8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 19: Review and Discussion of Problem Solving

PROFESSOR: The topics for the exam are on the web. You click on schedules and you can see the topics as we cover them at the various lectures. There is no way that I can cover all of this during one exam, and there's no way that I can cover them during this exam review.

What I do not cover today can, and probably will, be on the exam. And not all that I cover today will be on the exam. I have to make a choice, and I make my choice.

I want to start with a traveling wave in a string. Let the string has tension T, mass per unit length mu, and let the traveling wave have an amplitude A. So here is a traveling wave, and this amplitude is then A. And the speed or propagation is v.

And this v is the square root of T divided by mu. That follows from the wave equation, but I'm not going to address that issue now.

I want to deal with energy. And at first we'll write down the equation for the traveling wave. If this is the x direction, and this is the y direction, then y as a function of x, t. Would then be the amplitude A that you have there. And then you can write this down in many ways. I will write it down as the sine of k times x minus vt. If you want to write down for k, v omega, that's fine too. I have no problems with that.

So the idea now, what is the energy in one wavelength say? There are two parts to it. There's kinetic energy and there is potential energy. The kinetic energy is not because the wave is moving in this direction. There is no material moving in this direction. The material is just moving up and down. That's all it's doing.

So the kinetic energy is due to the motion in the y direction. There is potential energy due to the fact that I have to give this straight wire, or this straight string, a shape. And in order to give it that shape, I have to do work. There's a tension and I have to stretch the string. That's why I have to do work to give it that shape.

So it comes in two parts. And I will only do the kinetic energy part, and I will leave you with the potential energy part. I slice out here a section dx. And so the mass in that section dm is mu times dx. And so the little bit of kinetic energy, K stands for kinetic energy, in this small part is one half times the mass-- one half mv squared-- and v is the speed, the velocity, in the y direction.

If the wave is traveling in this direction, this material here is going up. A little later it will more to the right, and so this is going up at this moment in time. So I can write for this, one half times mu times dx. And for vy I can write then, dy dt squared.

And so the question now is what is dy dt? Well that's easy, because we have the function there. So dy dt, equals. So first I get A, then out pops a k. Then out pops the-- minus-- v. And then the sign becomes a cosine. Cosine k times x minus vt. But I have to square that. So I have a square here, and I get a square here.

So dK can now be written as one half times mu. And I'll leave the dx all the way at the end. And so I'll get A squared. I will get k squared. I will get v squared. And I'll get the cosine squared of k times x minus vt. And then at the very end, I get my dx.

And I now want to know how much kinetic energy there is in one wavelength, because the total amount of energy of this is infinitely long is of course infinitely high. So I'm interested in the amount of kinetic energy for one wavelength.

And so that K then is the integral of this whole thing from zero to lambda. So that now is no longer K, but that now is K provided that you realize that it is K, kinetic energy per unit wavelength. So I only do it for one wavelength.

All right. So I get 1/2. I'm going to get here a mu. I get an A squared. I'm going to write down for k, two pi divided by lambda. So that becomes four pi squared divided by lambda squared. For v squared, I'm going to write down T divided by mu. Remember, the velocity was the square root of T divided by mu.

And now, I have to do the integral of this cosine squared, dx, between zero and lambda. And you will take my word for it that this is lambda divided by 2. And so I have to multiply. So the integral, that's the only one that I have to do, these are constant, is lambda divided by 2. And if you look now, you lose a mu. Your 4 goes and you lose even one lambda. And so you get A squared times pi squared times T divided by lambda. That is the kinetic energy per wavelength.

So I'll leave you with the potential energy. It's a similar derivation, but now you have to deal with work that you have to do. And what comes out is perhaps a bit of a surprise. That the kinetic energy is exactly the same as the potential energy. Not all obvious, but that's the way it works out.

So that means that the total energy per wavelength, so now I write down E total, again per wavelength. wl stands for wavelength. Not for Walter Lewin, but for wavelength. The total energy per wavelength is now twice that. So it's 2A squared times pi squared T divided by lambda.

Now suppose I am generating this wave. I'm standing somewhere and I'm wiggling this. Then I have to generate this energy for every period that I go through, because in one period I generate one wavelength. So if I divide this by the time for one oscillation, then I have the average power over one oscillation. Now the period of an oscillation we normally call T. But I don't want to have another T, because we already have a T, so I call the period of the oscillation 2 pi divided by omega. Which is the same as the period.

And so the power then, the average power that I have to generate if I am driving this travelling wave into the string, the average power is then E total. You have that here. Divided by 2 pi by omega. And this is in watts of course. This is joules per second.

Now we turn to standing waves. The situation is very different. So here, I have a standing wave. There are nodes here, and at those nodes the string does not move. Let's assume that the maximum displacement is A. Again, the tension is T. Mu is the same. But when it comes to a halt, the maximum displacement in the center here is then A.

And the question now is, how much energy is there in a standing wave? This was a traveling wave. This is a standing wave. Well at this moment in time, when it comes to a halt, there's no kinetic energy. There's no movement in the y direction.

So the total energy that is in the wave per wavelengths is the same as U maximum. But of course, it's also the same as K maximum. When this thing goes through its equilibrium, when the string is straight, then you have the highest velocity. This comes down and has a velocity in this direction. This comes up and has a velocity in that direction. That is then the maximum kinetic energy that you can have.

