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8.03SC **Fall 2012**

Notes for Lecture #16: Electromagnetic Waves and Conductors

In discussing the interaction of electromagnetic waves with conductors, we first note that no electric fields can exist inside an ideal conductor. This means that when an electromagnetic wave hits an ideal conductor, it will be reflected. How this happens is determined by boundary conditions.

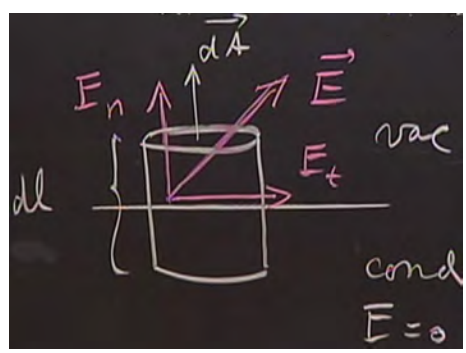
The fact that an electric field cannot exist inside a conductor also has implications for magnetic fields: although a static magnetic field can exist inside a perfect conductor, a changing magnetic field cannot. Since the curl of $\vec{E}$ is $-\dfrac{\partial \vec{B}}{\partial t}$, if $\vec{E}$ is zero and thus has no curl, $\dfrac{\partial \vec{B}}{\partial t}$ must also be zero.

Of course, real conductors are not ideal (conductivity is not infinite). Near the surface of a real conductor, the electric field falls off exponentially with depth into the conductor. The scale over which the field amplitude decreases by a factor of e is called the *skin depth*, denoted as δ (**2:00**). The skin depth depends inversely on frequency ω and conductivity σ as $\delta = \sqrt{\dfrac{2}{\omega \mu_0 \sigma}}$.

For an ideal conductor (with infinite conductivity) the skin depth is zero. For an extremely good conductor like copper (Cu), the conductivity is $\sigma = 5.8 \times 10^7$ $(\Omega \text{ m})^{-1}$ in SI units. With this conductivity and 1000 MHz frequency (30 cm wavelength), the skin depth is only 2 μm (2 microns). For optical light at 5×10^{14} Hz, one finds a skin depth in copper of only 3 nm, or roughly 30 atomic distances. This is much smaller than the typical wavelength of visible light ($\sim$500 nm).

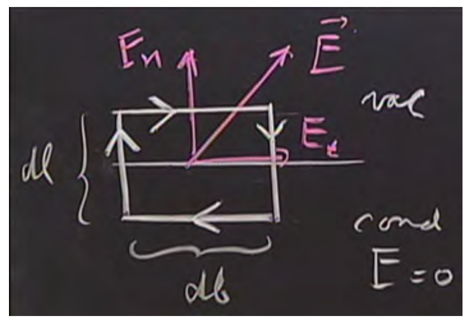
Consider the boundary conditions at the surface of an ideal conductor. The electric field follows $\nabla \cdot \vec{E} = \dfrac{\rho}{\epsilon_0}$ where ρ is the charge density in Cm^{-3}. In integral form, the electric flux emerging from any closed surface is related to the charge inside it by $\displaystyle\int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \int_V \rho \, dV$ where the "enclosed" charge is the integral of density over the enclosed volume, V (**5:00**).

Near a smooth enough surface, it is useful to consider a "pill box" for application of Gauss' Law. The pill box has its bottom inside the conductor and its top in vacuum. The height in total is taken to be dl. The outward-pointing perpendicular area vector is $d\vec{A}$. The electric field is decomposed into normal (E_n) and tangential (E_t) components with respect to the surface.



Gauss' Law for electric fields can now be applied to this pill box. The electric flux escaping is $E_n dA + \cancel{\phi_E}$, which represents the normal component out through the top cap of the pill box, plus the tangential flux out the sides, and already takes into account that there is no electric field inside the conductor, thus no electric flux through the bottom cap. The crossing out of the ϕ_E term represents the fact that we will take a limit in which the height of the pillbox goes to zero, so that the area of its sides also goes to zero (**8:00**). This leaves $E_n dA = \frac{1}{\epsilon_0}\rho dl dA$.

