Welcome back to 8.033!

Alexander Friedmann
1888-1925 (1922 paper)

Image courtesy of Wikipedia.

"As Copernicus made the Earth go round the Sun, so Friedmann made the Universe expand."

(1993 biography)

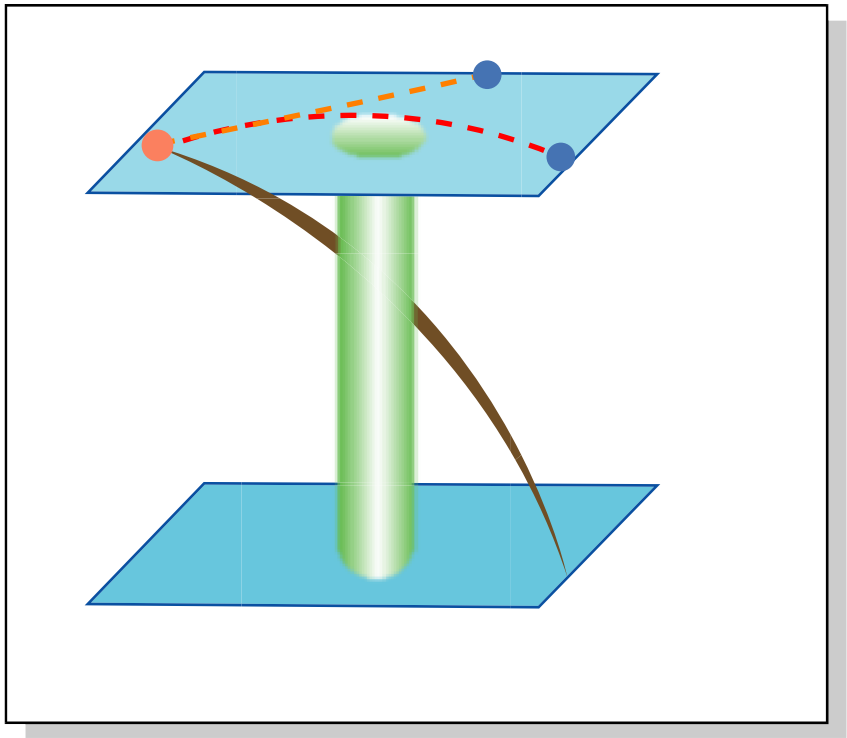


Figure by MIT OCW.

Summary of GR so far:

- **Unification:** gravitatation equivalent to acceleration

(accelerating frame indistinguishable from gravitational field)

- **Law of motion:** you move along geodesic
- **Key to problem solving:** working with metrics, variational calculus

- In GR, it's convenient to use units where $c = G = 1$, simplifying these metrics:

Implies special relativity

- Minkowski metric:

$$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2$$

Implies both gravitational redshift and Newtonian gravity

- Newtonian metric:

$$d\tau^2 = (1 + 2\phi)dt^2 - dx^2 - dy^2 - dz^2$$

(Why?)

- Minkowski metric in polar coordinates:

$$d\tau^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

- Friedman-Robertson-Walker (FRW) metric:

$$d\tau^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right)$$

- Schwartzschild metric ($r_s = 2M$):

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2,$$

- In GR, it's convenient to use units where $c = G = 1$, simplifying these metrics:
- Minkowski metric:

Implies special relativity

$$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2$$

- Newtonian metric:

Implies both gravitational redshift and Newtonian gravity

$$d\tau^2 = (1 + 2\phi)dt^2 - dx^2 - dy^2 - dz^2$$

$\phi = -MG/r$, so how much time elapses on a clock

- at rest far away?

- moving far away?

- at rest on the ground?

- in GPS satellite?