So E total is U max is K max. And so, we already know what U is. So all we have to say now is that is the same as A squared times pi squared times T divided by lambda. And so you see, a traveling wave has the same amplitude as a standing wave. A traveling wave has twice as much energy per wavelength.

And as far as power is concerned, well, you have two waves going through each other. And so you have a reflection from the other side. So you really don't have to do anything anymore to drive the system. You just let it sit. If there's no energy dissipation, the standing wave will just support itself.

So you do not have this power that you have to continuously put in, because in the case of a traveling wave you are continuously generating a wave that moves away from you. That is not the case with a standing wave. It's generated at one point in time. It reflects and it maintains itself.

Now let's turn to electromagnetic traveling waves. Electromagnetic traveling waves, I take plane wave solutions in its most general form. E as a function of x, y, z, and t can be written as an amplitude, but this is in three directions. So it has an x component, a y component, and a z component.

And then I could write down here say, for instance, the cosine of omega t minus k dot r. It's a dot product. And the meaning of k, k is called propagation vector, is kx f roof-- x roof plus ky, y roof, plus kz, z roof the magnitude of k is the square root of kx squared plus ky squared plus kz squared.

Lambda is two pi divided by that k, and omega equals k times v. And if it is in a vacuum, then v is the same as c of course. But v then is c divided by n if you have a dielectric.

The index of refraction n for dielectric is then square root of kappa e divided-- times kappa m. Most substances, kappa m is very, very close to 1.000. Except for ferromagnetic magnetic materials, kappa e itself is frequency dependent. And it can sometimes be extremely, strongly, dependent on frequency. That's where the dispersion comes in.

R equals x, s roof-- x roof plus y, y roof plus z, z roof. That is the position vector. If you wanted to know what the associated magnetic field is, associated with this electric field, then your best bet always is that the curl of E is minus dB dt. So, if you know that whole function, in x, y and z, you can take the curl of E. Sometimes it's time consuming. Sometimes it's fast, depends on the wave. And then you have to do an integral in time to get the B vector.

And what comes out of this is actually something that I do remember. There's very little in physics that I remember. This is one of the things that I remember so I don't have to apply every time Maxwell's equations when I solve these problems. And what I remember is what I wrote down earlier, point by point, on the blackboard.

For traveling waves, E is perpendicular to B. E and B are both perpendicular to k. So E and B are perpendicular to k. Therefore, E cross B-- let me give that unit vectors. E cross B is k roof. The Poynting vector, remember, goes in the direction of k.

The magnitude of B, at any moment in time in a traveling wave, is the magnitude of E divided by v which can be c, of course. And then, and this is key and I want to stress that, E and B are in phase with each other. In phase in space and in phase in time.

And what that means is the following. The location where E reaches a maximum is also the location where B will reach a maximum. And they reach it at the same moment in time.

So if you are somewhere, and you find that the E vector reaches a maximum, then you know that the B vector at that moment will also reach its maximum. If you're somewhere in space where E is 0, you can be sure that B is also 0. That is the case for traveling waves.

The Poynting vector, s, is E cross B divided by mu 0. And if there were magnetic material, you would also have a kappa m here but I will leave that out. For the magnitude of B you can always write down E divided by v. This Poynting vector, of course, is time variable. Because E itself varies with frequency omega. B varies with frequency omega.

So the Poynting vector is, of course, time dependent. And so the magnitude of the Poynting vector, you could also write down then, as the E squared value divided by mu zero divided by v. Because the B can then be replaced e divided by v. Since they're at 90 degrees relative to each other, I can ignore the cross because the sine of the angle is then 1.

If E is a sinusoidal function. For instance, E0 times cosine omega t. Then it's clear that you get here the square, E0 squared, times the square of cosine omega t. If you time average that-- this s is variable in time, but if you just want to know what the time average value is then the time average value of the cosine squared equals 1/2.

And so you can also write down then that s-- I'll write it down here-- s time averaged would then be E0 squared. This is now the amplitude divided by two mu 0 times v. And the two comes from the fact that the average value of cosine squared omega t equals one half. And this is then in watts per square meter. So that's the traveling wave.

Now I want to go to standing waves. A standing electromagnetic wave, just like the one on the string, has a separation of space and time. Let us suppose we take a specific example. We have here a coordinate system. I will call this x. Call this y. Call this z.

Suppose we have linearly polarized radiation, that I'll make it a little bit more difficult than normal, I will linearly polarize it in the yz plane. In other words, the E vector would be like this then, and oscillating back and forth in this plane perpendicular to k.

So the standing wave, E, as a function of x, y, z, and t, can then be written as this vector. This here would be E0y at its maximum. This would be E0z at its maximum. So that's the one that comes first. So you get E0y in the y direction plus E0z in the z direction. That is the direction of that e vector.

And now you get your spatial part, in the x direction, for which you can write down either the sine of kx times x. Or, if you prefer, cosine. I have no problem with that. And then we have here cosine omega t.

There is no dependence of E in y and z, the plane waves. So if you take any plane perpendicular to x, infinite in size, independent of y and z, the E vector will be the same. And so, therefore, k of y is 0 and k of z is also 0.

So there's only a k of x. And so k then equals k of x. And so, lambda is two pi divided by this value then. And omega divided by that k, omega divided by this k value-- I write an x but you can leave the x out-- would then be v or it could be c of course. If you're interested in knowing what the B field is you know that the curl of E is minus dB dt.