If the electric flux through the side goes to zero when dl goes to zero, does the right hand side not also go to zero? In fact, the charge is expected to be concentrated on the surface, so that the "enclosed" charge is more precisely determined by the surface charge density (ρ_s, often called σ, in Cm^{-2}) rather than ρ in Cm^{-3}. When $dl \to 0$, $E_n dA = \frac{\rho_s}{\epsilon_0} dA$ or $E_n = \frac{\rho_s}{\epsilon_0}$. This is our first boundary condition (**9:00**). Note that this relation holds for changing electric fields: the surface charge will immediately change to compensate for a change in electric field. Similarly, due to the absence of magnetic charges, the normal component of a changing magnetic field at the surface must be zero (the second boundary condition): $B_n = 0$.



Faraday's Law tells us that $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or the integral form $\oint_{\text{closed loop}} \vec{E}\cdot d\vec{s} = -\frac{\partial \phi_B}{\partial t}_{\text{open surface}}$. In this, the "open surface" is *any* open surface attached to the closed loop but is usually taken to be a surface in the plane of the loop (**12:00**). As before, the electric field is decomposed into E_n and E_t. A closed loop of length dl perpendicular to the surface, length db parallel to it, and centered vertically on the surface, is considered. We can proceed around the loop starting at the left middle, taking $\vec{E}\cdot d\vec{s}$. The left hand side contributes $E_n \frac{dl}{2}$ from the upper vacuum side. On the top, it is E_t that contributes along the length db, giving $E_t db$. On the right hand side, E_n is antiparallel to $d\vec{s}$, so that the contribution is $-E_n \frac{dl}{2}$. The remainder of the loop is inside the conductor where the electric field is zero, so does not contribute to the integral. Thus $E_n \frac{dl}{2} + E_t db - E_n \frac{dl}{2} = -\frac{\partial \phi_B}{\partial t}$. If we consider the case $dl \to 0$, then the loop has no area, so the flux through it must be zero (and thus so must its rate of change be zero). Further, note that the terms involving E_n cancel out. This leaves simply $E_t = 0$ as the third boundary condition. In reality, the electric fields exponentially decay over a characteristic length which is the skin depth (**15:45**).

To handle the case of the curl of $\vec{B}$, one does a closed loop integral of $\vec{B}\cdot d\vec{l}$. This gives rise to the final boundary condition, that $|B_t| = \mu_0 |J_s|$, where J_s is a *surface current* density with units A/m. We saw before that the charge density will adjust to meet the boundary condition on the normal electric field: this last condition means that there will also be a varying surface current density related to the tangential magnetic field. There is a lot going on at the surface of a conductor when an electromagnetic wave hits it! We can summarize the boundary conditions as

$$E_n = \frac{\rho_s}{\epsilon_0} \qquad B_n = 0$$
$$E_t = 0 \qquad |B_t| = \mu_0 |J_s|$$

We start with the simple example of an electromagnetic wave propagating along the z direction and polarized so that its electric field is along the x direction (**18:00**). The magnetic field will be along the y direction so that $\vec{E} \times \vec{B}$ is in the direction of propagation. The incident wave is taken as $\vec{E}_i = \vec{E}_{0_i} \cos(kz - \omega t)\hat{x}$ so that the wave vector k is along the $+z$ direction. This is infinite in all directions, but the electric vector points along x, its value changing with z and with time. For a perfect conductor at some point along the z axis, the incident electric field will only have a tangential component which must be zero at the conductor. This is similar to the boundary condition for a wave in a string incident on a fixed end, in which a change of sign in the reflected wave took place: "a mountain came back as a valley".

