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PROFESSOR: I'm Wit Busza, professor of physics at MIT. I'm joining my colleague, Professor Walter Lewin, to help you understand the physics of waves and vibrations. Now you may well ask, why spend so much effort on waves and vibrations. And the answer is very simple. If you take any system, disturb it from equilibrium, from a stable equilibrium, the resultant motion is waves and vibrations. So it's a very common phenomenon.

Not only is it that very common, understanding waves and vibrations have very important practical applications. And furthermore, the fact that they exist, that this phenomenon exists, has tremendous consequences on our world. If waves and vibrations were different or didn't exist, you wouldn't recognize our universe.

What is the role I am playing in this course? To answer that question, I have to remind you what is the scientific method. In essence, the scientific method has two components. The first, you look around and you describe what you see in the one and only language that can be used, or we find that can be used for the description of nature. That this mathematics, in terms of mathematical equations.

The second aspect is since the universe is describable in terms of mathematical equations, we can solve those equations. And that means predict result of situations, of experiments, which we've never seen before. Again, this is important for two reasons. One, practical-- to be able to predict what will happen. But the other far more important is that it is the way we have, the objective way we have of checking whether our understanding of the universe is correct or not. If the predictions do not give the right-- do not correspond to what one actually sees, you know, your theory, your understanding is wrong.

My role is related to the second part. In other words, what I would want to help you learn, take a given situation, convert it into mathematics, solve it, and predict what will happen. We call that problem solving. OK. Let me immediately start with a concrete example. What I have here, describing a situation which we would like to understand.

Imagine you have an ideal spring, a spring that obeys Hooke's law. As I've shown here is the spring constant k, length, natural length l0, and your suspend it from the ceiling. Imagine that you take a mass, a small mass, m, and you attach it to that spring. At some instant of time, and in the proceeds of attaching it, you may stretch the string. So the spring may at this instant not be stretched. But let's assume while you're attaching and you've stretched the spring a little bit, you're holding it, all right.

At that instant, it's velocity is 0, stationary. You let go. The question is, what will happen? Can you predict what will be the motion of that particle? You know, you've seen this often. But a priori, it's not obvious what will happen. The spring may pull the mass up. The mass may pull the spring more down. It may oscillate. Everything, until you've understood what's going on, you cannot predict the outcome.

So let's assume that at some instant of time, we call that time t, it's as shown on the right. In order to be able to describe this, I have to tell you where these masses are at these various times. So I will define a coordinate system. This is a one dimensional situation. So I only need one coordinate. And I'll call it the y. My y will be up. All right.

Now I also have to measure things from some location. So I need to define what I mean by y equals 0. And I will define y equals 0, the position where if the mass is at that location, the force of gravity pulling it down and the spring force pulling it up cancel, so that there is no net force on the particle. So y equals 0 is the equilibrium position. And then the position of the spring when it has no mass attached is the distance y0 from that at t equals 0, the position I will say is y initial. That's some number. So that's a known quantity. All right. And that any other instant time, I defined it as y equals t.

That is the physical situation I wish to understand. I want to know what happened with that spring. So now I will translate that into mathematics. I will now try to give you a mathematical description of that situation. So we know that we are dealing with forces and masses. So to describe that, we use Newtonian mechanics. So here is now my mathematical description. The mass is a point m, of mass m, on which two forces act. There is the force fs due to the spring and the force fg due to the fact that this mass is in the gravitational field, and therefore there is a gravitational force on this mass, fg. OK.

We call this a force diagram. Or some people call it the free body diagram. Now this mass, because of the force acting it, its motion will change. And it will have an acceleration, which I will call a of t. And by the way, that of course is the second derivative of y with respect to dt. It's a vector. It's in the y direction. And in order so I don't have to write things over and too many things in these equations, I will define the symbol y with two dots on it as the second derivative of y with respect to time. y dot is the first derivative-- in other words, the velocity, et cetera.