The fact that we have here the spatial part, separated from the time part, means that there are locations for x which never change provided E field is always 0. That's typical for a standing wave. And in this case, there will be nodal planes. The whole plane perpendicular to the x direction, I will draw one here, and I will draw another one here.

The E fields in that whole plane, at all moments in time, will be 0. That's the case when this sine, or whatever the cosine you've chosen, happens to be 0. And so these are nodal surfaces. And of course, the anti-nodal surfaces fall right in-between.

It is not so easy to draw now for you what you will actually be seeing in terms of this sinusoidal wiggle that goes like this. That is not so easy to do, because the wave will be in the plane that is coming out like this. And it's difficult to show you that in a three dimensional way. I will make an attempt, nevertheless.

So it will be in this plane, that is tilting forward, that you'll then have nodes and anti-nodes and they oscillate like this. Any plane perpendicular to x, at any moment in time, will see exactly the same thing.

Now it's interesting to compare our list we suspend with the traveling wave. Again, with the standing wave, E is perpendicular to B. It is not too useful to talk about direction of propagation, because there are really two waves going through each other. So the whole idea of k vector is a little bit bizarre.

B however, the magnitude of B, the maximum value will be E divided by v. The maximum possible value of B is E divided by v. But now comes the big surprise. E and B are 90 degrees out of phase in space and time.

So if you have here the nodal planes for E those are the anti-nodal planes for B. And where you have the anti-nodal planes for E you would have the nodal planes for B. So they are 90 degrees out of space and out of time. So no surprise, of course, that the average value of the Poynting vector is then going to be 0. And that, of course, can also be seen.

When you think of it that there's a traveling wave in this direction and a traveling wave in this direction. So there's sort of energy flow in this direction and energy flow in this direction. And the time average will obviously end up to be 0. But if you simply take that E cross B, and you time average it, you will see that immediately. This is a consequence of the 90 degrees out of phase and 90 degrees-- in space and time.

All right. Now I would like to pursue this. I can either clean the blackboard or let me go to the other side. I'm going now to accelerated charges.

So here we'll call that the y-axis quite arbitrarily, and here we have the z-axis. And I accelerate a charge q here with acceleration a. And I do that just in the z direction. And I am watching at a certain distance. My position vector is r and I'm watching here. And this angle is theta.

If this charge is a positive charge, and there is a sudden acceleration, a little later in time I will see an electric field. There's a traveling wave going in my direction. It's not a plane wave, but there is a traveling wave going in my direction in the direction r. And so, that E vector then is in this direction. I will return to this.

It's in the opposite direction as a perpendicular. This vector, which is the component of a, perpendicular to r is called a perpendicular. If q is positive, then this E vector is in the opposite direction direction of a perpendicular.

The E vector that you experience at time t is then minus q times a perpendicular at time t prime. Because the acceleration took place earlier than when the signal arise where I am, because it takes time to travel. In other words, t prime equals t minus rc. This is the time that it takes for the signal to reach me. And so, t prime is earlier than t.

And then we have this divided by 4 pi, epsilon zero, r, and I believe there is here c squared. Yes there is. And this is in volts per meter.

Now a can easily be some amplitude times the sine of omega t. In other words, I can oscillate this one up and down harmonically. Clearly, when I do that, I will get an electric field that will also oscillate here harmonically.

You really do not get plane wave solutions, because in any direction that you look you can change the r vector. And you have here the answer of what you see, the E vector. So there's really no such thing as a really a plane wave solution. If you're far away you can probably approximate it by a plane wave.

The associated B field has all the ingredients that we are familiar with. It's perpendicular to E. And the B field must also be perpendicular to the direction of propagation. Which, in this case, is r. And E cross B will also be in this direction.

And if you take all that into account, then you can write the B vector as the unit vector in the r direction times E divided by c. If it is the speed of light, c, if it is in vacuum then that is then an r, t. But that's the connection between E and B.

So now comes an interesting point that we have stressed before. And that is that a perpendicular depends on theta, a perpendicular is a times the sine of theta. It is this component. And so you see that the magnitude of the E vector, that you will see when you look at that charge, will depend on where you are in space.

If you happen to be at theta 90 degrees, you will see the maximum E possible. If you happen to be in this direction, where theta is 0, you won't see any electric field. And so it has a very strong dependence of the magnitude of the E vector in the direction.

And the same is true for the B vector, because the B and the E vector are always married to each other. So they get the same sine theta. So the Poynting vector is now going to be proportional to sine squared theta, because you have an E and you have a B.

I would like to summarize for you, in the same way that I did that here, what I would like you to remember. It's simple. I will raise this later again, because I want to work at a height so that you can see what I'm doing.

So in summary, which is very good to remember, is that E is in the plane through r and a. In the plane does not mean into the plane. It's in the plane. So look here, the plane through a and r is the blackboard. And therefore, the E vector is in the blackboard in this case.

That is key. E is in the plane through r and a, not into the plane. And E is perpendicular to r. Notice I have that. And E amplitude is proportional to the amplitude of E perpendicular. And therefore, it is proportional to the sine of theta.

E itself is proportional to one over r. Notice that, not one over r squared but one over r. Which is obvious, it has to do with the conservation of energy. If you make your sphere around it, then the energy that flows through the sphere must be the same no matter how large your sphere is.

And since that energy comes from the Poynting vector, as an E cross B, E is inversely proportional with r and B is inversely proportional with r. So that energy is conserved. So the Poynting vector will then be inversely proportional to r squared.

