PROFESSOR: So welcome back. And today, we will solve one type of problem, a very important problem, basically, the following. Suppose all that you have is a single charge, but it can move.

Here is the origin of coordinates. This is the charge, Q. Its position is given by some vector, z of t. And the charge can move. It can have a velocity. It can have an acceleration, et cetera.

And the problem is, somewhere in space, at a point, p, what is the electric field? What is the magnetic field at all times? That's the problem.

Now the differences between different particular problems will be where we are and what the charge is moving et cetera. In particular, we will consider problems where this charge is oscillating in a simple harmonic motion. It's along a line.

And then, in another problem, we'll consider it's rotating with uniform velocity. But in principle, we could be doing anything. So how do you calculate the electric field? How do you find out what the magnetic field is everywhere?

Also, if there is an electric and magnetic field, there will be flow of energy and energy flux or pointing vector. What will that be at that location? And finally, if a charge does accelerate here and it does radiate energy, and the question is, what is the average total power radiated by such a charge?

Now this kind of problem is extremely important. It's important just from the point of view of understanding how the world works. There are charges all over the place. And they are moving. And they do produce electric magnetic fields. A problem like this gives you insight how the phenomena takes place.

But let me tell you immediately, in general, this is a horrendous problem. You know that, if you have charges in space, and you have electric and magnetic fields, the charges, the currents, the electric and magnetic fields have to satisfy Maxwell's equations, those four Maxwell's equations. So you can imagine how complicated it must be to find electric and magnetic fields, in general, for some arbitrary motion like this.

And the answer is, you would never be asked to do that. And you would never have to do that, in general, analytically. If you really had a problem like this that you had to solve, in general, you would do it on a computer. All right?

But let me continue. But it turns out that, under certain conditions, this problem is not so hard. And yet, it contains all the physics and gives you all the insights you need. And that is the following situation.

If you have the charge moving in a confined region, which is much smaller, whose dimensions are much smaller than the distance at which you are interested to calculate the electric magnetic fields, suddenly, this problem does not become hard, or in fact, not hard at all, relatively speaking.

So in some sense-- and this often happens in physics when you are given the problem-- you have to have an understanding of the situation in order to understand what the person is asking you to solve. So in a situation like this, the buzz words, or what you would find is, for example, say, there is a charge. It's accelerating. Calculate the electric field at a distance far from the charge, or the far field, as it's called, rather than the near field.

And the reason for it is the following, in this case, that the hard part of this is calculating the field relatively near the charge, because there, you have a superposition of this static field, for example, the Coulomb field. If a charge is moving, it's like a little current. And so it produces the magnetic fields through the Biot-Savart Law, et cetera.

But these fields drop off, like 1/r, from the charge. And so, if you are far away, then, in general, in most situations where you're interested in understanding what's going on, you can ignore those fields, because the further you are away, the smaller they are. And they become negligible.

There is another contribution to the field, to the electromagnetic field, a very important contribution. And that is-- and in fact, you know that, in vacuum, without any charges there, you can have electric and magnetic fields present that come from propagating electromagnetic waves.

And it turns out that, if you have a charge which is accelerating, that does produce an electromagnetic wave. And that does not drop off as 1 over r squared, but as 1/r, much more slowly. So if you are far away, you are dominated by the propagating electromagnetic wave. Now it's not that this is trivial, but certainly, the propagating electromagnetic wave is much easier to calculate without the presence of these other fields.

Now Professor Walter Lewin has done that in his classes. He has shown how, if you have somewhere a charge which is accelerating, how it generates an electric field far away from that charge and a magnetic field. I won't derive it for you, but I'll summarize what you have learned.

Now for a second, I'll just want to point out why the problem of electromagnetic waves is more difficult than, say, that of a string. The analogous problem we want to recount for a string is the following. Suppose you have a string, you hold it at one end. Right? And suppose I move this one end sinusoidally?

You know that a string like this is a line. And in there are oscillators, infinitely close to each other, infinite number of them. All right? Each one moves along a single line, say, up and down. So for each one, it's a single number tells you what the displacement is.

You know that the string satisfies the wave equation. If you put a boundary at one end, the oscillator, the first one, is somehow externally driven. That excites the next one, et cetera. And you know that the wave propagates.

Now in the case of electromagnetism of this kind of a problem, the situation is very similar, but you have this extra complication. In the string, each oscillator was on the straight line and only moved up and down in one direction. For a charge, every point in space you could imagine, there is an oscillator.