And by the way, you may notice I'm going very slowly here. I'm doing that intentionally. I'm going to go here in gory detail every part, you know, because often I know that when one goes to a lecture, or studies in a book, et cetera, you look at some step from one step to another. And you can't figure it out. The reason for it often is not that you are not smart enough to do it, is but the because the teacher or whoever wrote the book, et cetera, is so familiar with the material he will do several steps in his head or her head, and you don't know about it.

For this first example, I will try to avoid anything of that kind. Later on, I'll go faster. And I'll do the same as everybody else. But the moment, as I say, I'm going in gory detail. OK. So this is the diagram, this free body diagram, of the situation. And I know from Newtonian mechanics that if there are forces acting on that mass, that mass will have an acceleration, which will be equal to the net force acting on it, divided by the mass, the inertia, of that system. So that is what a will be.

I further know the force is vector, so the net force acting on this mass is the sum of those, the vectorial sum, of those two forces. So f is the sum of the force due to the spring and due to gravity. Next, we also know something about the spring. I told you at the beginning that I'm considering an ideal spring. So for the purpose of this problem, I'm assuming I have this fictitious thing, a spring which essentially has no mass, massless, which obeys exactly Hooke's law.

And here I can't help digress and point out to you that that's a terrible misnomer. There is no Hooke's law of nature. It is an empirical relation which tells you the force that the spring exerts when you stretch it a certain distance, all right. But anyway, it's stuck historically. It's Hooke's law.

So Hooke's law, from Hooke's law, I know what will be the force, fs, when the situation is as shown over there, all right, at time t. So at this time, this extension of this spring will be, of course, y0 minus yt. OK. And so I get that the force due to the spring will be the spring constant times its extension at that instant of time. It is a vector. And y0 is a bigger number than yt, this is a positive number. Therefore, the stretched spring will pull the mass up. So this is in the y direction. This is plus.

How about the gravitational force? Well, that is, of course, the minus mg, the force of the gravitational field on that. And it's minus in the y direction here, because it's pulling this mass down. OK. Now what else do we know? We know that we could get everything done very carefully. We know that we defined y equals 0 to be the equilibrium position.

Therefore, when y is 0, we know that the second derivative of y is 0. It's not accelerating. So that's a condition we must not forget. Another thing we know that initially, in other words, at t equals 0, the position of that mass is y initial. Finally, I told you that the velocity of that mass was 0 at t equals 0, stationary. So this is the beginning of our translating all the information we gathered here into mathematics.

Let me continue now using this information and try to reduce it to the minimum set of equations. From a equals fm, from this, I get that the acceleration is the total force divided by m. I can now replace these two forces from the information I wrote over there. And so that is equal, 1 over-- this is f over m. It's 1 of m times the net force, which is the force due to the spring minus the force due to the gravity, OK.

So from this, I can now actually write an algebraic equation, rather than the vector one. Notice - ha ha. I have noticed myself even something. Here this has to be in the direction of y. OK. This is a vector equation, but all the parts are in the same direction, in the y direction. Therefore, I can rewrite this just the equation for the one component and not bother to write the y hats throughout.

So this equation I've rewritten now just removing y hat. So this is how the mass will be accelerating. Unfortunately, it's a single equation, but I have more than one unknown in it. Because I don't know y0. And I don't know y of t. Clearly, I won't be able to solve that equation, all right. But at that stage, I go back to the information I told you at the beginning. We defined y equals 0 to be the place where a y double dot, the second derivative, is 0.

Therefore, I can write that at position. When y of t is 0, this is 0. So 0 is equal to 1 over m, into ky minus mg. And I immediately from this get that ky0 is mg. Therefore, I have found what y0 is. OK. Great. So there's only one unknown here. So using this information in here, I end up immediately with this equation, that the second derivative of y with respect to time, the acceleration of the mass is equal to minus k over m times y of t.

At this stage, I will really find this quantity, k over m. For the time being, you can look at it as just for convenience, less to write on the board. But later, you see this will help us understand how to deal with different situations. But for the time being, you can just think of this as a convenience, so I can write less on the board. And I end up finally with one equation. The second derivative of y with respect to time is equal to minus a constant, that's k over m, right, times the value of y times t.

