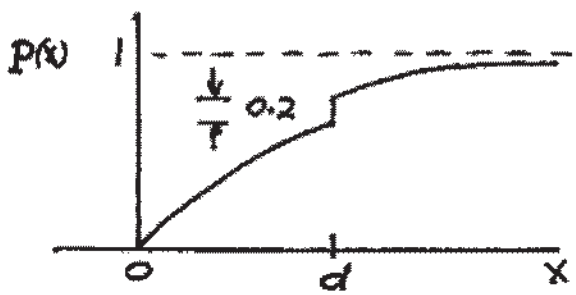
8.044 SOLUTIONS EXAM #1 2004

1 a)



b) $\langle x \rangle = \int_{-\infty}^{\infty} x\, p(x)\, dx = 0.8\ell \underbrace{\int_0^{\infty} \left(\frac{x}{\ell}\right) e^{-(x/\ell)} d(x/\ell)}_{1} + 0.2d$

$= \underline{\underline{0.8\ell + 0.2d}}$

c) $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 p(x)\, dx = 0.8\ell^2 \underbrace{\int_0^{\infty} \left(\frac{x}{\ell}\right)^2 e^{-(x/\ell)} d(x/\ell)}_{2} + 0.2d^2$

$= 1.6\ell^2 + 0.2d^2$

$\mathrm{Var}(x) = \langle x^2 \rangle - \langle x \rangle^2 = \underline{\underline{0.96\ell^2 - 0.32\ell d + 0.16d^2}}$

d) $\langle e^{-x/s} \rangle = \int_{-\infty}^{\infty} e^{-x/s} p(x)\, dx$

$= 0.8\left(\frac{1}{\ell}\right) \underbrace{\int_0^{\infty} e^{-x\left(\frac{1}{\ell}+\frac{1}{s}\right)} dx}_{\frac{1}{\left(\frac{1}{\ell}+\frac{1}{s}\right)}} + 0.2e^{-d/s}$

$= \underline{\underline{\frac{0.8}{(1+\ell/s)} + 0.2e^{-d/s}}}$

2 a) $p(E_B) = \int_{-\infty}^{\infty} p(E_A, E_B)\, dE_A$

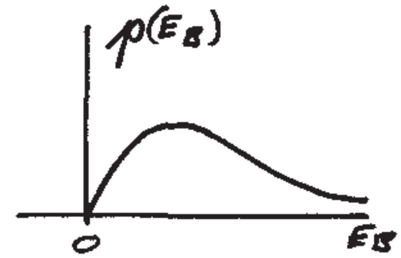
$$= \frac{4E_B}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} E_A e^{-E_A/\Delta}\, dE_A$$

$$- \frac{4E_B^2}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} e^{-E_A/\Delta}\, dE_A$$

$$= \frac{4E_B}{\Delta^2} e^{-E_B/\Delta} \left(1 + \frac{E_B}{\Delta}\right) e^{-E_B/\Delta}$$

$$- \frac{4E_B^2}{\Delta^3} e^{-E_B/\Delta}\, e^{-E_B/\Delta}$$

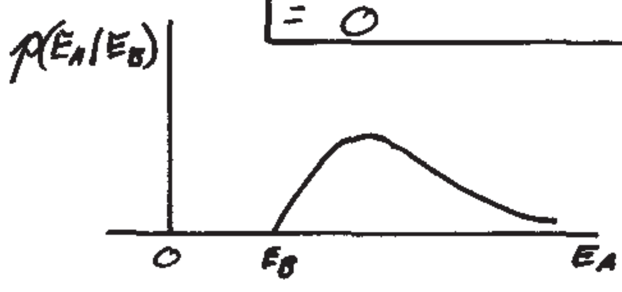
$$= \underline{\underline{\frac{2}{\Delta}\left(\frac{2E_B}{\Delta}\right) e^{-2E_B/\Delta} \qquad E_B > 0}}$$



b) $p(E_A | E_B) = p(E_A, E_B) / p(E_B)$

$$= \frac{1}{\Delta}\left(\frac{E_A - E_B}{\Delta}\right) e^{-(E_A - E_B)/\Delta} \qquad E_A > E_B$$

$$= 0 \qquad \text{ELSEWHERE}$$



c) NOT S.I. BECAUSE $p(E_A | E_B)$ DEPENDS ON E_B.

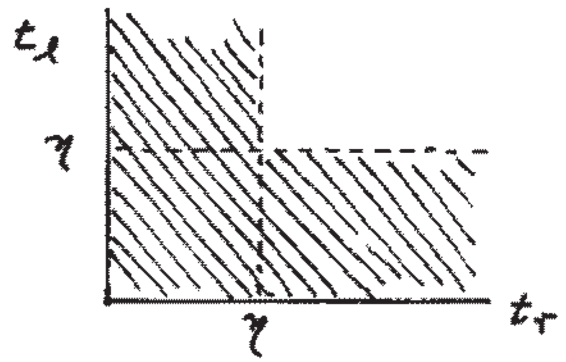
d) POISSON PROCESS $\langle n \rangle = f \times 10^6 h$ $\quad h \equiv$ TIME IN HOURS

REQUIRING $\sqrt{\mathrm{Var}(n)} / \langle n \rangle = 10^{-4} \Rightarrow \sqrt{\langle n \rangle} = 10^4$

$\Rightarrow \langle n \rangle = 10^8 = f \times 10^6 h$

$$\underline{\underline{h = \frac{100}{f} \text{ HOURS}}}$$

3



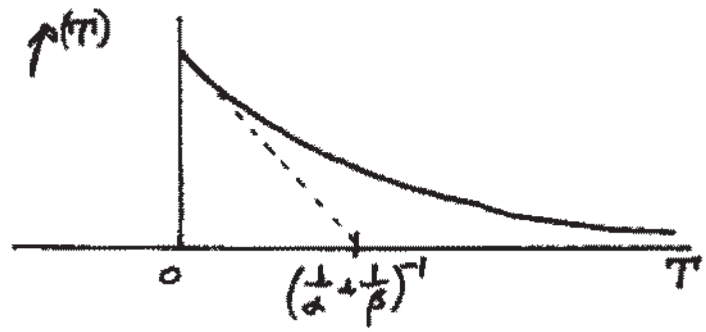
T < η IN THE SHADED REGION

a) $$P_T(\eta) = \int_{\text{SHADED}} p(t_l)\,p(t_r)\,dt_l\,dt_r = 1 - \int_{\text{UNSHADED}} p(t_l)\,p(t_r)\,dt_l\,dt_r$$

$$= 1 - \left(\frac{1}{\alpha}\int_\eta^\infty e^{-t_r/\alpha}\,dt_r\right)\left(\frac{1}{\beta}\int_\eta^\infty e^{-t_l/\beta}\,dt_l\right)$$

$$= 1 - e^{-\eta/\alpha}e^{-\eta/\beta} = \underline{\underline{1 - e^{-\eta\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)} \quad \text{FOR } \eta > 0}}$$

b) $$p_T(\eta) = \frac{dP_T(\eta)}{d\eta} = \underline{\underline{\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)e^{-\eta\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)} \quad \text{FOR } \eta > 0}}$$



8.044 Statistical Physics I
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