If you're interested in the total power-- so you're not interested really in the fact that the Poynting vector is a strong function of theta, namely sine squared theta-- but if you integrate the Poynting vector over one sphere that you choose and you take the local Poynting vectors and you multiply that by the local areas, which becomes an integral, then you can actually come up with a power.

This is now in joules per second. In other words, you tell me what q is. You tell me a is. You are creating electromagnetic radiation. I will tell you how many joules per seconds work you have to do.

And that then becomes-- that's called the Larmar result, the Larmar equation. And that becomes q squared times a squared divided by 6 pi epsilon 0, c to the power of 3. Notice there's no r anymore, because clearly it's independent of where you are in space. That's the how many joules per second you have to generate.

It's obvious that it is proportional to q squared because E is proportional with q. So B is also proportional with q. So the Poynting vector will be proportional to q squared. It's also obvious that there is an a squared, because E is proportional to a perpendicular but nevertheless a. And B is also proportional to a. So it's no surprise that you get there upstairs a q squared and an a squared.

Now if a is oscillating, if is a0 sine omega t or cosine omega t, then you can calculate what the mean value of the power is during one oscillation. And then you will get here the average value of the sine. Squared omega t then becomes 1/2 of course.

This is the energy that you have to generate per second in order to create electromagnetic radiation. It is not the kinetic energy that you have to put in the mass of the charge, the 1/2 m v squared. That's a whole different story. This is the price you pay for creating electromagnetic radiation.

Several students have come to my office and asked me, why is it-- apparently I wasn't clear enough-- why is it that when you have Rayleigh scattering, and when the light scatters at 90 degree angles, that even if you have unpolarized light that you start with. Why is it 100% linearly polarized if, and only if, it scatters over 90 degrees.

And so that is indeed an important point. So I'll expand on that in a way that is perhaps easier for you to digest. Maybe I went over it too fast when I cover it in class. I demonstrated if you remember. I did it with a smoke signal and I did it with a sunset.

So we have unpolarized radiation. Let's first agree what is unpolarized radiation. There's a beam of light coming straight at you and it's unpolarized. The first plane wave, I think of it still in terms of a very classic 19th-century idea of plane wave solutions. The first plane wave is linearly polarized like that. The second one is linearly polarized like that. Then there's one like this, then there's one like this, then there's one like this, then there's one like this, one like this, and one like this. It's a zoo of everything that's unpolarized radiation.

OK. I pick one of those right here, this one, and he happens to be fine dust particles which have electrons. And these electrons are shaken up and down by that electric field. So what I'm drawing now here is the motion of that electron. The electron is going to be accelerated in a harmonic fashion.

And where are you? Well you are here. You're looking, and this angle is theta. The same angle theta that I had there. What will you see? E is in the plane through r and a. So it's in the plane of the blackboard.

So radiation comes straight at you. But here is all of a sudden this electron, a bunch of electrons, that go like this. And I'm looking. I happen to be sitting in the plane of the blackboard, because that's 90 degrees right? When the radiation comes like this then the blackboard is 90 degrees. Whether you here it's 90 degrees. This is 90 degrees. This is 90 degrees. That's also 90 degrees.

This is not 90 degrees, but that all is 90 degrees. Radiation comes in like this and I'm looking there. But it is in the plane through r and a so it is in the blackboard. It is perpendicular to r. That's nice. So it's linearly polarized. Do we agree? Linearly polarized?

Now there's another one that comes in. This one comes in. OK. There it is. This one comes in. Starts to shake the electron in this direction. I am still where I was before. I'm here, same location.

This angle now is theta. For a second plane wave comes in like this, what do I see here? E vector is in the plane through r and a. That is the blackboard. E is perpendicular to r. So I see this.

Do I see the same strengths of the E vector? No, it's much less because a perpendicular is much less because theta is much less. But I will still see E vector in this direction following my recipe.

And then there's one electromagnetic wave that comes in which happens to be in this direction. Well, in that case I will see nothing. That is tough luck. But any other direction you will always see electric vector perpendicular to your line of sight. And that is only true for 90 degrees.

In other words, if I made a circle here-- and I have unpolarized light coming in here straight at you-- when you're here and you look down you will see the E vector like this. When you're here, and you're looking down, you will see an E vector like this. If you look here, and you look in this direction, then you will see the E vector like this. And this is only true if you are at 90 degree angles.

I can easily make you see that if you change the angle that it is not 100% linearly polarized. And the best way for me to show that is here are these electrons which are being shaken like this. One comes in. Shakes like this. And let's look now at forward scattering, not 90 degrees. This is 90 degrees. Forward scattering, here's the electron going up and down. You're sitting there in the audience. If this one comes in you'll see an electric vector like this generated by this charge which goes like this.

Now the next one comes which goes like this. You'll see an E vector like this. The next one will come in like this. Forward scattering, you will see like this. So you see, if unpolarized light comes in forward direction it remains 100% unpolarized. In any other direction it will be partially polarized, but in the 90 degree direction it is 100% linearly polarized.

That is the reason why the sky 90 degrees away from the sun is 100% polarized. And I check that almost every other day to make sure that physics still works. It's great fun. You take your linear polarizer, I always carry a handful with me, you look at the sky 90 degrees from the sun. And indeed, the sky is practically 100% polarized. Quite amazing.