This is the equation of motion for this mass. It tells me in mathematical form how the motion of that mass changes with time. I can now actually predict what will happen in this particular situation. Because I know what was the motion of it at time 0. I know that at time 0, the position was y initial. And the velocity was 0. These three lines are completely equivalent from the point of view of understanding the motion of the mass to our original description. This is a physical description of the situation. This is a mathematical description of the same situation.

So we've achieved step one. We've translated a physical situation into a mathematical one. Let me now try from this, I should be able to predict what this mass will do. OK. I'm now switching into the world the mathematics. As I just am repeating here, I've gone away from a physical description to a mathematical description. This is pure mathematics.

I have an equation, a mathematical equation, for y of t. It's a second order differential equation. I had the boundary conditions, or initial conditions. I can solve that using mathematics. OK. Let's do that. So I'm now doing pure mathematics. I don't want to teach you math. That's the role of the math department, all right. So how do I solve that equation? And let me tell you how I solve it.

I am make use of the so-called uniqueness theorem. I know, or the mathematicians have told me, that if I find a solution to that equation which satisfies-- if I find a solution which satisfies that equation, and if it has the right number of arbitrary constant, then I have found the one and only general equation, which is a solution of that. Let me be concrete. y of t equals to a cosine omega t plus phi, where A and phi are arbitrary, are arbitrary. They are some arbitrary numbers.

But a number is there. This can be 7 and this can be 21 degrees, or whatever, but any number. This equation satisfies my differential equation. If you don't believe me, try it. Differentiate this twice, all right, for any value of A and phi and you'll satisfy that equation. So this is a solution which satisfy that equation. It has the right number of arbitrary constants, that two arbitrary constants in here. And therefore, this is the only solution in the universe of that equation. OK.

Now, so being a physicist, I don't care how I got the solution. Once I had the solution, if I know it's the only one that exists, I'm home. Now you can say well, I suppose I didn't guess it. Well, there are many ways. You go on the web and find it. You go in the book and find it. You ask your friends what it is, all right. That's mathematics. And once you've found the solution, we can go on. All right, so this is the solution of that equation. Next, if that's y, what is y dot? What is the rate of change of t? That's going to equal minus omega 0 A sine omega 0 t plus phi, OK.

Can I predict what will happen? All right. I still need, in order to be able to predict what will happen, I need to find out what are the values of A and phi which satisfy the other information right here. See, I told you that we reduce that physical situation to a differential equation, the equation of motion for this mass, including the information about where it was at some instant of time, how it was moving, et cetera. So I need to make sure that this equation satisfies these boundary conditions. In other words, it its thees boundary conditions which will determine what are the A's and phi for the particular problem that I had there, OK.

And so what I do is-- let's, for example, takes here. Because I see 0. Y dot t is 0, all right, at t equals 0. Well, when t equals 0, this is minus omega 0 A sine phi. OK. Therefore, I immediately conclude that phi is 0. OK. Next, I know-- so now that I know that phi is 0, I can go back to this equation. This is now 0. And we know that y at t equals 0 is y initial. But that t equals 0 cosine of 0 is 1. Therefore, A is y initial. And so I get finitely y of t is equal to y initial, all right, times cosine omega 0. Let me now replace it with the 1-- well, let me leave it as omega 0 t pluse 0.

That, and I can rewrite this, putting all the numbers that I have, y initial cosine. And I'll now even replace omega 0 by k over n, square root of k over n, times t. Notice there are no unknown quantities in here. This tells me two things. At any instant of time, I can calculate where this mass will be. It's given by this equation. Secondly, I can describe the kind of motion it does. What is this equation? As a function of time, this corresponds to an oscillating position y. So this mass, when I let go, will oscillate.

What will be the period? How long will it take before it comes back to where it started? Well, the period t will be how much time do I have to add to this t, so that the angle here changes by 2 pi? Well, that's obviously 2 pi root of m over k. OK.

