

MITOCW | 10. Interference of Electromagnetic Waves

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PROFESSOR: So welcome back. And today, I'm going to do some examples of what happens when you have more than one charge radiating. This is a very important example, because whenever you have many charges radiating, if it's coherent, you get the phenomenon of interference, and it plays a very important role in many fields of physics, in particular, in optics.

You may have heard terms like interference patterns, diffraction, et cetera. All that is to do with when you have more than one source radiating.

So the example I'm going to consider is the following. Suppose you have two charges, both q . Each one is oscillating. It's oscillating with an angular frequency 2π over λ . This tells you the position of the charge as a function of time.

So it's centered to distance d from here and oscillating within amplitude a as a cosine 2π over λt . Well, λ is, of course, the wavelength of the radiated electromagnetic wave.

So this one is oscillating up and down like that, and there is another one over here oscillating just for fun in the other direction. This one is oscillating in along the x -axis, and this one is oscillating along the y -axis.

And here is the formula, which tells you what is the position of this one at the given instant of time. They're both given in terms of the same time, with the same time. t equals 0. Therefore, these are two coherent sources.

The question is, what will be the resultant field anywhere in space? At some point we want to know what is the resultant field.

Now, each one of these-- you know what it does. It radiates a spherical wave with a

certain angular distribution. Alright? And the waves overlap in space.

Now because the electromagnetic waves are solutions of the electromagnetic wave equation, which is a linear equation, if we have a coherent source of radiation at any point, we simply add vectorially the electromagnetic waves. The electric fields you add vectorially all the magnetic field.

Now in principle, we should go-- to first principles. Consider Maxwell's equations. Consider what happens when a charge is oscillating. And in fact, consider both charges, what they're doing at any instant of time, and solve in all of space Maxwell's equation to see what the electric field is everywhere.

We build on our experience, and what we will do is we will assume that we've solved the problem of what happens when this single charge is accelerating. We've done this several times in the [? PAR sensor ?]. Professor Walter Lewin has done it in the course.

Whenever a charge is accelerating, it radiates. And this is the formula, which tells you what is the electric field from the given charge, which is accelerating at some position, r and t .

I'm just reminding you that what we find is that the electric field at any position at some time t is given by, or related to, the perpendicular component of the acceleration of the charge. It's perpendicular to the direction of propagation, but at a point t in space, the electric field there is related to the acceleration at an earlier time. At the time, t prime, which is earlier than the time when we are considering by the amount r/c .

All that this does-- it reflects the fact that if I have some with an accelerated charge, at that instant, the electromagnetic wave starts to propagate, and it takes a time r/c to reach a distance r . That's all this is telling us.

The other thing this is telling us-- the wave front is a sphere. The amplitude of the electric field drops off like $1/r$. And it is not uniform in all direction, as we've seen in the past. Right?

For example, if I take this charge as it's oscillating up and down, the maximum amplitude radiated is in this plane where we are looking, where the perpendicular component of the acceleration of this is the maximum. And its minimum upwards is 0, because this acceleration has no component of x perpendicular component to this direction. And similarly, for this.

The other thing which I wish to point out is that the polarization of the electromagnetic wave, the electric vector, is parallel to the perpendicular component of a . So from this, the electric vector, when it gets to here or here, will be in the direction of x , while from this one, it'll be polarized in the direction y .

The other thing which I want to stress, that we will in the other examples-- always consider a situation where the point where we're looking at the electromagnetic field is far away from where the charges are. The formula, which I just showed you here, applies only in the far field. Nearby, the electric field and the magnetic field are very complicated. They can be calculated, but they cannot be summarized in that simple form. But if I'm far away from the accelerating charge then, it is possible to write it in this simple form.

So our discussion is only valid if the wave length radiated is much smaller than the distance from where we're considering. This distance is also much smaller than that. And this is to allow us to approximate $\sin \theta$ by θ , et cetera. And that's the case, alright?

OK. With that, we can now immediately solve the problem. Simply consider the acceleration of each charge from our knowledge of where the charge is. We can differentiate it twice, and we can get the acceleration.

So we know that the acceleration of the first charge, which is oscillating along the x -axis, is given by $-\omega^2 \cos \omega t$ in the x -direction. I want to emphasize, because this is in the same direction as the perpendicular component of a_1 .