- In GR, it's convenient to use units where $c = G = 1$, simplifying these metrics:
- Minkowski metric:

$$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2$$

- Newtonian metric:

$$d\tau^2 = (1 - 2\phi)dt^2 - dx^2 - dy^2 - dz^2$$

- Minkowski metric in polar coordinates:

- Schwartzschild metric ($r_s = 2M$):

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) d(ct)^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2,$$

MIT Course 8.033, Fall 2006, Lecture 17

Max Tegmark

Today's topic: Cosmology 1/4

- The FRW metric
- Expansion of the Universe
- Age of the Universe
- Brief history of the Universe

Key formula summary

- FRW metric:

$$d\tau^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right)$$

- Hubble parameter:

$$H \equiv \frac{\dot{a}}{a}$$

- Dimensionless current Hubble parameter:

$$h \equiv H_0 / (100 \text{km s}^{-1} \text{Mpc}^{-1}) \approx H_0 \times 9.7846 \text{Gyr}$$

- Friedmann equation:

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \\ &= H_0^2\left[\Omega_\gamma(1+z)^4 + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm k}(1+z)^2 + \Omega_\Lambda\right] \end{aligned}$$

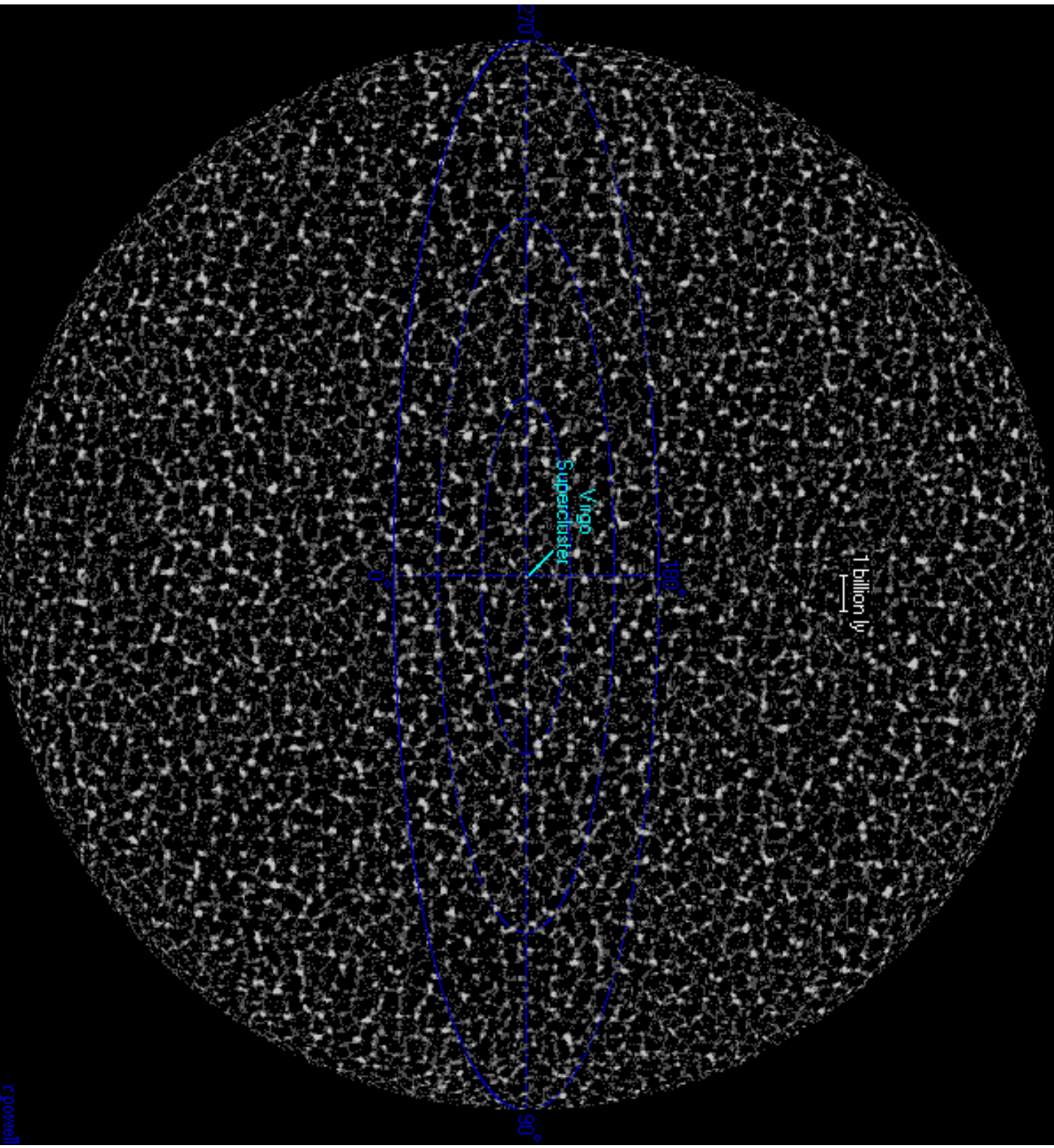
- Cosmological parameter measurements (2005):
  - $\Omega_b \approx 0.05$,
  - $\Omega_d \approx 0.25$,
  - $\Omega_\Lambda \approx 0.7$,
  - $\Omega_{\rm k} \approx 0$,
  - $h \approx 0.70$,
  - $\Omega_{\rm m} \equiv \Omega_b + \Omega_d \approx 0.3$,
- Age of the Universe at redshift z:

$$t(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

KEY FACT 1:

The (observable) Universe is homogeneous & isotropic (on large scales)

Image courtesy of Richard Powell, www.atlasoftheuniverse.com. Used with permission.



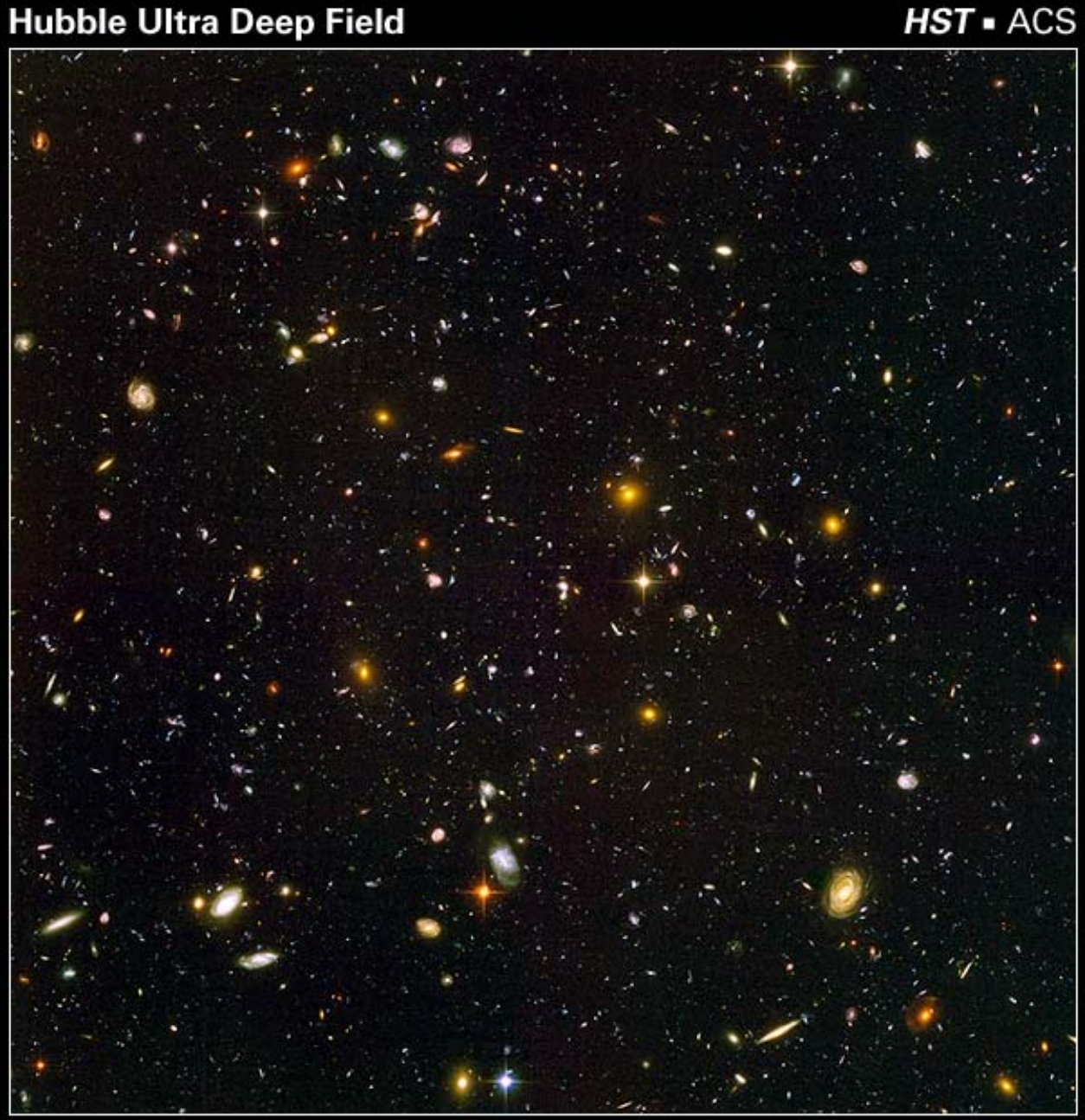


Image courtesy of NASA.