The excitation of that oscillator is the electric and the magnetic field. That's the analogous to the displacement. But notice, already, two quantities, electric and magnetic.

Here, for the string, you had the displacement, a single number in one direction. Here, the electric and the magnetic fields are vector quantities. So not only do you have them spread in three dimensions, instead of along the string, the displacement is a vector quantity. So is the directions in three-dimensional space, et cetera. So that, in principle, is the same, but it is much more complicated.

But coming back now, fortunately, if you analyze what happens, if you take a charge, if it's stationary, it does not radiate. It has the Coulomb field, which, as I told you, drops off like 1 over r squared.

If it's moving with uniform velocity, for example, it does generate magnetic fields. But again, they drop off. It doesn't radiate.

But if it accelerates, then it does produce, far away, an electron, a field which propagates. And as Professor Walter Lewin has showed in his lectures, you find the following. That if I have a charge which accelerates, at the time when it's accelerating, it produces an electric field, which I can summarize with this formula. So it's the following.

Suppose, at the origin, somewhere I have a charge. And it accelerates, say, in some direction, up. And I am interested what is the electric field that propagates outwards in some direction, r.

What one can show is that this charge will propagate in all directions, including this direction. But there's only one component of the acceleration of the charge which gives rise to the production of the field which propagates. And that is the component which is perpendicular to the direction in which I'm interested to calculate the electric field.

So if the charge, for example, is accelerating upwards, like this, then it's only the component of this acceleration which is perpendicular to this line that gives rise to the electric field. And one can calculate it. And its formula is rather straightforward.

You find the electric field at some distance, r, from the charge. At time t, is related to, as I say, this acceleration of the charge, but only the component, which I call aperp, the component of a perpendicular to this line, that's this quantity. And what is interesting is the field here that is produced is proportional to this perpendicular component of the acceleration.

But there is a difference in time because, as this charge accelerates-- it's analogous to take the string. When I took the end of the string and I moved it, at that instant, the kink got produced which propagated along this string. Same happens here.

If I have a charge which accelerates, it produces an electric field which propagates. And it propagates with the velocity of light, c. So the field over here is given by the acceleration of the charge at a time, t prime, which is earlier.

If you're interested in the field here at time t, then you have to know what was the acceleration of the charge here at the earlier time, t prime, which is equal to t minus r/c, which is the time the electric field took to get to that point. That's the tricky part. This is the only tricky part of this equation.

So the electric field at the position r at time t is related to the acceleration of the charge, the perpendicular component of it, at an earlier time times some constants, the charge and a constant. And it drops off like 1/r. OK?

So this is the crucial equation. It tells us that, as I say-- I'm repeating this, because it's so important. Whenever I have a charge which is accelerating, it, at that instant, produces an electric field which is moving outwards at the speed of light. Its amplitude is dropping like 1/r, all right? And goes on forever.

Now you know from Maxwell's equations, if you have an electric field which is propagating like that, associated, there will be a magnetic field. This is now just the same as you've always seen for electromagnetic waves. I like sort of visualizing.

If an electromagnetic wave moves in that direction, and the electric field is this direction, at that instant, there will be a magnetic field perpendicular to it. And the B is proportional to E. And they both move forward like that. OK?

So if this is E, then you can get from a B. They are proportional to each other. There's just that constant, C, because the units. And they are perpendicular to each other.

OK. So if you understand this formula, all the problems that you would be normally asked can extremely easily be done, OK? Once you know E and B, of course, you can calculate at every location. The Poynting vector is just a cross product between those. And I just summarize them on the formula we may need later.

So I'm now going to start from scratch by taking a concrete example. All right? So if this refresher didn't help, maybe the problem itself will. So now let's take a concrete example. I'll take a single charge, Q. OK?

So I have a coordinate system. Here is my origin of coordinates. And I'll take a charge here, charge of magnitude, Q, moving up and down, up and down sinusoidally.

So it's given by the amplitude z0. It's moving along the z-axis, cosine omega t. This is the motion. Of course, something must be moving it. But the assumption here is it doesn't in any way affect the electrical and magnetic fields. So that's the only thing that we have to consider in the universe. And the question are, at different places in space, what is the electric and magnetic field?

So for example, one place is a distance, R, along the z-axis up. So it's not along the axis. So the position where we are interested, where p is, has coordinates 0, 0, R. In other words, it's a distance, R, along the z-axis.

And we are all asked, at this time, R divided by C, what is the electric and magnetic field at that location? OK? That's the first part of the question. Well, that, I hope you've already done in your head.

