PROFESSOR: Today Is a big day for the Physics Department and for MIT. Professor Frank Wilczek sharing the Nobel Prize in Physics for his seminal work when he was a graduate student on quantum chromodynamics. We are now ready to tackle the normal modes in continuous mediums, and I will do that starting first with a string which is fixed at both ends. The one way you could do that is you can go back to the results of last lecture when we have capital N-bits, you can make n go to infinity. And the shapes that you get for your continuous media, for the strings when they're fixed at both ends, are sinusoidal motions. This would be the first harmonic, the lowest frequency, and then you get higher harmonics with higher frequencies.

The question now is, what are those normal mode frequencies for those continuing media. And I will derive these today-- I will derive these normal modes frequencies using a different approach than the n go to infinity route. And I'll find the same result, of course. So imagine that I have a string here and the tangent is T, and the mass per unit length is mu. And I wiggle one side and I generate in there a wave. So I'll move this up and down with angular frequency, omega, and I do that with an amplitude A.

I will then generate waves whereby this, is what we call in physics, the wavelength. That is the distance that the disturbance travels in one oscillation time. If I call this the positive direction, this wave will propagate with speed, v, in that direction. And we know what v is. v is the square root of T divided by mu-- we derived last time. And so y as a function of x and t-- y being in this direction, x being in this direction-- is that an amplitude A, times the sine-- or if you wish, a cosine, that's fine with me-- times 2 pi divided by lambda times x minus vt.

Let's first make t equal zero. Then you see what you have here-- the sine 2 pi over lambda times x. If you take x zero, then you find zero. And if you make x larger, by an amount lambda, that you get zero again. So that's why lambda is called the wavelength-- The pattern repeats over a distance lambda. Clearly this must be a solution to the wave equation because any single valued function, I told you, which is a function of x minus vt, satisfies the wave equation. And so this one does too.

Now lambda is v-- which is this v-- times the period of one oscillation, for which I will write a P today, not a T, because I don't want to confuse you. And that P is obviously 2 pi divided by omega-- this is my omega with which I am shaking. And so omega, which is then my frequency, is 2 pi times v divided by lambda. I will introduce, for 2 pi divided by lambda, a new symbol which I call k, which is called the wave number. It has dimensions, meters minus 1. This is done in most books. It's unfortunate that French calls k 1 over lambda. It's not his fault because in the old days it was always done that way.

I will follow the convention of Bekefi and Barrett, and I will call k always 2 pi over lambda. If I use that new k, then I can rewrite this in a form that you see very often-- but there's nothing wrong with that form-- that would now be A times the sine of kx minus omega t. Notice the 2 pi lambda over lambda becomes k, so that is kx, but omega is always k times v. And so this form is nice in the sense that if you have these two numbers, you know immediately what the frequency is. That's non-negotiable. And you know immediately what the wavelength is. It's 2 pi divided by that number.

And you even k now that the ratio of those two-- omega divided by k-- is your v, which is that v. So it's a nice way of writing things down. Now I'm going to not only generate a wave on this side that moves in this direction, but I'm going to generate one that goes in this direction. And then I want to see what they do together. And so this is now the wave that goes in the positive x direction-- I call that my positive direction. And now I'm going to have another one, y2 x, t-- same amplitude, same frequency, therefore same wavelength-- but now it's going not in this direction, but it's going in this direction.

Notice there's a plus here but it is a minus there. And so I want to know now what the sum of these two is because one is going like this, another one is going like this. So I want to know what the superposition of those two waves do. And so y, which is then the total displacement, is y1 plus y2. And so I get the sine of alpha plus the sine of beta. That is twice. So that becomes 2 A times half the sum-- the sine of half the sum. Half the sum becomes kx times the cosine of half the difference.

Half the difference would be minus omega t, but minus and plus for cosine is the same thing. So I will just write down cosine omega t. And this now is a very unusual wave. All the spatial information is here-- so this is all the spatial information, and all the time information is here. And so whenever the sine is zero, if the x is such that the sine is zero, it's tough luck. Then that location x never moves. The cosine omega t could never make it move. It always stands still.