And, I'm sorry. I misspoke. The answer-- what I said is correct but for the wrong reason. In this case, by definition, the acceleration of the first one is this because the displacement of the first one is in the x-direction. Similarly, the acceleration on the second one is in the y-direction, because the displacement is in the y-direction. Alright?

So these are the two accelerations. This is just some-- occasionally I'll use omega, occasionally lambda just to remind you of that.

OK. So all I now have to do for each of the charges is calculate this quantity at the position where I want to calculate the electric and the magnetic field. So, first of all in this problem, we're asked to do it in two places. We're asked to do it at position p1 and at p2. p1 is along the z-axis, and p2 is off the z-axis.

First, let's do it for p1. So, the electric field, due to the first charge at position p1 at time t, is given by this formula if I insert in this for the distance r, which is L. OK? And I put in the acceleration of this one and the perpendicular component of the acceleration perpendicular to the line drawing my two charges. And the position p1 is-- in fact, that line is along the z-axis. So the perpendicular direction is, in fact, in the x-direction.

OK. So I don't even have to take the-- the actual acceleration is the perpendicular component. OK. And I have to calculate it at a time which is t minus the distance p1 is from in my charges, which is L. So it's minus omega L over c. I calculate t prime in here.

Notice, I have not tried to calculate the distance between the charge and p1 exactly, because I told you that the distance d is very much smaller than L. And so the overall distance is negligently different from being just L. So that's the electric field due to the first one.

Similarly, I can do the same for the second one. Everything is the same. The only difference in that case is that now the electric vector-- the acceleration is along the y-axis and therefore, the electric field is polarized along the y-direction.

So at the point p_1 , the two electric fields are the same. They're both oscillating in phase with the same frequency, et cetera. But this one is pointing in the x-direction. This in the y-direction.

Now, as I mentioned at the beginning, if at any point you have electric field due to two sources, they simply add because the system is a linear system. So we simply have to add the electric fields.

Electric fields are vectors, therefore we don't add them algebraic as scalars, but we have to add them vectorially. And so, we take this vector and that vector and add it. And that gives us the total electric field at point p_1 . And simply from here, I get d_0 like that, which I can rewrite in terms of a unit vector.

Alright? This is a unit vector, which is at 45 degrees. In other words, if that's the z-direction x and y, it's at 45 degrees to both the x and y-axes. That's the unit vector. This is the amplitude of it. And this is d_0 -- tell us what is the oscillating frequency and what is the phase.

Notice, although each one of these sources radiated there an electric field of magnitude E_0 , when I add them, I did not get to $2E_0$, because as I mentioned, electric fields are vectors, and we have to add them vectorially. And that's why you get here the root 2.

So this is the first example, which I've done. Now I want to take the same geometry but calculate what is the electric field. What we've just done, we've calculated here. Now I want to do the same situation-- two sources oscillating coherently. But I want to look at what is the electric field at the point p_2 .

And I've taken p_2 to be in that the xz-plane, but in a direction that's an angle λ over a to d . d is this distance. λ is the wavelength of the radiated electromagnetic wave. And I've taken this crazy number simply to make the arithmetic come out easier at the end.

And so we now want to calculate the electric field here. And we do exactly the same

except we go a little faster. What I will do is I will calculate the electric field here.

Due to this charge oscillating, I'll then calculate the electric field here. Due to this charge, this one is oscillating like that. I'm reminding you this one is oscillating like that. And we will vectorially add the two.

Now, from the point of view of the amplitude, the difference in distance between this and here or this and here or this and here is insignificant. And so I have ignored that in the previous calculations, and I'll do the same here. But when you add to the two, we have to worry about the relative phase of the alternating the electric fields. And the difference of this distance compared to that distance is no longer insignificant when it comes to the relative phase of the two radiations.

So, in this formula when I'm calculating the electric field, this r -- it doesn't matter whether I call this L or L plus a little bit or less. This doesn't change very much. But in calculating the phase of the radiation, I cannot ignore that.

In the previous case, the phase was the same because this distance and that distance are exactly the same by symmetry. For this position, that distance and that distance are not the same.

And so I what I will do is I'll use as reference that distance, and when I'm calculating this distance, I will calculate by how much I have to subtract from here to end up with this length and for this one, by how much I've add to it to get to here. Pay attention to that when I'm doing that in a second. And the rest is very straightforward and similar to what I've just done.