Image of Kitt Peak observatory, courtesy of Wikipedia

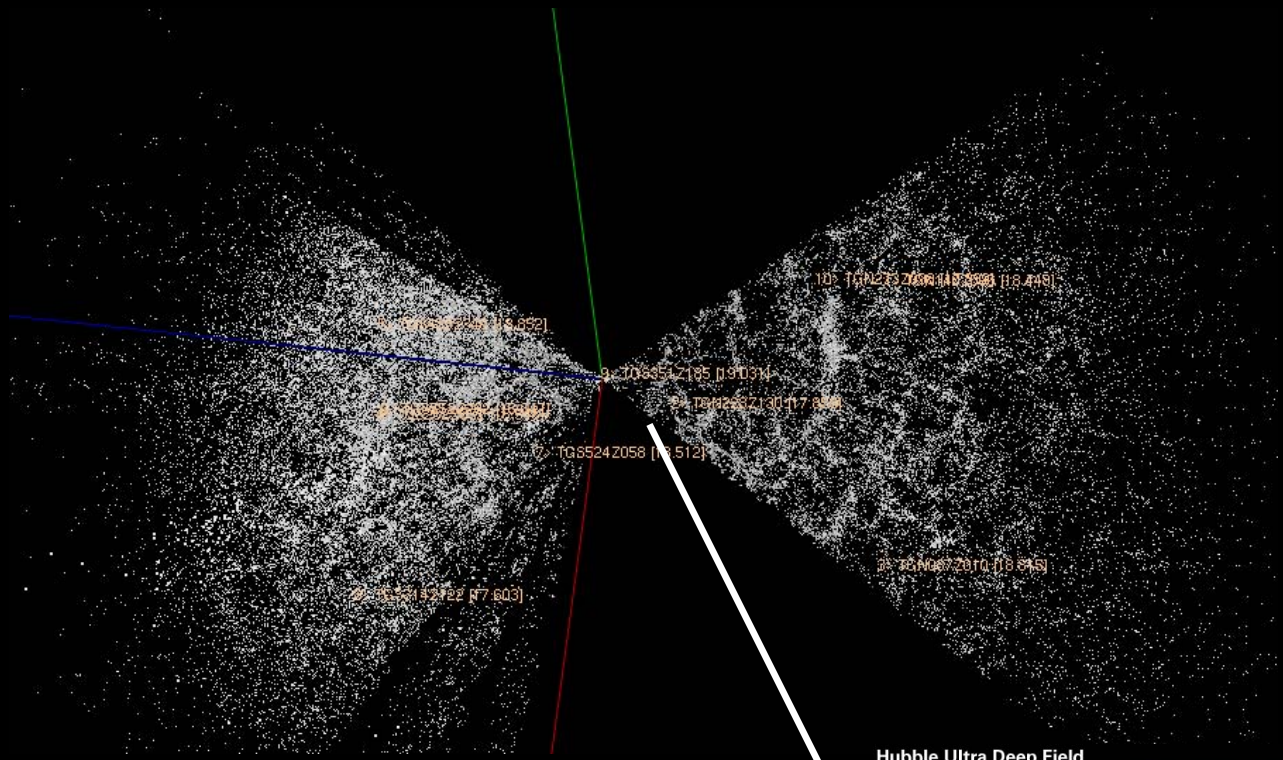