We call those nodes and I will show you, of course, some examples. This has a name, a very nice name. It's called a standing wave. Whereas those that I have there, we call travelling waves. Now I'm going to make the string, fix the distance, and fixed at this end. Well I may shake it a little bit here. You will see a demonstration of that.

And now, at this x equals zero, and at this x equals L, I now have boundary conditions that I have to meet. y cannot be anything but zero there because it's fixed. And so I now demand that, at x equals zero, and at x equals L, that y must become zero. And so the only way that that can be done is that you only allow certain values for k to exist. And those values for k, which I will now give an index n, as in Nancy, which is going to be the normal mode-- n equals 1, 2, 3, 4-- is now going to be n pi divided by L.

Clearly if x is zero-- sorry, we're here-- if x equals zero then surely we are here, then the whole thing is always zero. That's not an issue. But you see now if x equals L, then you get n times pi. And so again the sine becomes zero. So I've met this boundary condition at this point-- cannot move. So lambda then, which is 2 pi divided by k over n-- that's the way that I defined k over n-- then becomes 2L divided by n-- n being that 1, 2, 4, 3, 5. Omega n, which is always k over n times v, therefore becomes n pi v over L. And the frequency in hertz, if you prefer that, is 2 pi smaller, would become n v over 2L.

And so what are you going to see when you plot this function-- this standing wave function. Well it depends on the n number. Let's take n equals 1. So the length is L.

If you plot that sinusoid for n equals 1, it looks like this. And then the cosine term will make it do this. That's the task of this cosine term. The sine term is just this thing, which would then have an amplitude that could be different, of course, for the different modes, but it would be this amplitude that you see here.

We call this one the fundamental. It's the lowest normal mode but we also call it the first harmonic. I will do both. I will sometimes refer to this as a fundamental because I'm more used to that, but I will also refer to it as the first harmonic. Lambda 1 is 2L. L. That's just staring you in the face. In order to make a full wave out of this, you have to add this. So lambda 1 is 2L . And of course when you look here, lambda 1, is 2L divided by 1. That's exactly what we have there.

So now we go to n equals 2 which is called the second harmonic. So let me write this down. So this is called the fundamental, which is also called the first harmonic. And this is then called the second harmonic, whereby the point in the middle stands still. And then the cosine term will make it change shape like this. It is unfortunate that there are books that call this the fundamental, and this the first harmonic.

That's enormously confusing because now you have to start from zero which is the fundamental, and then n equals 1 is the second harmonic. I will never do that. This is always will be first harmonic, this is always the second harmonic. And so for n equals 2, you see immediately that lambda 2 is simply L. And that's exactly what you see here. And then you can put in the third harmonic. And I will demonstrate this very shortly. So here we have an omega 1, and here we have an omega 2.

And they follow this pattern. Omega 2 is exactly twice omega 1, because when you make n equals 1, you have omega 1, and when you make it 2, you get twice that much. So now you see that the ratio's-- omega 1, omega 2, omega 3, relate as 1:3, to 4 to 5. And so you could also write down here then that omega n, this n times omega 1, and therefore that f of n, which is the frequency in hertz, is also n times f1. So it's very easy to think in terms of the series of these harmonics. They are also sometimes called overtones. When we talk music, next Thursday, I will often use the word overtone.

So if the lowest frequency in this mode were 100 hertz, then the second harmonic would be 200 hertz, the third harmonic would be 300 hertz, and so on. Now when I'm deriving this string at one end, I will have a wave going in and have a wave coming back. That is exactly the recipe that I need for a standing wave. I get reflection here. And so I have one wave going in and I have one wave coming back. And if I derive these at these discrete frequencies, which are set by the boundary conditions, then the system will react very strongly-- will go into resonance.

You build up a huge amplitude because you keep feeding in waves and they keep coming back. So the whole thing starts building up. And that is the idea of resonance. And resonance and normal modes are one and the same. So that's the normal mode overseas strings, fixed at both ends. We also refer to them sometimes as natural frequencies. These points here have a name. They're called nodes. So this is a node, and this is a node.

