# Massachusetts Institute of Technology OpenCourseWare

**8.03SC** **Fall 2012**

## Problem Set #6

### Problem 6.1 (Bekefi & Barrett 3.3)[1] – Electromagnetic plane waves

A magnetic field of a uniform plane wave is given by: $\vec{B}(\vec{r},t) = B_o f(\vec{k}\cdot\vec{r} - \omega t + \phi)\hat{y}$ where $B_o$ is a scalar constant, and $f(\vec{k}\cdot\vec{r} - \omega t)$ is any function of the argument $\vec{k}\cdot\vec{r} - \omega t$. The vectors $\vec{k}$ and $\vec{r}$ are $k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ and $x\hat{x} + y\hat{y} + z\hat{z}$, respectively.

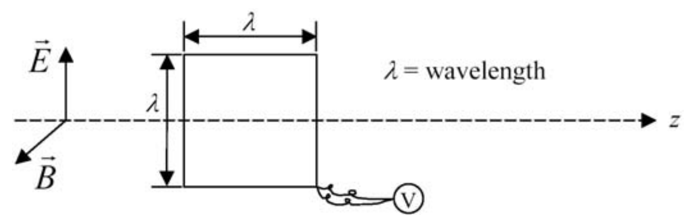
a) Give <u>all</u> the conditions imposed on $\vec{E}(\vec{r},t)$ and $\vec{k}$ by Maxwell's equations and the wave equations.

b) If $k_z = 0$, give an expression for $\vec{E}(\vec{r},t)$ similar to $\vec{B}(\vec{r},t)$.

### Problem 6.2 (Bekefi & Barrett 3.5) – Maxwell in action

The $\vec{B}$ field of an electromagnetic wave in vacuum is given by $\vec{B}(x,y,z,t) = \hat{x}B_0\sin(\omega t - kz)$

a) Use Maxwell's equations to deduce the equation for $\vec{E}(x,y,z,t)$ for this wave. A square single-turn loop of wire, with sides of length equal to $\lambda$ is used to pick up the signal from the wave by detecting the voltage $V$ appearing at the two ends. This will be of the form $V = V_0\sin(\omega t + \phi)$




b) The loop is placed as shown, with the two sides parallel to $\vec{E}$ and the other two sides parallel to $z$. What is the value of $V_0$ in this situation?

c) What is the maximum possible value of $V_0$, and how should the loop be oriented to obtain it?

### Problem 6.3 – Polarized radiation

Write down the electric field and associated magnetic field in vacuum for traveling plane waves. The amplitude of the electric vector is $E_0$ and the frequency is $\omega$.

a) The radiation is linearly polarized in the $y$-$z$ plane at an angle of 45° with the $y$-axis, and it is traveling in the $+x$ direction. There are two solutions.

b) The radiation is circularly polarized in the $y$-$z$ plane, and it is traveling in the $+x$ direction. There are two solutions.

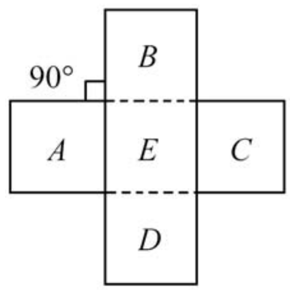
### Problem 6.4 – Linear polarizers - Malus' law + absorption

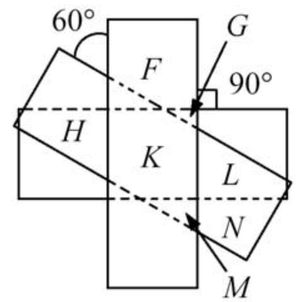
Note that you will be able to do these experiments yourself only if you have polarizers. However, you should be able to *predict* what will happen in all case knowing only the properties of polarizers.

If 100% linearly polarized light of intensity $I_0$ goes through an ideal polarizer (properly aligned), 100% linearly polarized light of intensity $I_0$ will emerge. Such a polarizer is called HN50. In real-

[1]The notation "Bekefi & Barrett" indicates where this problem is located in one of the textbooks used in 8.03 in 2004: Bekefi, George, and Alan H. Barrett *Electromagnetic Vibrations, Waves, and Radiation*. Cambridge, MA: MIT Press, 1977. ISBN: 9780262520478.

ity, polarizers are not ideal, and they come in various types with different degrees of absorption. Consider polarizers of type HN30. This means that if you do the experiment as described above, only 70% of $I_0$ emerges. Thus if we let the 100% linearly polarized light of intensity $I_0$ go through two of them (both correctly aligned), only 49% of $I_0$ will emerge. Look through one of your linear polarizers at a light source. Then place a second linear polarizer on top of the first (same orientation). You will notice that the light intensity decreases. Place two polarizers at right angle (see the figure on the left). Let us define the light intensity of the 100% linearly polarized light coming through the areas $A$, $B$, $C$ and $D$ as $I_0$. The light intensity through area $E$ is now zero. Now stick your third linear polarizer between the two (see the figure on the right), and rotate this third polarizer in its plane while keeping the original two at 90 degrees. The darkness at $E$ will disappear. Try it!!!




What are the light intensities through the areas: $F$, $G$, $H$, $K$, $L$, $M$, and $N$? Notice the angle of 60 degrees. Take into account that your polarizers are of type HN30.

**Problem 6.5 (Bekefi & Barrett 4.1) – Radiation from an accelerated charge**

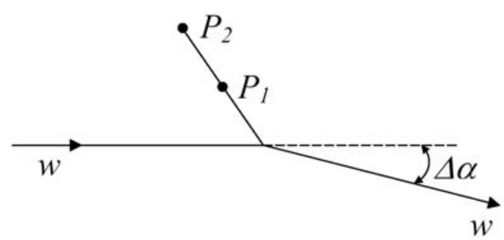
A charge, $q$, initially at rest, is given a brief acceleration along the $z$ axis. Let this occur at the origin of the coordinate system at $t = 0$.

a) Give the arrival times, the relative strengths, and specify the directions of the radiated electric field seen by three observers in the $y$-$z$ plane at a large distance $R$ from the origin. One observer is on the $y$-axis, one on the $z$-axis, and one is on a radius making an angle of 30° to the $z$-axis.

b) Describe the orientation and magnitude of the associated radiated magnetic fields.

**Problem 6.6 (Bekefi & Barrett 4.2) – Radiation from an accelerated charge**

A point charge $+q$ has been moving with constant velocity $w$ along a straight line until the time $t = t_0$. In the short time interval from $t = t_0$ to $t = t_0 + \Delta t$, a force perpendicular to the trajectory changes the direction without changing the magnitude of the velocity. After the time $t = t_0 + \Delta t$ the charge again moves with velocity $w$ along a straight line forming a small angle $\Delta\alpha$ with the initial trajectory.




a) What is the direction of the electric field caused by the acceleration, at the distant point $P_1$?

b) In what direction is the radiation from the accelerated charge most intense?

c) Where is it least intense?

d) Point $P_2$ is twice as far from the bend in the trajectory as $P_1$. By what fraction does the amplitude of the magnetic disturbance decrease as the radiation pulse moved from $P_1$ to $P_2$?

e) What is the total energy radiated?

Make careful sketches in answering parts (a), (b), and (c).

MIT OpenCourseWare
http://ocw.mit.edu

8.03SC Physics III: Vibrations and Waves
Fall 2012

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.