If the charge is moving up and down like this-- oh, and by the way, I have to emphasize, this problem, in general-- except this one part-- you are able to do only if we satisfy what I said here, that the region within which the charge is moving or located is small, compared to the distance where I'm asking what the electric and magnetic fields are. So here, it says for R very much greater than Z0. So whatever I say from now on, I'm making the assumption I'm very far away. I only have the radiated field. I can ignore the complicated fields like Coulomb's field and Biot-Savart, et cetera.

OK. So the first part, even without this, I can do because imagine we are asking what is the electric field along the z-axis. And so suppose I am at that location, and I'm looking down at the charge. What do I see?

I see the charge going towards me and away from me, and towards me and away from me. So it does have velocity at some stage and suddenly has acceleration. But the acceleration is towards me and away from me.

So what is the perpendicular component of the acceleration in my direction? 0. At no time do I see the charge accelerating sideways. I only see it going towards and away from me.

So the aperp is 0. And then I go back to my equation here. If aperp is 0, it doesn't matter what you do with it, E will be 0. If E is 0, B is 0, because they are proportional to each other. So at all times this will be 0, OK?

The next part of the question. And let we quickly tell you, in the next part of the question it says, now let's have this charge not move up and down, but along the y-- sorry. No. No. I misspoke.

Let's leave this moving that way. But instead of looking along the z-axis, let's look at the point along the y-axis. So at the point 0,R,0, or in other words, at distance R from the origin along the y-axis, again, at this time.

And then the third one, it asks let's look at it at 30 degrees. So now, in this case, we look at in the direction where it's in the y, z plane, but at 30 degrees to the z-axis, so in some direction there where the coordinate is this. OK? And the whole purpose is to just get you a feel how accelerated charges radiate in different directions.

So the first one I've already done for you. OK? But here, let me just repeat myself or point out. In each part of this question, we are a distance, R, from the charge.

Now you can say, well, the charge is moving a small amount. But the whole essence of making this a doable problem and getting a situation which you can solve is where all the motion of the charges over distances which is very much smaller than where you are. So because R is very much greater than z0, from the distance of the accelerated charge, from where you want to calculate the electric field, you can assume is constant in distance R. So in all parts of this problem, you are the same distance away, but at a different angle. And in all cases, we are asked to find the field at some time, t, when this is oscillating.

Let's go to the second part. How about in a direction which is along the y-axis? All right? So in the y direction, if something is accelerating in the z direction, that is perpendicular to the y-axis. And so for the second part, the acceleration of the charge at time t prime, is simply I have to take this and differentiate it twice, all right? And so we've done it. This is this.

And this acceleration, that has a component in the z direction which is perpendicular to the y direction. So in our formula, this is Eperp. And so we can plug it into that formula. And we get that the electric field at position R along the y-axis is given by this.

And I just took that formula-- let me just remind you-- I have just taken this formula and plugged in the aperp. And I end up with this equation. And so I see that, if a charge is oscillating up and down like that, it does radiate perpendicular to it.

This is the nearest analogy to that string case. If the string is moving like that, it propagates a wave along the string. And here, if a charge is accelerating like this, it does radiate along that direction. This is the magnitude. And as a function, that's what it is.

The actual problem was not in general what it is, what will it be at the time R/C? Well, at t R/C, this is R/C minus R/C. This is 0. So this is cosine of 0, so this is 1.

And so we get, finally, that at that time, the electric field is equal to this. It's just that times 1. This is the electric field. What is the magnetic field? Well, as I told you, if you know the electric field, B is perpendicular to it, proportional to it. And there's just a 1/C. So they here have it C cubed.

And the magnetic field will be in the x direction. The electric field is always parallel to the accelerated charge. The magnetic field is in the perpendicular direction.

Also, it is important to remember the minus sign. I come back to this formula. This minus sign is very important. What it's saying is, at the instant when the charge is accelerating in that direction, the electric field it produces is in the opposite direction.

So when this is accelerating, at the instant this accelerates, it produces an electric field which goes in this direction. And that field propagates like that. OK? And the magnetic field is perpendicular to it.

And so that is the Part 2. And now we come to doing the third one-- it's actually the same problem-- where we are considering the radiation at the slightly more complicated angle. So far, we've looked at the charge is moving up and down. We said there's no electromagnetic field radiated upwards, or downwards, for that matter. Along the y-axis it is. And of course, if we took any other location in the plane of our x, y, in the horizontal plane, in all directions, it will radiate in a similar manner as along the y-axis.