So I've achieved what I wanted to do. I've taken a physical situation. And I have predicted if I let go what will happen. This is the motion it will experience. This is the period. I can predict the time, et cetera. At this stage, let's stop for a second and consider what we've done. Because it's the essence of-- this is a good example of the essence of the scientific method. We have taken a physical situation. We've described it in terms of mathematics. Then we made an act of faith that if I take the mathematical equations and I solve them, that the resultant answer will actually correspond to what nature will do.

If you stop to think about that, it's amazing. Nobody understands that fact. Why that's true. Why it happened. In other words, nobody understands why nature can be described in terms of mathematics. OK. But it is that fact which makes the scientific method possible. Finally on this note, let me give a quotation from Einstein which beautifully summarizes what I've just said. And that is the following, "The most incomprehensible thing about the universe is that it is comprehensible." The fact that we can follow this procedure is amazing.

OK. Let me at this stage go and take another example, all right. So let's take another example. Consider the following situation. I take something like a ruler, a uniform rod. And I put a nail through it, some kind of a pivot. There is some pivot. I pivot the ruler on it. And it's hanging like this. OK. Let's assume the mass is m of the ruler. The length is l. It's a uniform ruler, a rod of some kind. And at t equals 0, I give it an impulse. I give it a little impulse, so we are now at t equals 0.

We give it an impulse. At that instant, the ruler is still hanging vertically. Let me, just so that when you look on the board, you may be confused in which plane I am. This is the vertical plane. So this is up. So I give it an impulse. So at that instant of time, this ruler will have an angular velocity which I will call theta dot. This is at time equals 0. And it has some number as a result, depends how big an impulse I gave it. And so that you remember what I'm talking about, I like to give this, instead of using a symbol, I'll call this angular velocity at t equals 0.

So this is some number, so many radians per second. That's at t equals 0. And I'm now going to follow this method again. I want to know what will be the motion of this. What's going to happen to this ruler. Is it going to start spinning around this, like this forever? What will happen? So I will try to translate this problem into mathematics. Because of the mechanical constraint, at some instant of time, the ruler may be doing this. Let's call this the time t. This is time t. And time t is like this.

And I've got to define some coordinate system. So I'll take this angle from the vertical. And I call that theta at time t. That's why I call this theta dot. This is the rate of change of that. So at some instance of time, it will be at this position, all right. At that instant of time, it'll have a velocity in this direction. And we'll have an acceleration in that direction. So for example, the acceleration will be theta double dot at time t.

And just so that at this stage, I will still to remind you that's alpha, alpha time t. Because often alpha is used as the acceleration. So at the moment, I just want you so you can easy for you to see what I'm talking about. So at some instance of time, that is the physical situation. I would like to now convert this into mathematics. Follow the same procedure as before. I need to write the equation of motion for this. And I need to write down the initial conditions. So how do I do with that?

So now I start off by the free body diagram. Here is the pivot. That's the route. This angle is theta t. There will be a force acting. We're now dealing with rigid body motion. So today we did Newtonian mechanics for masses and forces through a single point mass and forces. Now we are doing a Newtonian dynamics for rigid body motion. You know that if a rigid body is in the gravitational field, the gravity acts force fg. We can analyze it, as if there was a force fg g acting through the center of mass here of the body. So this length now is l over 2.

So there will be a force fg acting. And as a result, there will be torques about this point. Now let me say the following. We are dealing here with motion, rotations, in a single plane. And so we are dealing about rotations, about an axis through this point p. We're not dealing with three dimensional rotations, but simple situation where all the motion is about a single axis, which is perpendicular to this point, p.

There will be a torque about p because of the gravitational force. And as a result, there's going to be the acceleration, which as we've said over there, is theta double dot of t. Now, we know that torques gives rise to angular acceleration. Let me define that we will take clockwise motion, clockwise motion, clockwise rotations to be positive. So any rotation, this angle, for example, I am sorry. I meant anti-clockwise. Anti-clockwise is positive.