And these points here, also here and here-- the ones that have the largest amplitude-- are called antinodes. In Dutch we call them tummies, very strange. We call them barker, and barker is this. So the boundary condition leads to discrete values of the resonance frequencies. And if this were quantum mechanics, we would call these eigen solutions and eigen states. So if you want to write down now the situation for your n's mode, in it's most general form, then you would get y as a function of x and t. And the n's mode would have its own amplitude, A of n, whatever that is, you can pick that for different values of n.

And then you have here the sine of n pi times x divided by L. And here you have the cosine of omega nt. That then meets the boundary conditions for two fixed ends. And any linear superposition, any combination of various values of Nancy, n, and various values of A, will satisfy the wave equation. So this string can simultaneously oscillate in a whole series of these normal modes. And when we do music next Thursday, you will see that actually. I will demonstrate that to you.

What I want to show you now is that if I take a string-- I need again assistance-- even though it has spring-like qualities, we will treat it as a string. Then I will show you that if I drive this at the end at proper frequencies, that I can generate the resonances, and I could make you see the normal modes. So who is willing to assist me this time? You were trying last time also right, but Nicole won the battle then. OK. So hold it in your hands firmly. Just walk back and do nothing. Just walk back, walk back.

We need a little tension on there. OK, that is fine. So I'm now going to wiggle this-- this is really a fixed end. You will see when I hit resonance that my hand is hardly moving at all because I keep pumping waves in and they keep coming back at me and the amplitude will build up. And then I have to search for these resonances. And when I hit one, I know I hit one. You will see why I know it. I feel it in my stomach. I feel it in my tummy. I feel it in my brain, in my hands, everywhere. My whole body knows I hit resonance. And you will see that.

Ok, there we go. This is the lowest mode. This is n equals 1. And notice that my hand is hardly moving at all. So my hand, for practical reasons, is really a fixed end. So you get that solution. So I will now try to find the second harmonic, which would end up then as a node in the middle, in addition to nodes at the end. Is that it? And again, when I hit that resonance, which is a normal mode, notice that my hand is hardly moving.

And boy, do I feel it. I really know that I am on resonance. I can try the third one, in which case you would see two nodes in the middle, and two at the ends. Is this the third one? Wow, I'm good today, am I not. You see, very clear, these points stand still in normal modes. Oooh, you'e not supposed to do that. Oh man, you're ruining my demonstration.

So now I will try to generate the highest frequency normal modes that I can. And so you count the number of nodes in the middle. And if you find five, then that's the sixth harmonic. You make-- I think I could do better than that, but it's not so easy to get a very high frequency. So you do nothing because you would really ruin it. Yeah, yeah, yeah, yeah. I'm getting them, getting them, getting there. Got it, got it. Oh my God. Yeah, no no, no. You shouldn't have moved, you see. No, it was not your fault.

I'll try it again.

I got another resonance, I got a resonance, I got a resonance. Count, please count. How many did you see?

AUDIENCE: Five.

[INTERPOSING VOICES]

PROFESSOR: Five.

AUDIENCE: [INTERPOSING VOICES]

PROFESSOR: I counted 12 nodes in the middle.

AUDIENCE: [LAUGHTER]

PROFESSOR: What is your name?

AUDIENCE: Mike.

PROFESSOR: All right, so this is the way it works, that you see normal mode solutions which either result-- which result in standing waves. And the demonstration speaks for itself. I have here a rubber hose which we are going to excite in the fifth harmonic. Let's see how that works, whether we were close. If anyone touches the table, there then the tension changes. And immediately, of course, the resonance frequency changes because the resonance frequency-- notice-- has this v in it.

So the moment you change t it's different, but it's not bad. And so now I will strobe this one for you. Because you have no idea here that what is happening that is going like this. It is going too fast for you. And so I make it easy on you. Would you turn the lights off? Thank you. So I'm going to strobe it for you. And then I can, more or less, make the rope stand still. But I can also offset the frequency of my strobe light a little bit, so that you actually see the rope move very slowly.