So now, as I say, I want to calculate the electric field at the point p_2 , which is this. This is the x -coordinate, y , and z -coordinate. And I pointed it out to you a second ago. Alright.

So I don't have to rewrite all those q 's and ω 's, et cetera, by analogy, what we've done-- the electric field due to the first charge at position p_2 will be that the amplitude that we've got is the same for the previous case of P_1 . So it's-- E_0 is the amplitude. It's pointing in the x -direction as before.

And we know that the description of the electric field will be given by $\cos(\omega t_1')$ where t_1' is-- so far I've always talked about t' , because I only had one when I was considering just one charge. But here the t_1' is different to t_2 .

So quickly, this term comes from the perpendicular component of a right as before. And t_1' is equal to the time when I'm looking at the electric field minus the time it takes for this signal from the charge to get to the point where I'm looking at the electric field to point p_2 .

So, it will be-- as I told you a second ago, I'll take L as the reference. And I'm subtracting from it that distance $d \sin \theta$. For a second, let me go back to that picture because you can easily get confused. So I'm coming back here.

What we're trying to calculate-- the distance from here to here. So I'm taking this distance, which is L , and I'm subtracting from it this distance, which is $d \sin \theta$. So that's what I'm calculating there. So this is $L - d \sin \theta$ divided by c .

Then the electric field is then immediately followed from this d_0 in the x -direction, $\cos(\omega t - \dots)$. And if you assume that this angle is small, and I told you that that's given in the problem, the sine of an angle is equal to an angle. Approximately, they're small. And if I multiply this out, I simply get an angle here. That's $\pi/4$.

So I find that the electric field at point p_2 is pointing, due to the first charge, is pointing in the x -direction, as you'd expect, because of the direction which the charge is moving times $E_0 \cos(\omega t - \dots)$ plus a phase like that.

How about from the other charge? If I take the other charge, everything will be the same except now this charge is oscillating in the y -direction, so the electric field will be in the y -direction. And this time, it's $\omega t_2'$ where t_2' will be very similar to t_1' but now the distance is greater by this amount.

For a second, let's go back to this picture so you see what I'm talking about. If I calculate this distance, it's the same as that distance plus $d \sin$ this angle, which is $\frac{\lambda}{8d}$, and I'm subtracting. And I want to emphasize that what I'm focusing on here, what is important, is the difference between this and that and not the absolute value.

So when I'm approximating-- if you do this calculation for yourself, you'll see I'm doing a slight approximation to calculate that. But this and this is the same quantity. But the important thing is the difference of this phase here is plus $\pi/4$. Here is minus $\pi/4$, because here it's minus that little bit of a distance, and here it's plus a little bit of the distance.

So we finished. The total electric field at p_2 and time t is the sum of the electric field due to the first charge plus due to the second charge. And as I mentioned a second ago, electric fields are vectors. Thus, we have to add these two vectorially. This is in the x -direction. This is in the y -direction.

And so what we have at that point is two electric oscillating electric fields. They're oscillating coherently but out of phase. This is a plus $\pi/4$ phase. This is a minus $\pi/4$ phase. So the difference of phase between those two is $\pi/2$. It's 90 degrees.

So this is out of phase with, one with respect to the other, by 90 degrees. They're coherent, but 90 degrees out of phase.

The magnitudes are the same, but this one is pointing in the x -direction and this in the y -direction, and you know what that corresponds to. If I have at the location two electric fields, one doing that and the other one doing this, if they're out of phase by 90 degrees, and I add the two, what do I get? I get a constant size of a radius, and the thing is rotating. That is the description of a rotating vector of magnitude E_0 .

In the other case, at P_1 , there was not this phase difference, so the two were-- one was oscillating like this. The other one was oscillating like that. But they were oscillating at the same phase.

So this is what they were doing. And if you add those up, clearly you get a line

diagonally. And that's why before, the result was a linearly polarized electric field there at 45 degrees to the x and y-axis.

In this case, these two are out of phase by 90 degrees. So when one is doing this, the other one is also but out of phase. And so when this one's at the maximum, this one's at the minimum, and so that's what's happening.

So they're out of phase. And then if you add the two up, you get something doing this, which we called circularly polarized light. And that's why I chose that crazy angle, because it came out like this.

So that's the end of that problem. I will now do another one, which to emphasize some technique-- a different technique of doing it. OK. Let's move.