And finally, just for completion, let's take one more direction. And what the problem said, let's consider going at 30 degrees to the z-axis. That's an angle like that. What is the electric field that is generating in that direction? Again, this is just to give us practice at calculating vectors, components, et cetera.

So I will do that now over here. So this is the solving the Part 3. So what we're interested in, we have this charge moving up and down. And we're interested in the radiation a long way away, a distance R away from this charge, but at some angle like that, with just 30 degrees to the z-axis.

So in order to use that formula-- it's all the time the same thing, I'm just using the same formula-- I need to calculate the component of the acceleration of the charge which is perpendicular to that direction, OK? Now there are many ways. You can do it in your head, if you are good at manipulating vectors.

Here, I wrote it, in case you wanted a complete formula. Here is how the perpendicular component is related to the acceleration, a. And at the beginning there, I summarized it for you. But you can do it any way you want. I did it this way.

And so I am taking this charge and calculating, at some time, t prime, what is the component of the acceleration of the charge in a direction perpendicular to the direction in which I am considering the radiation. And this is what it comes out.

If you remember, I know what is the motion of the charge. You've seen it over there. So I know what z is, a function of time. Therefore, I can calculate this. And that's what it comes out as, alright? Here, I formally use this equation.

So it's as before. We will be interested in the electric field at the position, R, so at a time which is later. In other words, t minus R/C is the time at which the prime time, t prime, at which I'm interested in this acceleration. And that's what it looks like, OK?

Now, I can simplify it. I'm not going to waste your time. You can go through this algebra. Simplify it here.

And then I take this acceleration and plug it into that equation, which tells me, if I know the perpendicular component of acceleration at this earlier time, I know what the electric field is at the time t. And this is what it comes out at.

So the electric field in a direction 30 degrees to the z-axis, or in other words, at a distance, capital R, from the origin. Or in other words, at the coordinates 0, R/2, R root 3 over-- and this is the y-coordinate, x-coordinate, y, and z-- at this time, I plug this into that equation. And I get this and the direction. If you do it formally, you immediately get the direction. It's not very hard. It'll be, in fact, it's 60 degrees to the z-axis.

All right. So that's what the electric field will be. Then, knowing the electric field, again, I know that E and B are perpendicular to each other. They are proportional to each other, except for the 1/C. All right?

And so, at the same location, I can calculate B. And I get this. The magnitude here and here are the same, except for one power of C here, C cubed.

The direction is different. B will be the x direction. And that, you could do in your head. If the charge is oscillating like this, at all angles, you'll still have the magnetic field, which will perpendicular in that direction. All right? And you can check for yourself. This half is simply the sine of 30 degrees.

OK. So I've gone very slowly, intentionally, and taken the simple situation where a single charge is oscillating up and down and I've calculated for you the electric field, the radiated electric field, up and down, at 0. In a horizontal plane, I've done it effectively, all locations. And I've done it at an angle, theta, where theta was 30 degrees.

I could repeat this for every angle, theta, every location. So in fact, just taking more and more cases, we have calculated the electric and propagating electric and magnetic field in all directions, all right? And we calculated the magnitude at a distance from the origin. OK?

It drops off like 1/R we saw. And like the other field, we drop off like 1 over r squared. And so this charge, which is oscillating up and down, radiates in all directions, but with different magnitudes, biggest in the plane perpendicular to the vector describing the acceleration.

OK. Now the next thing that I will do now is the following. Let's take the same situation and try to calculate the total power radiated by the charge. So I've taken this same problem.

There is the charge oscillating up and down with amplitude z0 cosine omega t prime. And what we see-- well, we know that it radiates energy. If it radiates energy, of course, normally, it would have to finally stop. But I'm assuming there is some mechanism which uniformly maintains the motion of these charges in a certain motion.

Energy will be radiated. And I want to calculate what is the total energy radiated per second by this charge. And the way we are going to do this-- so the question now is, for this particular charge, oscillating like we discussed, what is the total energy per second-- in other words, power-- radiated at the time t prime?

Clearly, if it's radiating energy outwards, by conservation of energy, the amount of energy coming out of this will be the same as the amount of energy crossing a sphere of radius, R, at a later time. So if I calculate the energy crossing a sphere at the time t, where t is this t prime plus R/C, at a later time, that will be the total energy per second radiated by this charge at the time, t prime. So I will now calculate the energy per second which is crossing the surface of this sphere at the time, t.