Look at this. If this rotates like that to this angle, this I take to be a positive number, it's an anti-clockwise rotation. Similarly, if this acceleration is a positive number, it's accelerating in this direction. Since we are dealing with rotations about a single axis, we don't have to go to the vector formulation. We can consider it just the magnitude. And we know that the acceleration is equal to the torque divided by the moment of inertia. Or you may have seen it the other way. Torque Equals I alpha. I prefer it this way. For me, it's more logical. The angular acceleration is a consequence of the torque. So I write it like that.

So this is the dynamic equation, which tells you how the motion of this mass changes with time. All right. So alpha is theta double dot of t. OK. What is the torque at that instant of time? Well, you know general torque is r cross f, all right. That's true in three dimension. So it will apply here. So the torque is going to be this force times this distance. OK. So it's going to be-- let's write it. The force is mg right, times l over 2 sine theta, theta of time t. OK.

That's the torque about this axis p on this rod, all right. And it's divided by I, where I is the moment of inertia of this rod about an axis through p perpendicular to the board. OK. Now we need to calculate the moment, in order to continue further, we need to calculate I. Since we know this mass of the rod. And we know it's a uniform rod. And we know it's length l, we can calculate it. You know how to do it. If you don't, you can look it up in the book on mechanics, all right. Or just look up the moments of inertia.

And you will find that the moment of inertia, you will find that the moment of inertia I for a rod like that is 1/3 the mass times the length squared. OK. So now I have to continue. But I've run out of board space. So I'm going to erase the board at the far end. And we'll continue from there. So I erased the board. And then so that you don't have to look backwards and forwards, I've started rewriting it and I realized that I actually missed the negative sign. So I'm going to correct it here. So that's why it's completely written out.

So let me just remind you. The situation we have is this rod, which at time t, we define this angle to be theta t, the rotation of the rod. It has an acceleration, theta double dot t. And we are considering rotations about an axis perpendicular to the board through this point here. OK. We know, that was the last thing we did, that the acceleration is given by the torque divided by the moment of inertia. All right. The torque is mg, l over 2 sine theta, I derived it for you before, divided by I.

But what I neglected to put a negative sign. And that you could do in your head, right. Consider we've taken all the rotations to be positive if they're anti-clockwise. So this angle is a positive rotation. This would be, this direction would be a positive rotation. But the torque if you look at this, there is a force acting down on this. So about this point, it's trying to rotate this in the clockwise direction. And so it's minus. And I didn't-- it would have naturally come out if I did the full vector calculation, the torque is R times F. It would have come out, the sign would have come out. So that's where this minus sign comes in.

OK, so this is where we got on the board over there. And now let's continue. We can replace I from here. And we get that theta double dot of t is equal to minus, all right, 3 halves, 3 l, 3 halves, not l. It's divided by 2. l is at the bottom. Sorry. 3 halves g over l times sine theta, theta of t. OK. I'm sorry. Sine theta of t.

OK, as before, to simplify it, I will write omega, I'll define. Let's define omega 0 squared to be equal to 3 halves g over l. With this definition, we get that theta double dot t is equal to minus omega 0 squared sine theta of t. OK. So this is our equation of motion for this problem. That's the equation of motion. And these are the boundary conditions. So these three equations are a translation of this problem in the language of mathematics.

If we now want to predict what will happen to this rod at some other time, we have to solve these equations. And admit now, I have a problem. If you remember, when we did it for the spring, the equation of motion was one where I guessed the answer. I don't know what the answer is of this. If you go into books, you will find that this is not one of the differential equations which you can analytically solve. It's, in fact, a second order differential equations with transcendental functions in it. So this is not something we know the answer to.

So the only thing if I want to now predict what will happen, I have to numerically solve this. And then I can-- I have enough information. I can numerically solve this equation with these boundary conditions and predict what will happen. That's not very instructive for the purpose of course at the moment. So let me do something else. OK, let me modify the problem.

Rather than take the problem we took, let me say, how about if I took this rod and gave it only a very tiny impulse. So this angle is small. Let me make the angles sufficiently small, such that sine theta of t is always approximately equal to theta of t. Depends how well you want to approximate this. But typically, if you use your calculator or computer, up to about 10 degrees, that approximation is pretty good.