We choose to put $z = 0$ as the point where the conductor is. Then the reflected wave, with the mountain becoming a valley, and moving in the opposite direction, is $\vec{E}_r = -\vec{E}_{0_i} \cos(kz + \omega t)\hat{x}$. The sum of the incident and reflected waves gives $\vec{E} = 0$ at $z = 0$, thus satisfying the boundary condition. The sum of the two cosies can be simplified using $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$ to give $\vec{E}_{tot} = -2E_{0_i}\sin(kz)\sin(-\omega t)\hat{x} = 2E_{0_i}\sin(kz)\sin(\omega t)\hat{x}$ (**21:30**).

This is a standing wave. All the spatial information is in the first sine term, and all the temporal information is in the second. If $z = 0$, $\vec{E}_{tot} = 0$ for all times as required. This being a standing wave, there are nodal surfaces perpendicular to the z axis in which the E vector is zero at all points. These are separated by $\lambda/2$, where λ is the wavelength. The conductor is a nodal surface (as required by the boundary condition) but on the side of the incident wave there are infinitely many nodal surfaces. In between the nodal surfaces, there is a sinusoidal spatial variation of the electric field amplitude, and its temporal variation is also sinusoidal. The electric field vector always points along the x axis, and is the same at all points (x, y) for any given z.

We have looked at the electric field of this standing wave, but there must also be a magnetic field. Recalling that in a free electromagnetic wave the magnetic field is in phase with the electric field, there is a surprising result that the magnetic field in a standing electromagnetic wave is out of phase, both in space and in time, from the electric field. For example, where there are nodal surfaces in $\vec{E}$, there are antinodal surfaces in $\vec{B}$, where the magnetic field is maximum. This result makes sense in terms of energy transport (Poynting vector) which should average to zero in a standing wave. If $\vec{E}$ and $\vec{B}$ are 90° out of phase, this will always be true. Applying the procedures used previously, you can find $\vec{B}$ separately for the incoming and outgoing $\vec{E}$ and then add them.

The result is that the *total* $\vec{E}$ and $\vec{B}$ fields are 90° out of phase, both in space and time (**24:20**).

A demonstration is done with the same 88 MHz ($\lambda \approx 3.4$ m) transmitting antenna and receiver with a light bulb used several times already. The demonstration uses the fact that the blackboard has metal behind it, so the electromagnetic waves from the antenna will reflect off it (and also off walls etc. in the room, in a way that is complicated and hard to calculate). The overall result is that there will be nodal surfaces in the lecture hall, where the $\vec{E}$ field is very low or zero. Near the blackboard, the electric field is shown to be near zero. Moving through the room, other places are found where the light goes out, as well as places where it glows, although the geometry of the room means that it is not as bright everywhere (**29:40**).

The next demonstration uses a different physical setup, namely a *transmission line*. Two copper wires (barely visible in the video) 425 cm long are driven at one end by the same 88 Mhz power supply. We expect a wave to propagate on the wires but the final result may not be what you expect. There cannot be any electric field inside the wires or potential difference along the wires since they are conductors, but there could be a potential difference between one wire and the other. Surface charge, positive on one wire and negative on the other, creates electric fields following curved paths from one to the other. This situation could alternate along the wire. As the propagating electric field goes by any point on the wire, the surface charge will change to meet the boundary condition. We expect the pattern to move with the speed of light (**33:20**).

If the far end (away from the signal source) is shorted out, no electric field is possible there. This gives again a “mountain comes back as a valley” situation, like for a string with the end fixed. The shorted end is a nodal line, and there should now be a standing wave on the wires. There will be other nodal lines, spaced by $\lambda/2$, where there is no potential difference. Since it is a standing wave, these nodal points do not move with time (**36:00**).

The far end of the wires could also be disconnected. Recall that strings with a free end had a different reflection, so that a “mountain comes back as a mountain”. This is a reflectivity of $+1$, and there should be a maximum or antinode of the E field at the open end. Everything shifts by $\lambda/4$, and the new nodal lines are where the maxima formerly were.