Well, how much energy is crossing the surface? We know that, at the surface of this sphere, there is, at every location, an electric field which is parallel to the surface, because this is perpendicular to R. The electric field is perpendicular to this. So at this location, for example, the electric field will be like this, crossing it, part of the surface of this sphere.

There will be a magnetic field perpendicular to it. And so, at this location, there will be a energy flux, the so-called Poynting vector, the energy per second crossing a unit [INAUDIBLE]. And that's this vector, s, the Poynting vector, which, as you know from Professor Walter Lewin's lectures, is E cross B. I forgot my vector sign on top of the B. These are vectors, of course, E cross B.

Now we know, for an electromagnetic wave in vacuum, that the E and B are perpendicular and proportional to each other, just the fact the C. So this will be the magnitude of E squared divided by this mu 0 times C, because B is E/C.

This is a scalar quant-- these were vectors. This is a scalar. So this is the energy flowing per unit area, per unit time at the surface of this. And the E will depend on the location on this sphere.

So what is the total energy that's leaving the charge at time t prime? It will be the sum of all the energies that are coming out per unit area along this surface at time t. So it's going to be the integral of SdA. What I'm doing, basically, calculating at every location S multiplied by the area there-- so that gives me how much energy goes through that area-- and adding them across the surface everywhere around the complete sphere. That will be the total energy leaving here.

So imagine this is oscillating up and down, radiating these spherical waves outwards. And they cross this surface. And at every place, there is energy flowing, the Poynting vector. And here, I'm adding them all. This will be the total power radiated by this charge at that earlier time, t prime.

OK. So I now have to just log through and calculate this. S, I have to calculate from this. And then I have to calculate the piece of area. Well, let me tell you what I will do.

From what we've learned from the earlier part, the magnitude of E only depended on this angle, theta, and this distance. So imagine, along the surface, I take a slice this surface, like this, where every point along this surface is at an angle, theta, to the z-axis, OK? And it's a thin slice. So what will be this distance?

I'll take this to be d theta. This distance is R, so that distance is already Rd theta. If I multiply that by 2 pi r sine theta, r sine theta is the radius of this circle. So this total area going all the way around this sphere is 2 pi r sine theta multiplied by Rd theta. OK?

So now along this, everywhere, S is the same. So if S is at that angle, theta, and I've multiplied by 2 pi R sine theta Rd theta, this is the piece of area, if I multiply by that, this gives me the total energy flux or power that is leaving the sphere along this part, all the way behind there, all the way around. OK?

Now what is the total energy per second leaving this complete sphere is the addition of pieces like that for the complete sphere. So it would be the integral of this for theta going from 0 to pi. And then I've covered the compete sphere. So that will be this integral, which is the power radiated by the charge.

OK, if you've understood that, then it's just pure algebra now. S, I can calculate, because I know it's the magnitude of E squared over C mu 0. But that, we've already done. We did it for 30 degrees. And now do it for theta.

So here, instead of having sine 30, which was 1/2, I have sine theta there. So that squared gives me this divided by C mu 0 that. And of course, it depends-- so this is what E is.

But it's oscillating as cosine omega times this. And we have to square it, because this is E squared. So this is what the Poynting vector is at, the angle theta, distance R away at time t, OK? And therefore, the total power is this multiplied by that integrated from 0 to pi.

I've just rewritten it, this quantity, plugging that into here. I end up with this. Everything does not depend on theta, except sine theta, which you have a squared from here and another sign from there. So I get sine cubed theta d theta.

OK, we're home. So that's the answer. Because this you can do, I won't waste your time.

The way to do it is you can take sine theta d theta to be d cosine theta, and then integrating sine squared theta d cosine theta. And if you do that, you get 4/3. So I plug 4/3 in for that integral. This is the answer, OK?

So what we see now, through any-- and it's independent of R, which I hope did not surprise you. It's a statement of conservation of energy, because you have this charge oscillating up and down, radiating. Through every sphere of radius R, the same amount of energy has to go through, or otherwise, you' d be gaining or losing energy.

So that's a check. You can check. If R came into this formula, you would have made a mistake. And the only place where it does, is only-- it oscillates, so at any position, R, the energy going through oscillates, because it's a wave going out, OK? So this changes the phase, but not the magnitude.

In the actual question, it asked for what is the time average that's radiated outwards. So we want to know the time average of this. And that's the time average of that is here.

You know that the time average of cosine some function of t is 1/2. If you take cosine-- sorry, cosine squared, which was in here, cosine. So the time average of cosine squared is 1/2.