So I will now change my problem. And I said OK, let's see whether we can predict analytically the motion of the rod where I give the impulse, which is sufficiently small, that this angle is always small. Under those conditions, note that my equation of motion becomes theta double dot of t is equal to minus omega squared times theta at t. Because sine theta t is always approximately equal to theta t if I take the angle small enough.

And eureka, I can solve that one. Because that's exactly the same equation we solved before.

OK, so we get the solution to that equation, is theta of t is some constant cosine omega 0 t plus phi. As before, A and phi are some arbitrary constants. And clearly, if it worked over there for that same equation, it works here. The only difference here is we have theta of t instead y of t. That's just different symbols, but the solution is exactly the same.

So we know that's the solution of this equation. We know the boundary conditions. Therefore, we can predict what will happen. Let's continue and do that. So from here, you get theta dot of t is equal minus omega 0 A sine omega 0 t plus phase. OK. And we have to put in the boundary conditions. OK. Now at t equals 0, OK, we get that this is at t equals 0. So at t equals 0, we get theta of t 0 is equal to A cosine phi. OK. Therefore, phi is pi over 2. That's a possible value of phi.

Now that gives me that if I is pi over 2 here, we get to that theta dot of t, which is equal to the angular, angular velocity at t equal to 0, all right, will be equal to minus omega 0 A sine omega) t plus pi over 2. OK, from which I can get that A is angular velocity over t plus 0. And so my final solution is that theta of t is equal to angular velocity at t equal 0 divided omega 0 sine, sine omega O t.

OK. And I want to make sure I'm not making a sign mistake again. I'm not. All right. And so, and omega 0 we know, and so the in terms of knowing quantities, the answer is angular velocity of t equals 0 over. And omega 0, we have found defined to be that, so the square root of 3 g over 2 l times sine square root 3g over 2l times t. Now this is theta t.

So we have completely solved the problem. And we have predicted the motion. So as before, following this process of taking the physical situation, describing it in terms of mathematics, solving the mathematical equations, including all the information we have about the problem, the boundary-- initial conditions or boundary conditions, we can predict what will happen to this angle as a function of time, and also the kind of motion this is an accelerating motion. I can also predict, as before, that the period of this will be 2 pi, 2 pi square root of 2l over 3g, et cetera. OK.

Now, one of the things you'll notice, that in some ways, it seems I'm repeating myself. We took completely different situations, and yet the result, the equations of motion, and the results, have good very similar form. Now this is part of the beauty of the scientific method. Because it turns out that very many different physical situations can be described by the same mathematical equations. So once you've solved the problem for one physical situation, you have automatically have solved it for an almost infinite number of other situations which are described by this same mathematics.

Finally, let me do just more as a question of practice, one more problem of this kind that apparently seems to be completely different. I'll take a problem from electricity and magnetism. Let me consider the following situation. So now we're going to a different problem. The physical situation is suppose I have two plates, two metal plates, and I connect them with a wire. Schematically, it consists of a capacitor C connected to an inductor. This is a schematic representation of two parallel plates connected, short circuited by a wire. I will assume for simplicity here that these wires have no resistance, superconducting, all right. Any loop like has an inductant L. And the capacity between these is C. So this is an L C circuit.

And I'm going to assume that at time equal 0, so this is now time equals 0, I have a charge here, minus Q0 plus Q0 here, OK. And let's assume that at time there's even a current flowing, so I is 0 here. So this is a system which is disturbed from equilibrium. And what will happen is a function of time. I will do the same and almost boring you to tears,

I'm going to, you'll see I'm essentially doing the same problem again. I will now consider this circuit at some arbitrary time t, derive the equation of motion for the charges in the current, therefore translate this physical situation, or describe this physical situation in terms of mathematics, deriving mathematical equations, solve them, and predict what will happen. So I just follow what I just did a second ago.

So at some instant of time, that same circuit L will have some current I of t to the charge minus Q of t plus Q of t, all right. This at time t. So from this, I can derive the equation of motion. Let me remind you about Faraday's law. You know that if you have current coil in the loop, it produces magnetic flux in that loop. The changing flux gives rise to an EMF around that loop.