So the next problem is the following. We're again dealing with several charges oscillating coherently. Their cohering sources. And we're asking, what are the electric fields? Somewhere in space? And if we found the electric field, of course, we could always calculate the magnetic field at that location using Maxwell's equations or our knowledge of the relation between electromagnetic field in a progressive electromagnetic way.

Now I'll take three charges. But that's not the hope. It's more different, but what this problem adds is I'll use it to show a technique that is often used, which helps in the solution of such problems.

So the problem now is the following. I have three charges. Each of the same magnitude. They're located along the x-axis at position 0 minus d and minus 2d. So these distances are the same-- each distance d.

And what the problem is-- up to t equals 0, these three charges are displaced from equilibrium. I put y is 0. So, at the height, y_0 here. Then, at t_0 -- don't ask me how. Magically, I get these three charges to start oscillating. Such that at time yt , the displacement is $y_0 \cos \omega t$. And they continue-- this continues like that forever.

The question is, as a result of this oscillation of charges, what will be the electric field a long way away from here at position L along the x -axis, so we call that position p . So the problem is, calculate E at position p for all times. Once we've calculated, we could calculate the magnetic field, But to save you time--I mean, you could do it for yourself.

You know if you calculate the electric field, there is a progressive wave over here. if E is like this, then b will be perpendicular to it of the amplitude which is just a dE over c . So that will be straightforward, so I'm not asking it.

Now, let's just think for a second what goes on in this problem. Initially, the charges are stationary, so there will be a Coulomb field around. But we are specifically asked to ignore static fields. So in the last problem too, I ignored static fields. We were only considering the time-dependent fields.

You can always have superimposed on the time-dependent field some static field that doesn't add anything. So what we have here is at time up to time t equals 0, the charges are here. So let's take one of the charges here. Then it starts moving.

It will have an acceleration, which is perpendicular to the direction in which I'm interested the propagation of the electromagnetic wave. So it will certainly radiate in this direction. So this charge, which was oscillating, will radiate.

Over here at the point p , the electric field will be initially 0. And it will continue being 0 until the electric field, which is generated here, propagates that distance. So, only after a time, L/c , will the electric vector get here? So up to the time L/c , there will be no electric field here.

How about this charge? This charge is initially is displaced at y_0 . Then it starts oscillating the same. Initially, it's stationary. It'll produce a Coulomb field, which is a static field. We're not interested in it.

But once it starts accelerating, it starts radiating. And that radiating progresses. So that will also produce a field over here. But since these two distances are different, the radiation from here, first of all, will get there a little later. But also, once it gets

here, it will have a different phase because the radiation traveled a different distance. And same for the last one.

So now, how do we do this? I almost sound like a broken record. As always, I can calculate the electric field from each charge, and it'll be given by this formula, which we've seen over and over again where t' is the distance from the charge to the point p .

Furthermore, I know the perpendicular acceleration and also the perpendicular component of it, because in this problem, the perpendicular component is the same as the actual acceleration. After time t equals 0, the acceleration of every one of these charges a_1 , a_2 , a_3 , is the same. It's the same direction. And it's given by that simply because we know what is the displacement y of t . If I differentiate it twice, I get the acceleration.

If I plug this into here for each charge, I will know what is the electric fields from each charge. I can then add the electric fields from each charge, and I'll get the total electric field. You have to add them vectorially.

So, once again, for t less than L/c , the electric field at position p will be 0. There was no earlier accelerated charge which radiated an electric field which got here before this time. So that's nice and easy.

How about for a later period, a period between L/c or just after L/c . For t just after L/c , the acceleration of the charge at x equals 0 would produce an electric field which propagated and would have got to my point p . So between that time L/c and the time when the radiation from the second charge got to the point p , I will have an electric field but only from the first radiation due to the first charge.

I can forget the second and third charge at minus d and a minus $2d$ but not the charge at x equals 0. So that will be I use this formula. I plug it in. I use the acceleration of a_1 . I put it in here, and I know that t' is t minus r/c .

r in this case is L . So this formula immediately tells me that's what it is and lets me save-- I don't want to write this over and over again. This-- I'll call that E_0 . So from

the first charge, this is what the electric field-- it's an oscillating electric field-- what it looks like at position p . But I'm reminding you, this is only true for this time window when the radiation from the charge at q equals 0 has got to point p , but from the other one's not.

How about from a little later? From the time L/c plus d over c -- in other words, now radiation over a distance L plus d has had time to reach my point p . So now, the second charge, the one that minused d , has had enough time to reach my point p .