So I can purposely give the strobe light a slightly different frequency. So that's what I'm doing now. You see too [INAUDIBLE] so you have no doubts anymore about the fact-- no doubts-- that indeed, if the middle goes up, the one on the side goes down and vice versa. What I can also do, to turn this into a work of art-- if I can turn this one off again. Yes I can. I can also-- I worked with an artist who actually liked these things a lot. So I can also strobe it twice as often.

So this is actually-- the frequency of the rope is about nine hertz, so my red light was close to nine hertz. The green light is 18 hertz-- and a little bit off. So now you see it twice per oscillation. Of course, to make it really wonderful, to make it very sexy--

AUDIENCE: [LAUGHTER]

PROFESSOR: --I can do them both simultaneously. So now you see the red one doing its own thing, and the green one doing its thing. And what I find interesting, it may not work for you, but there are moments, occasionally, that the red exactly overlaps with the green. When that happens I see white light. Let's see when-- whether we can have a case like that. It can only-- yeah, there was it, at the bottom. Did you see it? It's white. It's really amazing. And again, at the top. All right. So you can have some light again. Thank you, Marcus.

I can now do the same thing for sound. These were transverse waves but there's nothing wrong with doing the same thing for sound. In which case, you would have a tube which has a certain length L. Say it's closed on both sides for now. And I can now generate in there pressure waves-- make the air column oscillate. And the air particles, in the lowest mode, all I have to do is what is here in the y direction, for this is a transverse position. I now have to offset the molecules of the. Air in the x direction. So this is the x direction.

And the direction in which they move is in the x direction. But the displacement I will call psi-- how much they displace from equilibrium. So in the lowest mode-- that means in the fundamental, in the first harmonic-- in the middle the motion would be very large, like the y is very large. And then it's a little smaller, and a little smaller, and here it would stand still. And here it's a little smaller, here it's smaller, and here it stands still.

So therefore, in terms of psi, which is the position of the molecules, this would be a node, and this would be a node, and this will be an antinode, in the case of the first harmonic. And so I could write down that psi, in its ends mode, as a function of x and t, is then effectively the same as my y that I had before, except I have to replace y now by psi. So I give it some amplitude, A of n, and I get the sine of n pi x over L and I get the cosine omega nt. So the equation is completely identical. However, keep in mind that omega n, which is this value-- which is n pi v over L-- that is no different. That is again n pi v over L, but v is now the speed of sound. It is non-negotiable. v is 300 meters per second, and you're stuck with that. And that has major consequences, as I will show you Thursday, for the design of wind instruments. Whereas with string instruments, you can manipulate v and you can change t and mu. You can make the tension in the strings larger or smaller.

You do not have such an option with wind instruments. The only thing you can play with is L, for this is the approximate speed of sound. More often than not, do we express the standing wave, in case of sound, in terms of overpressure. So we don't look at the displacement of the air molecules, which I did here-- which is psi-- but we want to know where the pressure is larger than one atmosphere and where it is lower than one atmosphere. It's the same idea, but it's very often done. Now keep in mind that if these particles flow in this direction and pile up here, that the pressure here becomes higher. And the pressure here becomes lower because the particles move away from it.

So that means where you have a node in psi, you always have an antinode in pressure. So these are the locations where the pressure becomes high. And the pressure here, the overpressure, over and above ambient, is 0 here, because there's nothing that prevents the pressure from building up. These particles are free to move. And so if you write it down in terms of P, which is now overpressure-- it is not the total pressure, but it is what is over and above one atmosphere-- then you get some amplitude in pressure. I give it just a P of n.

And now I get the cosine of n pi x over L, and then I get the cosine of omega nt. And you understand now why you get the cosine-- because the antinodes are now here, at the ends, which is the consequence of the boundary conditions. If I drew a curve-- if I made a plot of the pressure-- I will do it here, if it doesn't become too cluttered. So here is now 0 and is L. If I put here the pressure, then in the first harmonic-- the fundamental-- here is a pressure node in the middle where there was an antinode from psi there is now a pressure node in the middle. And then the pressure will change in this way.

So this is now n equals 1. So the pressure here, at one moment in time, is high. Then it is 0 here. And then it is low here, it is below ambient pressure. And that, of course, is changing with time-- with the cosine omega t term. And again you see that lambda 1 is, of course, 2L. There's no difference there with the string. I can demonstrate this to you in really a very, very nice way.