I have a movie which is not the greatest movie. I tried to show this to you earlier. It's not the greatest. It's trying to make you see that when you oscillate charges back and forth that something happens in the E field. That you get in the E field kinks.

And students have asked me more than once, why do you only get kinks, and therefore electromagnetic radiation, if you accelerate them? Why don't you get kinks if you simply move them with a constant velocity?

And I think the best answer that I can give is the following. Think of the electric field lines as spokes, rigid spokes, which are attached to the charge. So they're attached to the charge. They are rigid, but they are fragile like spaghetti. But they are rigid.

And so they move with constant velocity. So all the spaghetti moves with the charge. They all have the same velocity. There is no stress anywhere on the spaghetti, because the whole thing has the same velocity. Now all of a sudden I take the charge and I accelerate it. Now the spaghetti feels the kink. The spaghetti feels, all of a sudden, it wants to break because the spaghetti is very fragile.

And that break caused a kink in the field line. So that's the best way that I can convince myself why a constant velocity does not cause electromagnetic radiation, but it's really the acceleration.

Some students thought when I used the word spaghetti that I meant cooked spaghetti. No, I didn't mean cooked spaghetti. I meant uncooked spaghetti which is very brittle. Which easily breaks just like this. And when you accelerate all of a sudden the charges, these field lines can break like uncooked spaghetti.

And this movie is making an attempts. It's not the best thing, but we'll make an effort. So Markos, if you can do the honors there then I will do the honors here. And you may not get much out of it, but at least it is an attempt. Were we going to give it TV? Or are we going to make it completely dark? TV?

If you wait a second then I think the part that I want to show. OK, so oscillating charges. So it oscillates but the speed is constant. So there's instantaneous fast acceleration, and then the speed remains constant. You see these shells here, which are then representative for the electromagnetic radiation. They're not harmonic yet. We're going to shake them harmonically shortly.

And so these lines that you see are the field lines, and they're just here in these shells. They are broken. It's broken spaghetti. Also know that this is like a spherical wave. It's nothing to do with plane wave solutions anymore. You've really got a spherical wave going out.

It's not so clear from here that the strength of the wave is very different for the different directions of theta. That's not so clear. So you here you have a simple harmonic motion. This is about one hertz electromagnetic radiation. Have you ever heard of that? One hertz? We're dealing normally with megahertz and gigahertz.

This is very slow motion, but it's simple harmonic. And you begin to see that, indeed, there are electromagnetic. These are the E field lines. That they are being distorted in the direction perpendicular to the direction of your line of sight. Is it a great attempt? No, but it is an attempt.

So I think this is a good moment for a break, perhaps a little bit early, and we will reconvene in five minutes. So you can warm up.

OK. I will now tell you what I will not talk about, which is a lot. I will not discuss Fourier analysis. That doesn't mean that it will not be on the test. Make sure you're familiar and comfortable with the examples that I had in the problem sets.

I will not cover today Doppler shift. Make sure you feel comfortable with Doppler shift. It's not a very difficult subject, but it is very far reaching consequences as we discussed including cosmology and black holes.

I will not discuss Fresnel equations, even though they are very crucial. They were at the center of my lecture earlier this week. I will not discuss Snell's law. I will not discuss today the Brewster angle, but don't be surprised, don't be shocked, if there is a problem related to the Brewster angle.

I will not discuss critical angles, total internal reflection, but it may be on your test. And I will not discuss today radiation pressure. That doesn't mean that it will not be on your test. It's simply not possible to cover all of this in the available amount of time.

Now of course, I have not left out purposely things that will be on the test. A lot of stuff that I have covered today will be on the test, of course. But there will also be stuff that I cover today that will not be on the test.

I want to discuss now one of the bizarre dispersion relations that evolve from the boundary conditions of electromagnetic wave on ideal conductors. And we spent a lot of time on the demonstration whereby we had two parallel conducting plates. And these plates were separated in the x direction by a distance a.

This is the y direction. This is the x direction. And this is the z direction. And I want to propagate through these plates, which are infinitely large in the y direction, infinitely large in the z direction. At least very much larger than the wavelengths.

I want to propagate electromagnetic radiation which linearly polarized only in the y direction. So that is what I want to do, and that's what I demonstrated also. Well, k-- this is beginning to be boring now-- is kx times x plus ky, y roof plus kz, z roof.

This by the way is zero, because there's no dependence of the E field in the y direction. So ky is zero. So k is the square root of kx squared plus kz squared. Omega is k times v. Let's assume this is in vacuum. So omega is k times c.

The boundary conditions demand that at x equals 0 and at x equals a, this component must vanish. Because you cannot have an electric field in the surface of an ideal conductor. That was one of the boundary conditions that we derived. In other words, E of y must become 0 for x equals 0 and x equals a.

And as a result of that, you're going to get a standing wave in the x direction and you're going to get a traveling wave in the z direction. And the boundary conditions demand now, in order to meet this, that the k of x is going to be m pi divided by a. Then you have your wave that you will see substituting for kx. This value will always give you then, if you have the sine of kx times x, always a 0 here and always a 0 there.

And so omega will then be c times k. So it's c times the square root of m pi over a squared plus k of z squared. And this is what we call a dispersion relationship. This equation is responsible for a bizarre behavior. And the bizarre behavior then can best be shown in a diagram which we call the omega kz diagram.