With strings one can fix the length so as to get resonance. In the case of a fixed end, one would have wavelengths at resonance of $\lambda_n = 2L/n$ for $n = 1, 2, 3, \ldots$ while for an open end this will happen if $\lambda_n = 4L/(2n-1)$. In both cases there is a frequency related to the wave number by $\omega_n = k_n v$, and here, since we are dealing with electromagnetic waves essentially in vacuum, we expect the speed to be c. For the demonstration, the length of the wires, $L = 4.25$ m, and the wavelength corresponding to the power supply frequency, $\lambda = 3.4$ m, combine to give a system that is $1^1/_4$ wavelengths long, and therefore in resonance when the end is open (**40:00**). Starting at the open end, there should be a nodal line $\lambda/4$ away, another nodal line $\lambda/2$ further along, etc. A light source with extremely high ohmic resistance is suspended between the wires. The high resistance means it will not disturb the field between the wires. It can be slid along, and will light up even with a minutely small current. Sliding the light along clearly shows the expected standing wave pattern. If the far end is shorted out, the resonance condition is no longer met. The former nodes now have a electric field, not as high as at resonance, but still present (**44:35**).

The next demonstration is a 127 m long coaxial cable, which has one conductor inside the other embedded in an insulator, wound up in a coil. Voltage pulses 100 ns long (100×10^{-9} s) are applied at one end. Again we expect that if the far end is shorted out, "a mountain will come back as a valley", whereas if it is open, "a mountain comes back as a mountain". Three readouts are shown on an oscilloscope: the input, the far end, and the return pulse (**47:45**). It can be seen that for a closed end, an injected pulse returns 1.3 μs later as a valley. If the end is opened, the pulse comes back as a mountain, and at the end of the cable, the voltage pulse is twice as high as the initial pulse. This is just like the open string in which the end moved twice as much as the incident pulse, since the reflected wave added to it. Given the length, the round trip should take $2L/c = 0.84\mu$s. Since it actually takes 1.3 μs, the speed appears to be only $0.65c$. The explanation is that the coaxial cable's insulator has a dielectric constant, so the propagation in the cable is actually $v = c/\sqrt{\kappa_e}$. The dielectric constant for the material used as an insulator is $\kappa_e = 2.3$ so, indeed, the speed is expected to be $0.65c$. Recall that $\sqrt{\kappa_e}$ is called the index of refraction (**51:55**).

Now, consider a 10 GHz (10^{10} Hz) radar wave ($\lambda \sim$3 cm), propagating toward $+z$ and polarized along the x axis, emitted and received using small 'horn' antennas which receive radiation polarized in the x direction. A 'comb' of parallel (but not connected) metal wires can be inserted into the beam of EM radiation, with the wires either along or perpendicular to x. If the wires are parallel to E, we expect the wave to be reflected, and if they are perpendicular the wave should go straight through. The expected behavior is demonstrated. (**56:25**). Interestingly, the conductivity in a human hand is high enough to generate a similar effect.

Edwin Land's invention of optical polarizing films uses this effect. In randomly polarized light (or any EM radiation) impinging on a 'comb' which conducts in one direction only, the part that is polarized parallel to that direction is reflected. This creates a linear polarizer since the part that passes through is purely polarized perpendicular to the wires. The comb is a linear polarizer for radar. In the case of optical polarizers, strings of molecules are aligned so that current easily flows along them, analogous to the wires in the comb.

A quite different topic related to electromagnetic radiation is *radiation pressure*. In modern physics, electromagnetic radiation is not thought of as plane parallel wave fronts but rather as bullets or "packages" called *photons*. Photons are produced by atoms or molecules decaying from an excited to a lower energy state. These photons carry momentum (**59:00**). If you throw a tomato at someone, it gives a push when it hits the victim. In the same way, light hitting an object produces a force. The energy of a photon, which we call E_n so as not to confuse with electric field, is $E_n = hf$ where h is called *Planck's constant*, $h = 6.63 \times 10^{-34}$ Js (Joule-seconds). For 10 MHz EM radiation (λ ~30 m), one photon has an energy $E_n \approx 6.6 \times 10^{-27}$ J. Even in the rather small unit of electron volt (1 eV $= 1.6 \times 10^{-19}$ J), this is the still rather small number $E_n \approx 4 \times 10^{-8}$ eV (**1:01:40**).