I have here a tube which is closed at both ends. And so here it is. But in one end I have a piston, and so I can change the length L. You'll see in a minute why I like to change that length L. We're going to drive the inside with a microphone-- so here's a microphone. And we're going to do that with a frequency-- we really wanted to do 803 hertz, but 803 hertz has a rather long wavelength-- I wanted to have it shorter. So I do it 3 times 803. So the frequency is 3 times 803 and you [INAUDIBLE] which is 2409 hertz.

And so the wavelength lambda, which is v divided by the frequency-- yeah, that's correct, the wavelength is lambda-- equals v divided by the frequency, is then very roughly 14 centimeters. So that gives you an idea, it's about this much. In here we have a microphone that we can move in and out. And a microphone measures pressure as a membrane and so it's sensitive to pressure. And this microphone is connected to a loudspeaker so you can hear it, but we also show it to you-- the signal of the microphone-- we will show it to you as we record it on an oscilloscope.

If I put the microphone at a pressure node, you hear no sound. And if I move it to an antinode, you will hear sound. I will create in here pressure nodes, nodes, nodes, nodes, nodes, nodes, nodes, nodes, nodes, which are always half a wavelength apart. Think about that, why that is. Here the nodes are half a wavelength apart. The same would be true for sounds. I'm then going to put this microphone somewhere at a node that we all agree you hear almost no sounds. And we see no signal from the oscilloscope.

And then I'm going to move it back and search for one, two, three, four nodes. And then I know that this distance is going to be 2 lambda. And I have therefore measured lambda. And the amazing thing is-- which actually surprised me-- that you can do that to an accuracy of about one millimeter. It is so enormously clear when you are at a node, that if you move your mic by one millimeter, you can really see that you are no longer at the node.

So we will know lambda probably to one millimeter accuracy. Once we know that, the speed of sound is lambda times f. But we know what f is. That's known to one part-- we couldn't be off by more than one hertz, so that's very well known. And so we now have a measurement of the speed of sound-- high degree of accuracy we measure the speed of sound. And so we catch three birds with one stone.

First of all I could show you the nodes and antinodes, so you see how they build up there. And then, at the same time, we can measure the speed of sound. The only reason why we make this end movable is that as I move it and bring the mic first at a node, I can make sure that the length is just at resonance. And so that's just my beginning-- to make sure that the node is as close to zero as it possibly can be.

And so if we now get the image up there. And then I think we're going to get the light situation like this. And I'm going to turn on the sound. Here is the sound. I can turn up the volume so you can hear it better. Is the scope connected, Marcus?

MARCUS: Microphone.

PROFESSOR: Oh yeah of course, thank you. I didn't connect the microphone yet. It has a-- has a switch. Ok, now we first move the-- oh, awful. This is 2409 hertz, by the way. I'll turn it down a little. Is that better? OK. So let's first move it around to a-- let's first bring it to a node. Boy, there nothing left anymore right? And now I'm going to change L. OK, I think this is as-- I cannot do much better. So what L is is not very important.

I set L so that I think the system is at resonance in a mode that you can calculate, if you know how many nodes there are. So I'm going to search for a-- oh, here you see the antinodes, by the way, you see that? I'm moving it in now. I'm moving it further it that way. And here you get to another node. You see that? Isn't that wonderful? And I go again to an antinode. And here comes another node.

And this node, I'm going to measure the position. I have a ruler in there, and the ruler gives me the position of the mic. And this one is 52.8. So 52.8 centimeters is this position here somewhere. And I can put a mark here for you-- not that it will help you very much because my accuracy is one millimeter and this is a very crude way. So now I'm going to pull it back. You're going to see an antinode and you hear it. A little bit more sound?

And you're going to see a node. That's number one, node number two, node number three, node number four. And I'm going to look-- and I'm going to look now at the reading, and I read 24.3. So I read 24.3 centimeters. So I subtract them. That is 5. 28.5 centimeters and this is 2 lambda. And I know this really to-- I would say to a millimeter. Certainly no worse than 2 millimeters.

