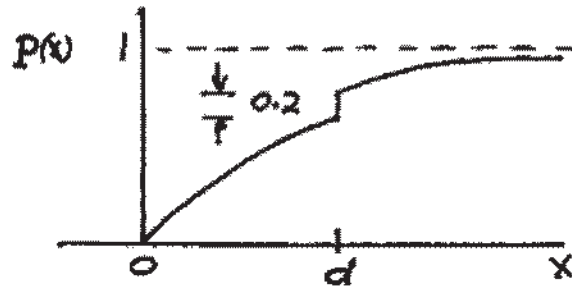


1 a)



$$b) \langle x \rangle = \int_{-\infty}^{\infty} x p(x) dx = 0.8l \int_0^{\infty} \underbrace{\left(\frac{x}{l}\right) e^{-x/l}}_1 d\left(\frac{x}{l}\right) + 0.2d$$

$$= \underline{\underline{0.8l + 0.2d}}$$

$$c) \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 p(x) dx = 0.8l^2 \int_0^{\infty} \underbrace{\left(\frac{x}{l}\right)^2 e^{-x/l}}_2 d\left(\frac{x}{l}\right) + 0.2d^2$$

$$= 1.6l^2 + 0.2d^2$$

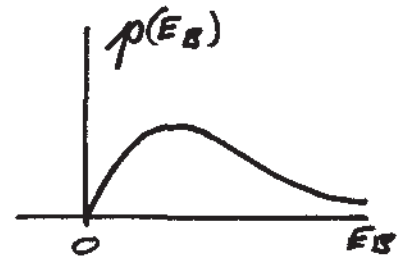
$$\text{Var}(X) = \langle x^2 \rangle - \langle x \rangle^2 = \underline{\underline{0.96l^2 - 0.32ld + 0.16d^2}}$$

$$d) \langle e^{-x/l} \rangle = \int_{-\infty}^{\infty} e^{-x/l} p(x) dx$$

$$= 0.8 \left(\frac{1}{l}\right) \int_0^{\infty} \underbrace{e^{-x\left(\frac{1}{l} + \frac{1}{l}\right)}}_{\frac{1}{\left(\frac{1}{l} + \frac{1}{l}\right)}} dx + 0.2 e^{-d/l}$$

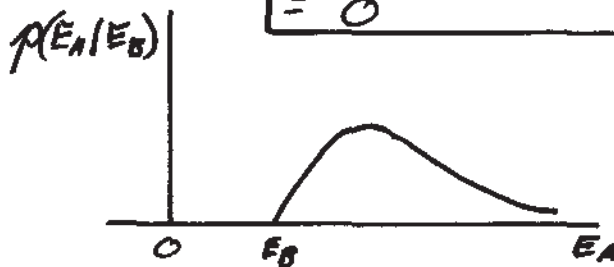
$$= \underline{\underline{\frac{0.8}{(1+l/l)} + 0.2 e^{-d/l}}}$$

$$\begin{aligned}
 2 \quad a) \quad p(E_B) &= \int_{-\infty}^{\infty} p(E_A, E_B) dE_A \\
 &= \frac{4E_B}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} E_A e^{-E_A/\Delta} dE_A \\
 &\quad - \frac{4E_B^2}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} e^{-E_A/\Delta} dE_A \\
 &= \frac{4E_B}{\Delta^2} e^{-E_B/\Delta} \left(1 + \frac{E_B}{\Delta}\right) e^{-E_B/\Delta} \\
 &\quad - \frac{4E_B^2}{\Delta^3} e^{-E_B/\Delta} e^{-E_B/\Delta} \\
 &= \underline{\underline{\frac{2}{\Delta} \left(\frac{2E_B}{\Delta}\right) e^{-2E_B/\Delta} \quad E_A > 0}}
 \end{aligned}$$



$$b) \quad p(E_A | E_B) = p(E_A, E_B) / p(E_B)$$

$$\begin{aligned}
 &= \frac{1}{\Delta} \left(\frac{E_A - E_B}{\Delta}\right) e^{-(E_A - E_B)/\Delta} \quad E_A > E_B \\
 &= 0 \quad \text{ELSEWHERE}
 \end{aligned}$$

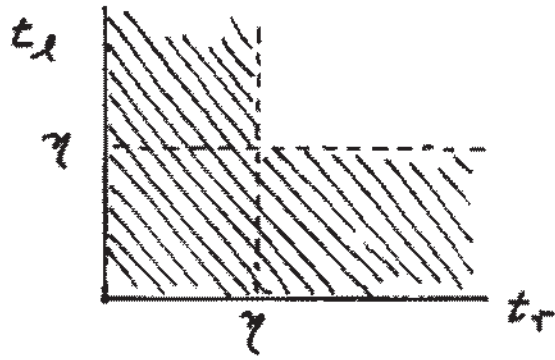


c) NOT S.I. BECAUSE $p(E_A | E_B)$ DEPENDS ON E_B .

d) POISSON PROCESS $\langle n \rangle = f \times 10^6 h$ $h \equiv$ TIME IN HOURS
 REQUIRING $\sqrt{\text{Var}(n)} / \langle n \rangle = 10^{-4} \Rightarrow \sqrt{\langle n \rangle} = 10^4$
 $\Rightarrow \langle n \rangle = 10^8 = f \times 10^6 h$

$$\underline{\underline{h = \frac{100}{f} \text{ HOURS}}}$$

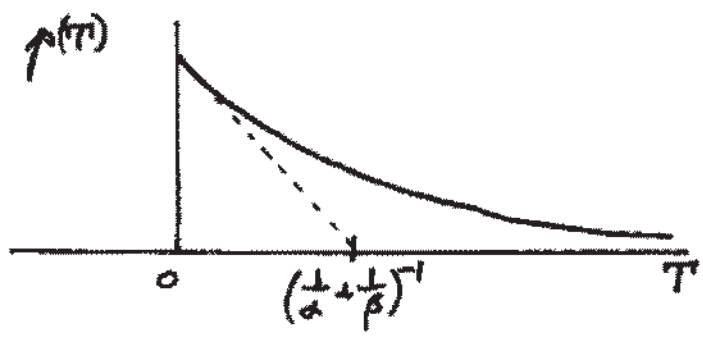
3



$T < \gamma$ IN THE SHADED REGION

$$\begin{aligned}
 \text{a) } P_T(\gamma) &= \int_{\text{SHADED}} p(t_1) p(t_r) dt_1 dt_r = 1 - \int_{\text{UNSHADED}} p(t_1) p(t_r) dt_1 dt_r \\
 &= 1 - \left(\frac{1}{\alpha} \int_{\gamma}^{\infty} e^{-t_1/\alpha} dt_1 \right) \left(\frac{1}{\beta} \int_{\gamma}^{\infty} e^{-t_r/\beta} dt_r \right) \\
 &= 1 - e^{-\gamma/\alpha} e^{-\gamma/\beta} = \underline{\underline{1 - e^{-\gamma(\frac{1}{\alpha} + \frac{1}{\beta})}}} \quad \text{FOR } \gamma > 0
 \end{aligned}$$

$$\text{b) } f_T(\gamma) = \frac{dP_T(\gamma)}{d\gamma} = \underline{\underline{\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) e^{-\gamma(\frac{1}{\alpha} + \frac{1}{\beta})}}} \quad \text{FOR } \gamma > 0$$



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