principles to the analysis of the situation depicted in Figure 2. Two rays of a plane wave incident at grazing angle α on a surface with an ideal periodic pattern of reflectivity - 100 percent reflectivity strips of width w separated by zero reflectivity strips of width $s-w$. The reflected light is projected onto a distant screen so that the optical intensity at a given point on the screen is proportional to the squared sum of the amplitudes of the Huygens wavelets reflected from all elements of the illuminated portion of the grating. According to Huygen's principle, the total amplitude A at the spot corresponding to reflection angle β is proportional to the sum of the amplitudes of the wavelets from each differential element of surface area. Each differential amplitude can be expressed as the real part of complex amplitude proportional to $\exp\left[i(\omega t - \mathbf{k}\cdot\mathbf{r})\right]$ where $\mathbf{k}\cdot\mathbf{r} = \frac{\mathbf{2\pi x}}{\lambda}(\cos\alpha - \cos\beta)$ is its phase retardation relative to that of the wavelet reflected from the point "a" in Figure 2, and x is the distance along the grating from point "a". The total amplitude is proportional to the integral over the surface (we suppress the dimension into the page):

$$A \approx \int_0^{NS} \exp\,[2\pi i \frac{x}{\lambda}(\cos\alpha - \cos\beta)]R(x)dx \qquad \text{(A1)}$$

$$A \approx \int_0^{W} e^{[2\pi i \frac{x}{\lambda}(\cos\alpha - \cos\beta)]}[1 + e^{[2\pi i \frac{S}{\lambda}(\cos\alpha-\cos\beta)]} + e^{[2\pi i \frac{2S}{\lambda}(\cos\alpha-\cos\beta)]} + \ldots + e^{[2\pi i \frac{(N-1)S}{\lambda}(\cos\alpha-\cos\beta)]}]dx \qquad \text{(A2)}$$

$$A \approx \frac{\sin U}{U}\frac{\sin NV}{\sin V}, \qquad \text{(A3)}$$

where $U = 2\pi\frac{w}{\lambda}(\cos\alpha - \cos\beta)$ and $V = 2\pi\frac{S}{\lambda}(\cos\alpha - \cos\beta)$.

After integration and substitution of the limits, the first bracket is evaluated with the aid of De Moivre's theorem, viz. $\exp(i\theta) = \cos\theta + i\sin\theta$. The second bracket is first evaluated as a geometric series of the form $1+z+z^2+\ldots+z^{N-1}$, and then with the aid of De Moivre's theorem. The intensity I, proportional to the square of the amplitude (the complex amplitude times its complex conjugate), is

$$I \approx \left(\frac{\sin U}{U}\right)^2 \left(\frac{\sin NV}{\sin V}\right)^2. \qquad \text{(A4)}$$

Note that as $w \to S$, i.e. as the width of the non-reflecting gaps approaches zero, $I \to \left(\frac{\sin NV}{V}\right)^2$ which is large only near $V = 0$. This means that as the gaps are reduced in width, the ratio of intensity in the nth spot to the intensity in the $n = 0$ spot (i.e. the ordinary specular reflection spot) falls more and more rapidly with increasing n until the diffracted spots disappear.