All electromagnetic radiation is made up of photons. A photon's momentum is related to its energy by $p = E_n/c = hf/c$. Force is related to momentum by $\vec{F} = d\vec{p}/dt$. When any object hits something and sticks, all its momentum is transferred to the object that was hit. This change of momentum over time creates a force on the struck object. For example, throwing 10 tomatoes of 0.25 kg each per second at 90 miles per hour (~40 m/s) creates an average force of $F = 10 \times 0.25 \times 40 = 100$ N at the place where they hit, easily enough to knock a person over.

This effect also applies for the absorption of light. For a beam of cross-sectional area A, the pressure P (force per unit area) is $P = \frac{F}{A} = \frac{dp}{dt}\frac{1}{A} = \frac{1}{c}\left(\frac{1}{A}\frac{dE_n}{dt}\right)$. However the rate of flow of energy per unit area, which is the part in brackets, is just the Poynting vector S (**1:05:20**). The average pressure from radiation is given by $\langle P \rangle = \langle S \rangle / c$. So, S can be thought of in terms of either $\vec{E} \times \vec{B}$ or energy flow due to photons. This equation applies for any absorbed EM radiation.

Earlier, we calculated the Poynting vector due to solar radiation reaching Earth as $\langle S \rangle = 1.4$ kW/m^2 (i.e. 1400 Js^{-1}m^{-2}). Knowing this value, one can calculate what would be the radiation pressure from absorbing this light. The result is minuscule: the force on a small area like a hand would be only 5×10^{-8} N, completely negligible compared to other forces typically present. Note that if the light is reflected, for example from a mirror, then the change in momentum, and hence the force and pressure, would be twice as large (**1:08:15**). In our daily lives, radiation pressure is never a noticeable effect. However, stars that are 20, 30, or 40 times more massive than the Sun radiate so much energy that their atmospheres are literally floating on radiation pressure, with the outward radiation force balancing gravity.

In a related effect, comets typically have two tails, an easily visible white one and a blue one that is harder to see. A comet has very small dust particles (a few micron diameter) moving with it. Radiation pressure on these particles blows them in a direction straight away from the Sun, creating tails up to several hundred million km in length. This dust tail is not blue as might be expected from Rayleigh scattering, but instead is white since the particles are fairly large compared to the wavelength of light. The blue tail is due to the solar wind, a stream of particles, mainly protons and electrons, emitted by the Sun. When trapped by the Earth's magnetic field, these particles cause the northern lights at night, but they can also interact with the carbon dioxide and carbon monoxide of the comet tail. When these molecules are excited and later de-excite, they emit blue light. Since the solar wind moves considerably slower than light, and the comet is moving rapidly in its orbit as it nears the Sun, this blue tail points at an angle from the white one (**1:11:30**).

As a test of these ideas (call it a brain teaser) consider a radiometer. This is a glass bulb with a low pressure gas inside (not complete vacuum but close). It has two crossed arms that can rotate with very low friction, and at the end of the arms are small vanes which are black on one side and white on the other. If we shine light on it, it starts to rotate. The puzzle is to figure out whether the white parts should move toward or away from the light. A demonstration shows that the white surface moves toward the light. This cannot be if the force causing the spinning is from radiation pressure because, if that were the case, the white surfaces (which reemit some of the light) would experience twice the force as the black ones (which absorb the light). The effect has nothing to do with radiation pressure! Students are encouraged to think about another possible cause...

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These viewing notes were written by Prof. Martin Connors in collaboration with Dr. George S.F. Stephans.

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