I will raise this later again, because I want to work over my head so that you can see what I'm doing. So this is kz and this is omega. And I'm going to plot for you this relationship. This line would be omega equals kz times c. That will be non-dispersive.

However, this is different because of this. And so now you have here a frequency which is the lowest frequency possible for which radiation can actually go between the plates. And this then is the case for Mary equals 1, and omega is then c times pi divided by a.

And so here for omega, it is the cut off frequency is pi times c divided by a. And if I plot then that curve you get something like this. So no frequency below that value can propagate through the gate so to speak, through the opening, because it cannot meet then the necessary condition that the E field becomes 0 here and 0 there. Which is non-negotiable. It has to become zero here and it has to become zero there, because it's an ideal conductor.

So k of x must obey this boundary condition. So let's assume that at a particular moment in time we have a frequency going through these two places. The frequency of this linearly polarized radiation in the y direction, and that this is the value for omega. That means that the associated value for kz is then this.

kz must adjust itself so that kx can remain what it has to be. This line, everywhere on this line, kx is the same value. And the kx value everywhere is pi over a. And so kz is being slaved to become the value that meets this dispersion relation. That's this line. So kz settles there.

The phase velocity in the z direction, v phase in the z direction, is omega divided by kz. Well, look what that means. Omega divided by kz.

So I can draw a line here. And so, this angle is an indication for the ratio omega divided by kz. Which is larger, this angle is higher than this one, and this was c, remember? And so you see that that value is always larger than c. You can just see that by looking at the graph.

The group velocity in the z direction is d omega dkz. And d omega dkz is the tangent along this line here. So at this point here, the tangent would be like this. I don't want to draw another line, because it becomes too cluttered. But you can see that this slope is smaller than the line here which indicates the c. And therefore, the group velocity is smaller than c. So this is smaller than c.

If you lower omega, and gradually reach your cut off frequency below which you can no longer propagate any radiation between the two plates, then you get a situation which becomes even more bizarre. That when you reach that point nothing will go in the z direction anymore. Nothing will come out. I demonstrated that.

The group velocity will therefore have to become zero. Well, you can see that because the tangent on this slope here becomes horizontal. So the group velocity, indeed right here, is zero. But the phase velocity is infinitely high.

Think about it, because kz now becomes zero. And so you get an infinitely high phase velocity. And I spent quite some time during my lectures to explain to you why that has no physical meaning. I don't want to go over that now.

There is no such thing as resonance frequencies. Often you people think that these are resonances. No, it's not a resonance at all. It is a mode. It means that if you have radiation at this frequency, that in the x direction, kx will adjust itself such that you get a standing wave in the x direction. In this case, you get something like this. Nice little sinusoid. Zero here and zero here. That's for m equals 1, and kz will then whatever it has to be.

And so when you change omega, kz will adjust all the time. If you go to very high frequencies, there is here another cut off frequency which is twice this number. Because that is when Mary equals 2. And when kz becomes 0, that becomes twice as high.

Then of course there are two different ways that you can propagate radiation in the z direction. Because now you have here the line, and if now your omega is high enough that you get an intersection with this line, and you get an intersection with that line. So that allows you for two different values of kx and two different values of kz.

It's not a resonance frequency. Nothing is resonating. It's a mode, but it's not a resonance mode. It's not a normal mode in that sense.

Now the most interesting thing for me is, and I stressed that when we discussed this, and I even demonstrated that. We did this with radar remember? With the 10 gigahertz transmitter the three centimeter waves. The most interesting thing is that if you reach omega equals omega c, and you can do that by-- we had a 10 gigahertz transmitter and we made a smaller and smaller and smaller.

And when a became less than lambda over 2, that means omega became less than omega c. Then no radiation will propagate through anymore. And so what we did to demonstration was we started out. We had lambda was three centimeters. And we started out with a gap of about two centimeters, and radiation went through. And the moment we get the one and a half centimeters it stopped abruptly.

The interesting thing now is that if your radiation is only linearly polarized in the x direction there is no such problem. There is no such thing as this crazy dispersion relationship, because an E vector being perpendicular to this plate, and being perpendicular to this plate, is no problem.

If the E vector oscillates with frequency omega always perpendicular to those two plates the only thing that happens that nature like crazy adjusts the local rho s's-- the local number of Coulombs per square centimeter, so that you always meet the normal boundary condition. But nothing ever has to become zero. And so therefore, there is no boundary condition that gives you rise to such a crazy dispersion relationship.

So if you had radiation linearly polarized in this direction, then it would follow this line. Non-dispersive, phase velocity is c. Group velocity is c. So now comes the interesting part.

Suppose you manage to get radiation which is unpolarized. And this you try to send through there. And you make a smaller than lambda over 2. That means that the vertical component of the E vector of each one of those waves cannot get through. Only the horizontal component can get through, but without any difficulty.

Speed of light, that means you have created linearly polarized light out of unpolarized. I wouldn't say the light in this case it is radar. So if you could manage to get unpolarized radar going into this direction, and you make a smaller than lambda over 2, all components in this direction are killed. Only this component can go through. And so you have created at the other end linearly polarized radiation.

And I realized that after I gave that lecture how cute that is actually. That this is a way, in principle, that you can create linearly polarized radiation simply by squeezing it through a very narrow opening, a very narrow tunnel so to speak.

The mother of all demonstrations was the sound box. That will go into history as one of the greatest demonstrations ever. This was the x direction, size a. This was the y direction, size b. And this was the z direction, and I gave that size c. Some of you thought that it was d but it really is c.