But if I limit to the window, this and that, in other words, there's still a difference of d/c in time between those. I will get at point p the radiation as from the first charge. That's exactly the same as this. That's obviously got there.

But now from the second charge, it has got there. And they're very similar. They are both-- the charges are accelerating along y -direction in phase. So they have the same frequency, and the amplitude is going to be the same, because in this formula, this r is the total distance between the charge and the point p .

Now, there is a tiny difference in that distance for the first and second charge. But if I take them here a tiny difference, this will not change very much. I'm ignoring that difference. So I'm calling both of them E_0 and ignoring that tiny difference.

But I cannot ignore the difference on the phase. So the t prime into two cases is different. In one case, it's t minus L/c . And the other case, it's minus L/c minus this little extra time where the radiation takes from the second to the first charge. And the third one, the radiation could not to reach the point p yet.

So that is now the total electric field from the two. And finally-- and I have to do this. I haven't done the addition. And finally, if we take a time which is greater than this, then we are in a situation where the oscillation on the last charge has had enough time to get to the point p . Let me go back to this picture and just repeat this so that you don't get lost.

So these charges have started to move all at the same time. After a time L/c , the

radiation from this one will have got to here. If I add to the time d/c , that's how long the electromagnetic radiation takes to get to from here to here. So a little bit later, the radiation from this plus the radiation from this is getting here. And a little bit later, the radiation from this also gets there.

So at the end, there is now radiation from all three charges getting to this point. And from then on, it'll continue forever as long as these are oscillating. So from then on, you have the radiation from the three. And the only difference between them is they all have a slightly different phase.

So you might be satisfied with just knowing that these are the electric fields as sums of the algebraic sum between those all pointing the same direction. So I don't have to worry about the vector addition, but I have to worry about the addition of the amplitudes.

If we want to find out what this sum is-- what happens when you add one or two or three oscillatory functions like this? What is the resultant oscillation? We have to algebraically, or trigonometrically, add these three or these two.

And I do this example in order to introduce a mathematical technique, which in situations of this kind, makes life much, much easier than just going and adding cosines. And it is by using the so-called complex amplitudes.

So the issue is, how do we add these three cosine functions and where they each have a slightly different phase? One way is brute force. Do it by using trigonometrical [? formulae ?].

For example, you could add the first and second by using the formula $\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$. And you could do that for adding the first to the second, and then later, once you've got an answer, you could add to that the third, et cetera. But you can see that if you have five, six, seven, eight sources, or many, many more, this becomes cumbersome.

There's a nice mathematical trick. And that is by using complex numbers. You know

that this is De Moivre's theorem that E to the j theta can be written as the cosine of an angle plus j sine of an angle.

I can use this mathematical trick to solve the problem of adding these. Remember that at this stage, this is pure mathematics. As always, we've converted an experimental situation into a mathematical problem. And we've got to solve this using mathematics.

You don't have to ask yourself what's the meaning of j sine theta something. It is a mathematical expression. And we're going to use mathematics to solve this problem.

Using this, I could always write-- that suppose I have the cosine of some function, I can always write it as the real part of E to the j that angle. So if cosine, for example, of $\omega t - kL$, and you'll see it somewhere up there for example. It can be written as the real part of E to the j $\omega t - j$ to the kL , but that is the same as multiplying these as E to the j ωt times E to the minus jkL , where here I'm just reminding you of $k\omega c$, et cetera, is.

I can do this for every-- if I want to do-- let's say we're doing this one first. I want this to that. I can write this as the real part of a complex number. And I can write this as a real part of the complex number.

What I will then do-- I will first solve this. I will do the addition by adding the two complex numbers, knowing full well that if I added the complex numbers, I will have in the process added the real parts and also the imaginary parts. And since you cannot have a real number equal to an imaginary number in any way, if I take during the process of addition, I would have continuously kept separate the real and imaginary part.

So for example, for this third part, this addition here, I can write the first term as the real part of each of the E to the j ωt , E to the minus jkL in the y -direction times E_0 . This is, of course, nothing other than $E \cos(\omega t - Lc)$.

The second one I can do the same. This is the same. And the only difference

between those two is that phase minus d/c , which is kd [? using ?] here. And so, the real part of this will be the answer to the sum of those.