Image courtesy of Enoch Lau/Wikipedia.

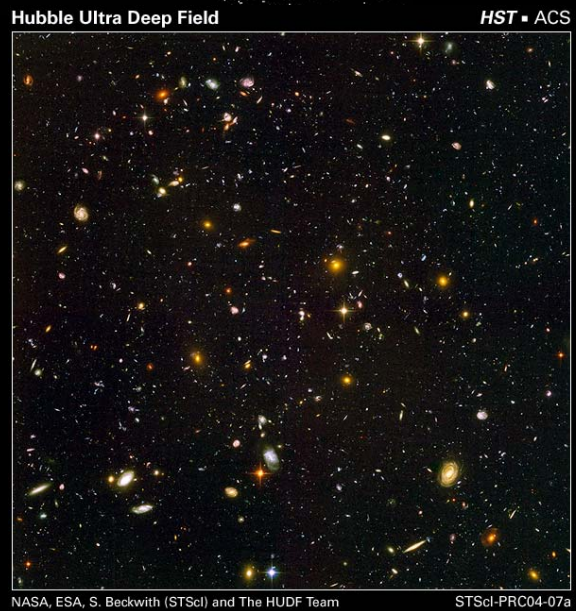
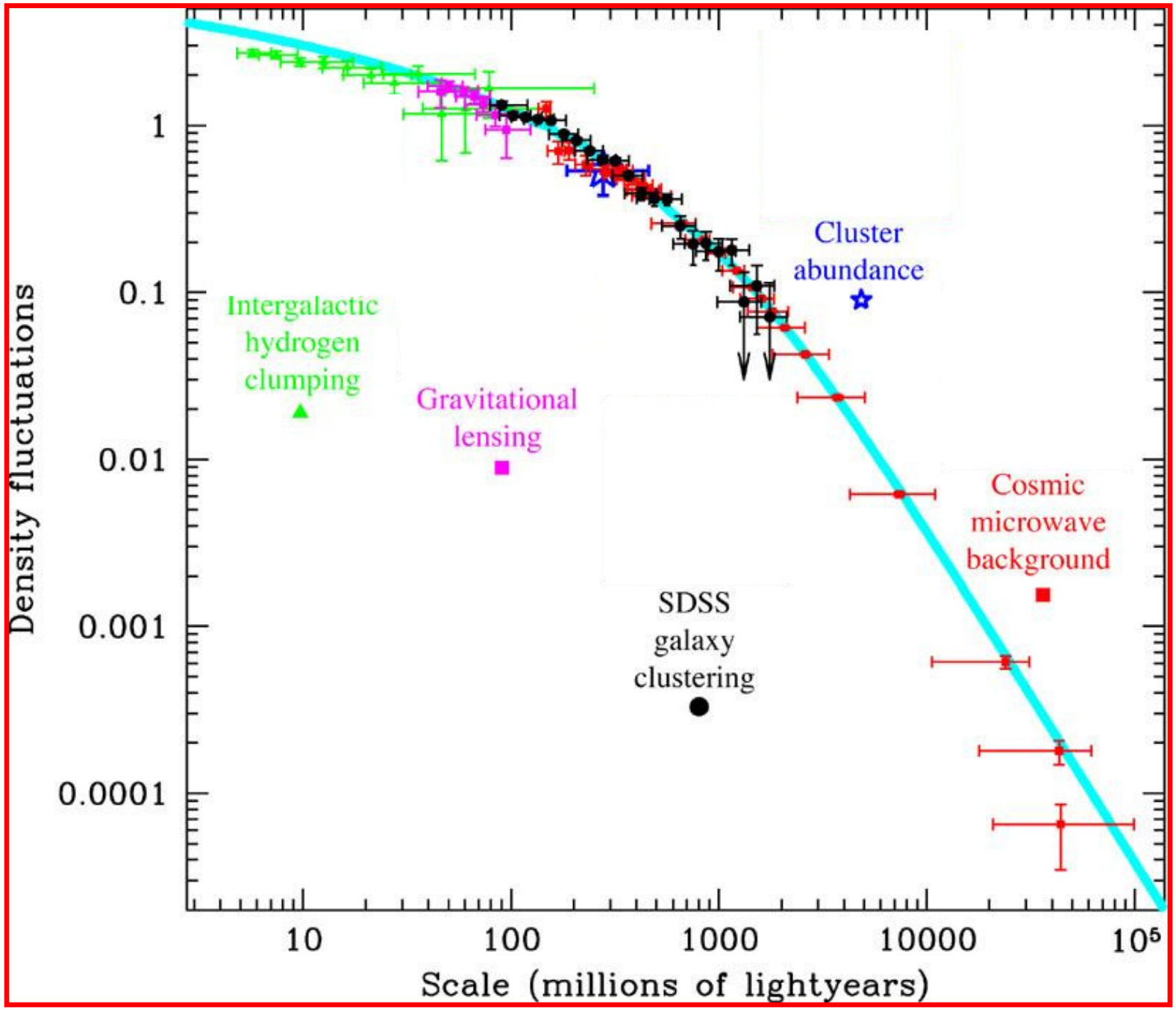