We have sound. And this box is closed on all sides. I'll write down an a here. So the box is closed on all sides. That means if we think of the pressure, there must be pressure anti nodes at all surfaces.

The particles, the air particles, cannot go beyond the wall. So they can push on the wall, they can suck on the wall, they can push on the wall and suck on the wall. That means anti-nodes in pressure. And there have to be anti-nodes on all walls.

So I can write down the pressure p, that is the pressure over and above one atmosphere, as the sum amplitude p0 times the cosine of kx times x times the cosine of ky times y times the cosine of kz times z times cosine omega t. And the reason why I already picked cosines is because I know that I want the anti-nodes when x equals 0 and when x equals a. I want the anti nodes when y equals 0 and y equals b and the same for z for the z direction.

And so the boundary condition demands that kx can only have unique values, discrete values, which is l pi divided by a. ky can only be Mary pi divided by b. And kz can only be Nancy pi divided by c. If not, then the boundary conditions are not met. And then I don't have anti-nodes at all the surfaces.

And l, m, and n can be 0 or 1 or 2 or 3. Including 0, make one of those 0, there's no sign here. If you have a sine there, you make it 0 and then everything becomes 0. But if you make a cosine 0 then it's just 1. So 0 is allowed. Accept they cannot all three be 0. You will see very shortly why.

So omega, which is always k times v-- v is the speed of sound. So omega equals v then times the square root of l pi over a squared plus m pi over b squared plus n pi over c squared. Omega has then unique values. Now we are dealing with resonance frequencies. Completely different from there. There were no resonances there. These are the unique discrete resonances l, m, and n.

And now you can see why you cannot make them all three 0. Well, because then you have omega equals 0 and that's not a very interesting thing. So now comes the question, what is the lowest possible frequency that is the lowest resonant frequency?

Well that depends on which dimension, a, b, or c, is the largest. And so if a, for instance, were 10 centimeters and if b is 20 centimeters and if c is 50 centimeters, which happens to be 0.5 meters, then clearly the lowest frequency is l 0, m 0, n equals 1. 0,0,1.

For a 0,0,1 mode I then get a frequency f. Which is omega divided by two pi. And that means I lose all these pi's here, by the way. And so I get v divided by two c. That is my c. Because look, if I make Nancy 1 my pi is gone. So I only have a 1 over c squared. This is 0. So I get 1 over c and the 2 comes from this.

So the frequency that is the lowest possible frequency is v divided by 2c, c being now half a meter, and so if v is 344 meters per second then the lowest frequency is 344 hertz. And I proudly demonstrated that to you. Our prediction was accurate to better than one hertz. We had exactly the 344 as the lowest possible frequency.

And then course you can look for higher order frequencies. And that depends then on the dimensions of a and b. Whether the next one is 0,0,2. Or whether the next one is zero, one, one. Or whether it is 1,0,1. And we ranked them all and I showed you eight of those frequencies. And we had a hand out, at least on the web, I showed you these wonderful resonance curves that we generated during our lecture. I think it was lecture number 16. You can still download it from the website.

Now comes an interesting question. What would happen if I knock out this panel in front and the panel in back? So I knock out both panels in the z direction. So I make it open in the z direction, open at both sides. What happens now?

Well, the pressure now can never be an anti-node at z equals 0 and at z equals c. On the contrary, it is connected with the universe. No pressure differential can ever build up. So the pressure has to become a node now for z equals 0 and z equals c.

Well, that we can easily do. We just change the cosine into a sine and nothing else changes. Because look, if now I make kz n pi over c, then you will see for any value for n that you choose this will indeed become a pressure node. You will get 0.

And so this now is the right solution then when the panels in the z direction have been removed. The only thing that you cannot allow n to be 0. Because if now you make n zero, then no mode could exist. Because if n is 0, then this would always be 0. So now you have a situation that yes, l can be 0. m can be 0, but n cannot be 0.

So what now is the lowest possible frequency? It's the same. 0,0,1. And so the lowest possible frequency when you break out, when you make it a tunnel, is again 344 hertz. And that was quiz number nine. I asked you for this one, which was of course a giveaway if you attended that lecture. You could not possibly have forgotten that. I was almost crying when I showed you this demonstration. So you couldn't have forgotten that I was crying.

And then I just wanted to test your insight. That you realized that the anti-nodes become nodes but that nothing changes. At least not in terms of resonance frequency. Of course something changes in the box of course. Because now you have four walls where you have pressure anti-nodes, and you have two non-walls where you have pressure nodes. But the resonance frequency is the same.

When Markos and I were working on this, we got this crazy idea to ask the question, what would be the resonance frequencies of a sphere? So we have a sphere. And of course, it's not a perfect sphere. It is more like this.

And it has a diameter. I call it capital D. And the diameter is about 28 and a half centimeters. Uncertainty is only a few millimeters. And so we got this crazy idea to put a speaker there, just like we did with the box, and to have a microphone inside. And then to see if we increase the frequency of the speaker whether we could predict, and actually confirm, the resonant frequencies of the motion of the air inside this box.

And so I said to Markos, I know the answer. It's easy. It's a trivial problem. If this were a perfect sphere, then clearly the wall here must become a pressure anti-node. A complete sphere must become a pressure anti-node because there must be spherical symmetry. A sphere is a sphere, right? There must be spherical symmetry.