And there is no uncertainty, for sure, in the f. If you can get me the light back. So I can now calculate what the velocity of sound is. So that is 28.5. I divide that by 2. That gives me 14.25 centimeters. This was just a rough number that I gave you. And now I multiply that by the frequency 24 0 9. And then I find 343. So v is 343-- I would say plus or minus-- maybe two meters per second.

So we've measured the speed of sound to a high degree of accuracy. And of course, at the same time, you've seen this wonderful resonance normal mode behavior-- where you see it in the nodes and the antinodes, in this case of pressure. Earlier you saw them here in terms of transverse motion. Now you have seen them in terms of longitudinal motion.

There is energy in a traveling wave. If I have here a traveling wave which is moving in this direction, has tension T. mu is the mass per unit length. Then I can write down y as a function of x and T. I can give it a certain amplitude A, times the sine.

And I could write this now in many different ways, but let's write it down in this form-- k times x minus vt. And we know this v. That v, let's call it v squared, is t divided by mu.

So this is a traveling wave that goes into the plus x direction. Is any matter moving in the plus x direction? No. But is anything moving? Yes. It is moving in the y direction. So since these particles have mass and they're moving in this direction, there is kinetic energy due to the motion in this direction-- not due to any mass that moves along. No mass moves in the direction of here.

Suppose I carve out here the section dx. So this is the direction of x and I take a small section dx. Then the kinetic energy, dE kinetic-- tiny little bit of kinetic energy-- is 1/2 times the mass dm times the velocity in the y direction squared. That is 801 right? Kinetic energy is 1/2 m v squared. Never confuse this v with that v. That is the velocity of propagation. This is the velocity of the string in this direction.

So I could write this down. dm is obviously mu times dx. If I have a length dx here and I have mu kilograms per meter so I get here 1/2 mu times dx. And for this I write down dy/dt squared. It is the velocity in the y direction and I use partial derivatives because I do it at a given location for x. So what is dy/dt? Well, that's easy. That's a piece of cake. There is my function.

So I get A. Then I get a k and a minus v. So I get a minus, and a k, and a v. And then the sine becomes a cosine. And so I get k times x minus vt. I should have put brackets around there too, but that's self-explanatory. And now I want to know what the square of this is. So I'm going to square this. Now I can calculate what the total energy is-- kinetic energy-- in one wavelength. All I have to do is integrate now from 0 to lambda to get the total energy in one wavelength.

So I'm going to do that. I'm going to write down here now E kinetic. Now follow me closely. I have 1/2, I have a mu, and then I get the dy/dt squared so I get an A squared. I got a k squared, I got a v squared. And then I got the integral from 0 to lambda of this function cosine k, x minus vt. And then I have my dx which is this dx. That's what the integral is in the direction of x. Yeah?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yep, thank you very much. Extra course credit. What is this integral? Maybe you don't remember, but I've done this often enough that I do know. The integral of cosine squared of that function is lambda divided by 2. And I leave you with that. That's a very easy exercise. And I can also take the v square out and write for that T over mu. So E kinetic now-- and this is in one wavelength-- this is my short hand notation. So all I'm calculating for you now is how much is in one wavelength.

So you're going to get now 1/2 mu. You get an A squared. The k squared I can write down as 4pi squared divided by lambda squared. For the v squared, I can write down T divided by mu. And then I have my lambda over 2. Well, this mu kills this mu, this 2 kills this 4, and this 2, and 1 lambda kills 1 lambda here. So now I have the final result that E kinetic-- in one way wavelength-- equals A squared times pi squared times T divided by lambda.

Now if you look at this and you ask me-- is that obvious? I would say not to me. There's a T, there is a lambda. My goodness. And I can also cocktail the whole thing. I can get the T out, and get a v back in again. And I will admit that I don't have a very good feeling for this function, except for one. I do know that always the energy in a wave is always proportional to the amplitude squared. You're going to see that when we do electromagnetic waves in 8 0 3. It's always proportional to A squared. So that's the only one for which I may not have a feeling, but I know it's got to be there.