To be specific, Faraday's law I can write. If I take this circuit of the wire, the integral of E dot dl around a closed loop, that is equal to minus du phi. 5. Now watch out.

The Greek alphabet has a limited number of letters so you'll find one constant is reusing the letters. But at the moment not to confuse you, I'm going to put here magnetic flux, total magnetic, total magnetic. So phi is the total magnetic flux linking this circuit, all right, dt.

So Faraday's law tells us that the integral of ED all around this loop will be equal two minus the rate of change magnetic flux. This is the dynamic equation which tells you how this behaves. It is the analogous to Newton's law f equals ma in the case of our mass, or 2 torque is I alpha in the case of rotations, et cetera. This is the non dynamic equations. So let me calculate this.

And now I'm going through the-- around this. And you find since this wire I'm assuming is superconducting, there can be no electric field inside it. So the contribution to this line integral is 0 when I go through the wire. So the only place where this line integral is non-zero is between the plates, all right. And that is simply the potential difference between those, which is q over c. OK. Q at time t over c, where c is the capacitance is. That is the integral of EDL around that loop.

And that's going to be equal to minus D dI at time t dt. All right. Because the magnetic flux, this is by definition of the inductance or first inductance is that the total flux linking the circuit when the current flowing in it is I of I, the total flux is L times I. Here, I have the rate of change of that flux, so it's equal to this. OK. Now we know by charge conservation, that the current I of t. What is the current? It's the charge is flowing per second will be equal to the number of charges per second that arrive at this plate here or the part from there is equal to dQ dt. OK. Or in other words, Q dot. OK, let me continue.

So from these two equations, right, the dIdt therefore, this is the second thing, so I end up from there that Q double dot, second derivative of t, is equal to minus right 1 over LC times Q of t. Eureka. We have once again the same equation. This I can define as before, omega O squared. If I define that as one over LC, all right, then what I have here is Q of t double dot is equal to minus omega) squared Q of t. Again, we have come to the same equation.

This is the same equation of motion as we came in the other two situations. So the answer will be the same. The variables will be different here. It'll be the charge that will be changing with time, while there in the one case was the angle. In the other case was the position of the mass. OK, and the solution to this problem, I can now write immediately, is Q of t is A cosine omega 0 t plus fe. Note that this is the fe is nothing to do with that phi. OK. And Q dot of t, which by the way is I of t, is equal to minus omega o A sine omega o t plus phi. OK.

Now as before, what actually happens depends on the initial conditions. And we look at that picture on the top board. We know that initially Q is Q 0. And we know that initially Q dot t is I0. To save time, I'll just immediately write. You can do that in your head. And if you the write that out, you find that if you've used those two conditions, you find that time phi this time is equal to minus IO over Q0 omega 0 and A is equal to Q0 over cosine phi.

I saved time without just solving algebraic equations. Take these two equations. Consider t equals 0, the values of those quantities, and just solve for the two unknowns and you get this. And so once again, we have predicted what will happen. And what I would like to just at this stage emphasize that although we have taken three different physical situations, in each case, we took the system, displaced it from equilibrium, let go and we wanted to see what will happen.

In all the cases, it turned out that the mathematical description, the mathematical equations are identical in form. And so they gave, not surprisingly, the same kind of motion. This motion that we see in all those cases, we called simple harmonic motion. It has the characteristic that if you displace the system from equilibrium, it oscillates with harmonic motion, meaning it oscillates as sine or a cosine of different phases, et cetera. If you tell me any one of these systems where it was at any instant of time, I can predict it forever in the future.

Now finally, the last few minutes. Some of you may have noticed that in each of these-- or I told you at the beginning that I can take a physical situation and describe it in terms of mathematics, and thereby predict the future. But in each case, I in some ways almost cheated. I said let's consider an ideal spring. We'll assume it has no mass, that it exactly obeys Hooke's law. Or when it came to that rod, I assumed that it's only displaced by small amounts, so that's sine theta equals-- I can approximate with theta.