Image courtesy of NASA


Density fluctuations
1
0.1
0.01
0.001
0.0001
10
100
1000
10000
10⁵
Scale (millions of lightyears)
Intergalactic hydrogen clumping
Gravitational lensing
Cluster abundance
SDSS galaxy clustering
Cosmic microwave background

Alexander Friedmann

1888-1925

(Russian; 1922 paper)

George Lemaître 1894-1936

(Belgian; indep. 1927 paper)

Arthur Geoffrey Walker 1909-?

(British; 1935 paper with H P Robertson showed that FLRW metric is *only* homogeneous & isotropic metric)

Key formula summary

- FRW metric:

$$d\tau^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right)$$

Let's derive some implications!

- Hubble parameter:

$$H \equiv \frac{\dot{a}}{a}$$

- Dimensionless current Hubble parameter:

$$h \equiv H_0/(100\text{km s}^{-1}\text{Mpc}^{-1}) \approx H_0 \times 9.7846\text{Gyr}$$

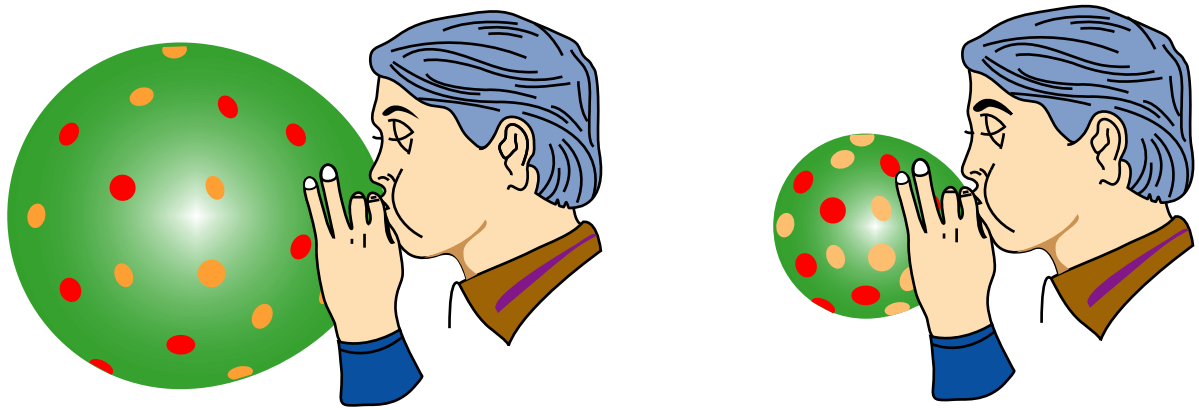


Figure by MIT OCW.