So finally, we get that the average, time average, power leaving this oscillating charge is given by this formula. It's actually a famous formula. It's called [? Lambor ?] formula. OK?

So that's the total energy radiated. Fine. We have a few minutes left, and so I'll do one more quick problem.

OK, just for variety-- so far, we've considered a charge which is accelerating up and down. And be considered all permutations and combinations about what happens to the electromagnetic field going outwards. Now let's consider something else, a charge which is going in a circle.

Let's consider that a charge, Q, which is rotating in the z, y plane with a uniform velocity, v, given by omega z0 where omega is the angular velocity of this. So we have a charge going like this, uniform velocity. What is the radiated field? What radiated electric and magnetic fields are produced?

Well, go back to where I started. The charge is moving with uniform velocity. Static charges don't radiate, but accelerated charges do. Does this charge accelerate? It's moving with a uniform speed in a circle, but does it radiate?

Well, you know that, if an object is going with a uniform speed, V, around a circle, it's all the time accelerating, right? It's accelerating inwards all the time. And so this charge is accelerating inwards. So at every instant of time, it's accelerating like this, so as this moves more.

So all the time, it has a constant magnitude of acceleration, but the direction changes all the time. So this charge, at every instant of time, will radiate. And it will radiate in all directions, like before.

For simplicity, so we don't get overburdened by mathematics, let me just think for a second what a-- let's just talk qualitatively. I can describe the motion of a charge going like this by, at any instant of time, it has a position.

This is Z0 vector is the position of this charge at time t prime. I could write it as Z0 cosine theta y plus Z0 sine theta Z, all right? That's the y-coordinate. That's the z-coordinate. If that is the position, I can rewrite this like this, because theta, of course, is given by omega t prime. It's rotating, right?

What is the acceleration? I differentiate this twice. So here is the acceleration.

So if we consider the two components, what I see is that this charge has a component in the y direction and in the z direction, like this. And so it will, at every instant of time, radiate. I can decompose the radiation, using this formula, into the two components. And what we see is that the two components are out of phase from each other by 90 degrees.

So for example, if I am straight ahead looking at it, I will see a charge which is accelerating up and down, like that, and out of phase, like this. So it will radiate straightforward a component like this, oscillating, and out of phase by 90, like that. And what is that? You know. That's called circularly polarized light.

There are many ways to look at it. You can either look at the components, or you could look at the rotation of the electric vector. At the moment, I am looking at this from the point of view of components.

So if I look at this, straight at it, what I will see, an electromagnetic wave with the electric vector polarized vertically, and another one polarized horizontally, out of phase by 90 degrees. And so, at any location in space, I'll see an electric vector which is rotating like this. Of course, there's a magnetic vector perpendicular to it. And that we call, circularly polarized light. So this will radiate in that direction circularly polarized light.

If I look from above, what do I see? The charge is going like this, so I just see this component from above-- you can look at the two components-- like that. So straight up, I'll see linearly polarized light. If I look from the side, I see it's going up and down. I will see also linearly polarized, but in a different direction.

So a charge like this, as before, will radiate in all directions. But the extra complication now is, in different directions, it'll be different polarization. For example, if I'm looking at this from here, this crazy angle, what I will see is I will see a component from the acceleration like this, and from that, but they have different amplitude. And so I'll get the two polarizations which are different amplitudes, corresponds to elliptically polarized light. So in this direction, I'll see an elliptically polarized light. And using this formula any place in space, you can calculate the electric and magnetic field, just doing the same as we did before, but take the full vector description of the perpendicular component of the acceleration of the charge.

So let me stop there and just sort of summarize. In principle, we've done just very little. What we showed today is, in general, if I have a charge which moving with some velocity, some acceleration, et cetera, it'll be surrounded by a very complicated field. There will be the Coulomb field, the Biot-Savart field, the radiated field, et cetera.

If you need to solve the complete thing properly, you will need a computer. You can't do it. But if you are only interested in the radiated field which is far away from the accelerated charge, it turns out the situation is sufficiently simple, you can do it almost on the back of an envelope.

All you need to know, if you are interested in the field in any location, is you ask yourself, at an earlier time, what was the charge doing? And by earlier, I mean at time that light had time to come from the charge to me.

So I take that distance, R/C, and at that earlier time, I need to know what the charge was doing. I ask myself, in which direction it was accelerating? I take the magnitude of acceleration, which is perpendicular to the direction in which I'm looking, and I calculate that and multiply by some constants that we've done over and over again, and that will tell me what the electric field is. From that, I can get the magnetic field, and I get the full radiation. Thank you.