8.08 Problem Set # 8

March 30, 2005
Due April 6, 2005

Problems:

1. Consider a gas of bosonic sodium atoms confined in a quardratic potential well $U(\boldsymbol{r}) = \frac{1}{2}m\omega_0^2|\boldsymbol{r}|^2$ where m is the mass of the sodium atom. The characteristic length of the oscillator potential is $r_0 = \sqrt{\hbar/m\omega_0} = 5 \times 10^{-3}$cm.
   (a) Ignore the interaction between the sodium atoms, fined the size of the condensed sodium atoms at $T = 0$. How does the size of the condensation depends on the number of particles?
   (b) For interacting bosons, the shape of condensation at $T = 0$ is determined by

$$\Big[-\frac{\hbar^2}{2m}\partial_{\boldsymbol{r}}^2 + (U(\boldsymbol{r}) - \mu) + g|\psi(\boldsymbol{r})|^2 \Big]\psi(\boldsymbol{r}) = 0$$

   In Thomas-Fermi approximation, we assume the wave function ψ is smooth and drop the $\partial_{\boldsymbol{r}}^2$ term. In this case the shape of condensation is determined by

$$[(U(\boldsymbol{r}) - \mu) + g|\psi(\boldsymbol{r})|^2]\psi(\boldsymbol{r}) = 0$$

   Now, how does the size of the condensation depends on the number of particles?
   (c) Fig. 15.2 of Huang's book shows measured shapes of condensation. The maximuum density is 10^{11}cm^{-3} for the shape near $T = 0$. Using the data provided by the curve, find the scattering length a of the sodium atom. (Note a and g is related through Eq. (15.3) in Huang's book.)

2. Problem 15.9 in K. Huang's book.