So I'll do this in a second, but then let me immediately go to the third one so I do both of them at the same time. And in this third case, the answer I'll want is the real part. This is the same as before. And here the three terms.

The first term is just E_0 , because all the phases are out here. The second one differs from that by minus d/c , which is minus kd , so it's E to the minus jkd . And the last one is E to the minus j times $2kd$, because here we have a $2d$.

Now, why did I bother to do this? Because this addition is trivial while the other one was not trivial. It needed hard labor. Why do I say this is trivial? Because I've converted this algebraic problem into a geometry one.

I can represent each one of these terms on an Argand diagram. So for example, here to here. Let's take the third case. We're adding these two vectors. This is only a real part, so it's E_0 .

That's a vector of length E_0 along the real axis. I'm reminding you on an Argand diagram, this direction is the real axis, and this is the imaginary axis. With that, so the E_0 is only real, and I'm adding to it E to the minus jkd , which is what? Has a magnitude of E_0 . And this is the angle, θ , with respect to the real axis, which in this case is minus kd .

So this is the angle, so it's minus. So I'm going down. So it's a vector, which is length E_0 , and it's pointing in an angle of minus kd .

It's this angle. I suppose, not to confuse you, I'll call it minus kd . This is minus and minus, just so not to confuse you.

With the distance between the charges of d , kd is 2π over 3 , which is a 120 degrees. So this term is a vector like that. And we have to add those two vectors. Well, this is easy to do.

This angle is 60 degrees. This is 120. OK? And this length is equal to that, so the result of this plus of that will be this red vector here, which has magnitude E_0 . And at what angle is it? This angle here 240 degrees.

I'm sorry. I need that later in the second for the next part. I don't need it at this stage. I'm sorry.

I'm adding this to that. The result is this vector. And this vector has a magnitude E_0 . And this angle here is, of course, 60 degrees, which is 2π over 6.

And so, so adding those two will give me just the real part on each of E to the $j\omega t$ to the minus $j\omega L$ in the y -direction. And adding those two is E_0 in the direction of 60 degrees, which is minus 2π over 6. Bracket right there.

So this describes this addition. And this, of course, I can now take the real part of this whole thing, and there is the answer. E_0 in the y -direction cosine ωt minus ω over c and with a phase of minus 2π over 6, which is of course, 60 degrees.

For the first one, so this is the answer. And we didn't have to add to any cosine. We just do a vector addition on the Argand diagram.

Let's do the last case. In the last case we have three terms-- one, two, three. If I write these as the real part of a complex amplitudes, by analogy with this, the first term is the same. The second term is the same. And the third one is almost the same as the second time, except the d has now become $2kd$.

This was d/c . Here it's $2d$ over c . So here we have minus $j2kd$. So now I have to add these three complex numbers. And again, this is much easier to do it geometrically than trigonometrically.

The first one I can represent by this on the Argand diagram. The second one by this. And the third one, of course, is at 240 degrees to here. This was 120. The next one is 240, which means that the last one, all that it is, I'd remove this vector, and

it's in this direction.

So now we're adding this to that to this, and even I can solve that in my head. If I add this vector to that to that, I get 0. I'm back where I started.

So this quantity is 0. And so the electric field will be 0. And see how relatively easy it is to do if you use this complex amplitude method?

One can do many problems to do with waves and vibrations using complex amplitudes. In general, it makes the algebra easier. But for most cases, I have not done that in order to-- that you don't have the double difficulty of trying to understand the physics and struggling with the mathematics that you may not be so familiar.

By the time we come to many sources of the radiation, it is so much easier to do using complex amplitude that I would urge you to learn it on simple cases like this, and then use it in more complicated situation.

Finally, I just want to save one word, and that is the following. Some of you may be surprised. How is it that I got-- I have three charges. Three charges oscillating, radiating. How is it that when you get to here, you get after a certain time, you get to [INAUDIBLE].

And the answer is-- pictorially you can see what happens-- at this point, the radiation from one of the charges looks like that. From the second one, it's out the phase by $1/3$ of the wavelength, and the next one by $2/3$. And so you're adding three waves, three oscillating motions on top of each other. And if you add these, the result is 0.

That is, pictorially, the same as what I did here in this diagram. You simply-- you do have three waves arriving from the three charges, but they add to 0 because each has a different phase from the previous one. And you can see it here what happens. Here I'm plotting as a function of time. The amplitude at the point p.