In the case of the electrical circuit, it maybe not so obvious what I assumed, but I certainly made assumptions about that the wire is perfectly conducting, et cetera. And I didn't discuss in detail what happens in between the plates or the capacitor where the fields are, et cetera. One is doing approximations. In reality, if you look at any physical situation in the world, it's always incredibly complicated. It's never that you have an idealized situation like this. So to what extent does what we have just done correspond-- is it useful at all.

And the way I'll answer it is by another example. This is the last thing I'll do on the top of a simple harmonic motion, last problem. Suppose I'm looking out of the window. And I see there is a tree and a branch and a bird lands on it. Do I understand what will happen? It's clearly extremely complicated. The mechanics of the branch is complicated. There is air friction. Nothing is simple about it. And yet you and I can predict what will happen. You know what will happen.

As the bird lands, it'll start oscillating and finally come to rest, very much like harmonic motion. I claim I can use the word, I understand what's going on. And the reason why I claim that is the following. That to understand something, all I would like to understand the general features of what's going on. I don't need to know what every atom in the branch is going on in the process of trying to understand what the bird is doing. If I want to understand what atoms are doing, that's a different story.

And so one of the important abilities we have to develop is to be able to, when you see some situation, model it in terms of the most important aspects of the situation. And let me be concrete. In this case, I can say look, I can model this approximately as the branch I'll treat as a spring, of some spring constant k. The bird I'm going to treat as a mass m. And I'm going to consider this situation to be modeled by a mass being placed on a spring and let go.

Now is that going to be exactly this? No. But from the point of view of understanding the general features of this, it will be a reasonable approximation. Now how can I check, this is the scientific method, that this is a good approximation is the following. Make a prediction. Suppose when I see the bird landing, it makes five oscillations, five oscillations in ten seconds. OK. I can predict approximately once the oscillations have died out how much the bird has compressed, distorted, this branch. In other words, from the moment it landed, what distance will the branch, its position, be lowered when it comes to rest.

So let's model it. I'll model it first of all, as I did here, as a mass and a spring problem using-- this is now the same problem we did at the beginning that is still on the board here. I can calculate that for this idealized situation, the period will be equal to 2 pi square root m over k, for this idealized model. In reality, there's friction. So this oscillation will be dumped out. And you would have learned from Professor Walter Lewin that if you have a damped oscillator, the frequency of oscillations does not depend significantly on how much damping it is, provided it is weak damping.

So I would make the assumption that the period will be given by that. I also know that at the time when the motion has been damped out and the bird has come to rest, at that instant, there's no net force on the bird. And so the force of gravity on it will be equal to the restoring force due to the spring. So mg is equal to kl. From this, I get that m over k is l over g. All right. But we know what the period is. We said five oscillations in 10 seconds. So the period is two seconds.

So two seconds will equal 2 pi divided by but this, which is l over g, the square root of l over g. Square this and calculate the one unknown l and you get one meter. So my prediction is that this bird will, after it's settled down, roughly be lower by one meter. It's certainly not going to be one millimeter. It's not going to be one centimeter. It's not going be 10 meters. And if you go and you measure it, you find this is approximately correct.

The fact that I can predict it is for me the same as saying I understand what's going on. I realize it's not exact. But with the approximations that I've made, I get an answer which is consistent with what is observed. Today I have tried to tell you what my role in this course is, what I'm trying to help you learn. I intentionally went very slowly. I used the word gory detail. I tried, in particularly in the first problem, not to miss any steps.

And what we covered today is the phenomenon of simple harmonic motion. It occurs whenever you have any system which is displaced from equilibrium where the restoring force is proportional to the displacement. And it illustrated that you could have very, very different physical situations which, when translated into mathematics, give essentially the same problem. So it's a beautiful example of the scientific method where we utilize this same-- well, once we've learned it for one system, we can apply the results to another system.

So as I said, today I did simple harmonic motion. Next time, we would be considering problems to do with simple harmonic motion, but which includes friction, damping. We'll then go on to talk about harmonic oscillators which are driven. So we have driven harmonic motion. And gradually in the course, we'll go to more and more decrease of freedom, waves, et cetera.