You cannot have an anti-node here then a node here, because nature doesn't know the difference between upstairs, downstairs, left, and right. So it must be a complete anti-node sphere. But this center must then also be an anti-node, because if the air flows away and pushes onto the wall and then sucks onto the wall it must always come back there. So the center itself must also become a pressure anti-node.

Knowing that, I said to Marcos well, that means that lambda must be 28.5 centimeters. Because from node to node to node is one whole wavelength. And so I made the prediction that the frequency f, which of course is the speed of sound divided by lambda, I made the prediction that would be 340/0.285. And that is something like 1,190 hertz.

And I even made the prediction what the second harmonic would be. You must again add a nodal sphere. Sorry, an anti-nodal sphere. Another anti nodal sphere where the pressure reaches maximum, minimum, maximum, minimum because of the spherical symmetry.

And so that means that now the diameter is twice the wavelength. So the wavelength is now half that value, because from here to here to here is one wavelength. From here to here the same wavelength. And so I predicted that the second harmonic would be twice this high.

And so we set it up. And full of expectation, I said to Markos why don't we just start the system somewhere at 800 hertz. And then we'll find these resonances. And Markos, a little bit more cautious than I am, he said well, let's start a little lower. I said, well why waste our time. He said, well let's start low. Let's start at 45 hertz.

That's what he did. You will see here the frequency of the speaker which is mounted there. And you will then see there, if that works, you will see the-- oh boy, Markos it does work. And you will see there at the bottom the driver which is the speaker. And you will see at the top, you'll see the response of the microphone which is inside. Sorry for that. Thank you.

And so we started to slowly increase the frequency which I thought was going to be a complete waste of time. 45 hertz, 50 hertz, 58 hertz. Oh, I forgot to turn on the microphone. Thank you Markos. There would have been no resonance at all.

OK, let's go back to the 45 then. 48, 51, 52, 53, 56, 58, 60, and there is at 61 hertz an unbelievable resonance. I couldn't believe my eyes. I said this is nonsense. This cannot be. It's impossible. How can you have a 60 hertz resonance frequency? What's wrong with my reasoning?

Well we were staring at each other in disbelief, and so we went on. And we said, well. I said to him, the 60 hertz are the lights in the building. That's what you pick up. It's some pick up. It's some crazy pick up. It's not a sound resonance. Whenever you see 60 hertz in Europe, in the United States, it's pick up. If you see 50 hertz in Europe you know it is some pick up. So I was willing to forget about it.

In any case, we went on. And we were trying to find the other frequencies. So I will quickly now try to go to much higher values. So I go in core steps and see where the first one is. I was hoping to see the 1190. And then by now it's 6-- 700s. I will now go a little slower. So watch. You see the frequency at the lower signal is increasing much higher than it was before. And here, you're actually beginning to hear it.

And so the 61 make me sick in my stomach, and I was not all too happy with 787 hertz. And so I insisted to at least start looking for the 1190. And so we started looking for the 1190. And I was beginning to be happy because I think it's coming. I think it's coming up there. I think it's coming up there. I'm And it fell a little short, but not embarrassingly short, 1164.

I went home and I couldn't sleep that night. I couldn't sleep. 60 hertz is absurd. Keep in mind that anything that is this size-- what am I doing? I turned off the microphone. Anything that is that size cannot have such a low frequency. I didn't sleep the first night. I didn't sleep the second night. And then the third night, I woke up.

And then I remembered when I was your age, when I was a student in college, that one day I had a little flask like this. And I was blowing air past it. And I said to myself, what? Something 10 centimeters in size? You expect resonance frequencies of the order of the speed of sound divided by the length that is three kilohertz. This is nowhere near three kilohertz.

And all of a sudden that image came back to me. And I remember what I did. I said, there has to be an explanation for that. So I went to the library. Looked in books, couldn't find a solution. Went to the library, found an article in the library. And that article told me what the resonance was. This one, only one resonance no harmonics, provided that you know all dimensions.

You have to know d. You have to know L, which in our case L is 99.5 centimeters with an uncertainty of about three millimeters. And you have to know this d, which in our case is 5.5 centimeters with an uncertainty of maybe one millimeter.

I did my homework. I turned the crank. I went to the Hayden library the next day. I found an article, maybe not the same that I found 50 years ago. But I found the article with the same solution. I plugged in the numbers. What do I find? 62 hertz. And I was very happy. I had recovered what I had lost 50 years ago.

Now you are now at about the same age that I was then. And I want to present this to you as a challenge, something that goes a little bit beyond the standard textbooks. If you present me, before December 7, with a proper solution for the 61 hertz? I will generously reward you with extra credit for 8.03. I am not too worried about the 787.

The reason for that is, it is not a perfect sphere. There is no doubt in my mind if that had been a perfect sphere, if this thing had not been there, you would not have seen the 61. Which is true. You would not have seen the 787. But the opening makes it very difficult to calculate. In fact, it's not even so bad that my 1164 does show up when my prediction was 1190. But I'm not even sure whether that is really the one that I predicted.

In any case, it's the 61 that is the bizarre one. And it has no resonances. There's only one answer. There's only one resonance frequency, no harmonics.

I will be here over the weekend to help you prepare for your exam if you need me. I will make half hour appointments. You have to indicate what you want to discuss with me, and I will then agree on a time with you. I wish you luck. And I'll see you then or Tuesday.