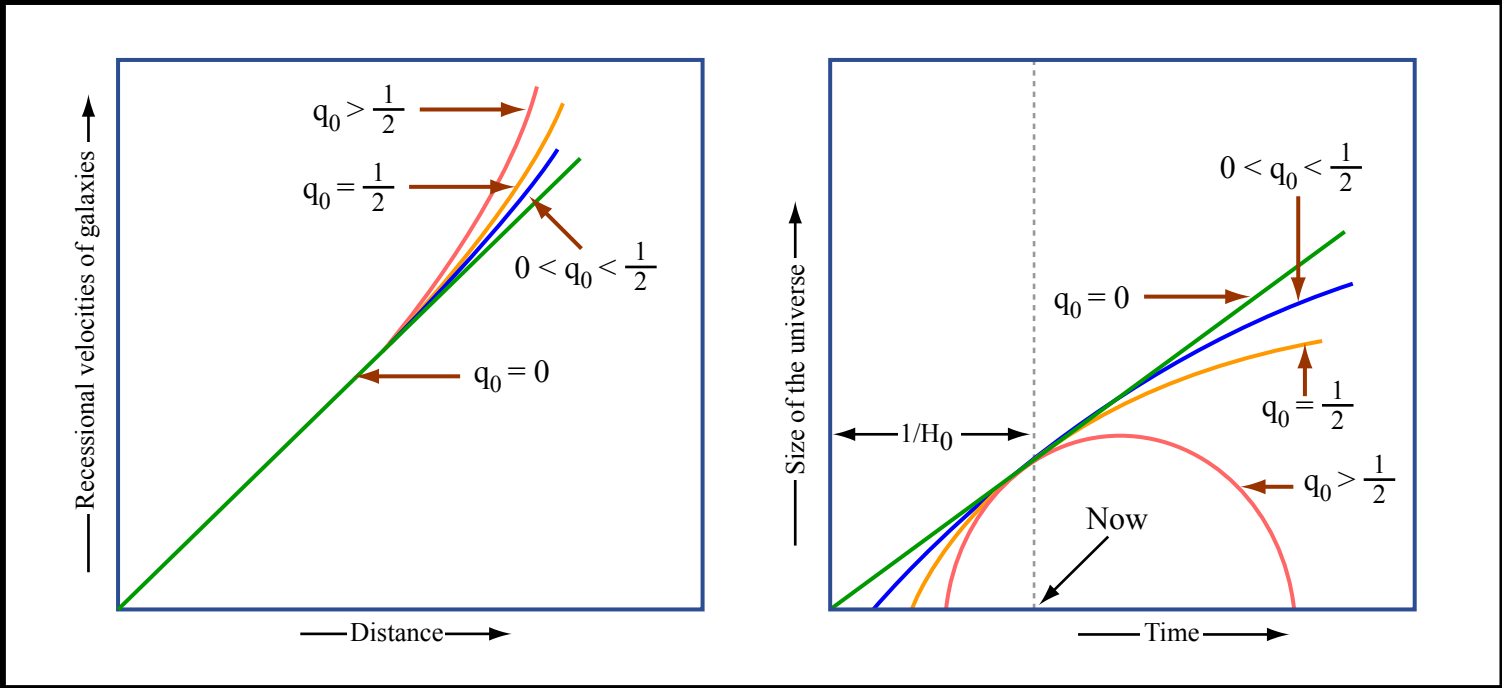


Figure by MIT OCW.

Figure 8 from "What is the Universe made of? How old is it?" by Charles Lineweaver.
http://arxiv.org/pdf/astro-ph/9911294