And the rest, I will leave you with that to see where whether, perhaps, you can talk yourself into understanding why you see the symbols where they are. Now clearly, there is also potential energy because it takes energy to make that straight line into a curve. And that means work that you have to do. You have to squeeze to stretch it. There's a tension and you have to stretch that. And so we have to do work to just get the shape. And that is potential energy. And the potential energy per wavelength, which I want you to do on your own.

It's worked out nicely in French. I will not do it today. Happens to be exactly the same as the kinetic energy, which is by no means obvious. So the potential energy per wavelength is the same as the kinetic energy per wavelength. And so what that means then is that the total energy in a traveling wave is twice this-- kinetic energy plus potential energy. That is the total energy in a traveling wave.

I can now make you see, in a nice way, an energy balance to compare travelling waves with a standing wave. If I have a standing wave-- and a standing wave is like this. And I do this-- I look at this picture at the moment that the wave stands still, that vy is 0. So it goes like this-- that's a standing wave. You've seen that. I do it at this moment-- there is no kinetic energy. There is only potential energy.

A little later there is only kinetic energy, and there is no potential energy. And a little later, there is only potential energy and no kinetic energy. If I pick that moment that it stands still, then I know that that's the total energy, because it's only potential, but it is the total. In other words, for a standing wave-- for a standing wave, if this has an amplitude A, it must be the same potential energy as you have in a traveling wave because it's simply due to the fact of the shape. It has nothing to do anymore with motion. So for a standing wave, the total must be that number. Where for travelling wave, you have kinetic energy plus potential energy. So you have twice as much.

Now I can convince you, and that was my plan earlier today, that if I have one traveling wave with amplitude 1/2 A, and I have another traveling wave, again with amplitude 1/2 A, I know that I'm going to make a standing wave with amplitude A. Right, because 1/2 A and 1/2 A , when we added them up you got twice. Here you've got this 2 A. But there must be a certain amount of energy in that wave that comes in with 1/2 A and certain amount of energy in this one. And when you add those energies up, you must exactly get the energy in a standing wave because no energy was lost.

So therefore I make the following statement now, which is testable, that two traveling waves, each with amplitude 1/2 A-- I'm going to compare them with a standing wave with amplitude A. There must be the same energy in a standing wave with amplitude A as there is in two traveling waves with amplitude 1/2 A. Did we follow that? Was that too difficult? Because we know that we can make that wave a standing wave. OK.

We know that the total energy in a traveling wave is this, but we have two of them. So in the travelling waves-- in two traveling waves-- we have 2. Then we get that 2 there, that's non-negotiable. Then we get pi squared, then we get T, and we divide by lambda. But the amplitude A that we have there is now 1/2 A. Yeah? Is that too difficult? I give it an amplitude 1/2 A. So now I get 1/4 times A squared. So this is the energy in two travelling waves. Do we agree? What is the energy in one standing wave with amplitude A? Well, we have the answer here.

In a standing wave, all the energy must be A, pi squared, T divided by lambda. Look at it. They are the same. This 2, and this 2, and this 4 eat each up. Just a second. So you see the consistency-- I'll give you a chance-- that two traveling waves making up one standing wave, in the exercise I did earlier, that indeed, the energy is conserved. The energy in the two traveling waves with half the amplitude gives you then a standing wave with A. Yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR: A squared, thank you very much-- always A squared and when I said energy is always A squared. Thank you very much. If I generate travelling waves, I, Walter Lewin, start shaking strings or some other instrument is making traveling waves. Then there's an energy flow in the direction of propagation because these wavelengths keep moving these waves. And so there's an energy flow. There's no mass flowing. Oh. There's energy flowing, the mass is only doing this. And so therefore, if I generate a certain amount of energy, then I need power to do that. Namely, power is energy per unit time.

And so I can now very easily calculate for you the power to generate a standing wave. That is utterly trivial because the energy in the standing wave is 2 A squared, pi squared, T divided by lambda. And all I have to do now is to divide it by the time that it takes me to generate one oscillation. For that's the period of the oscillation. And the period of the oscillation-- I try not to put too many P's in there-- and I hate to call it T. So the period of the oscillation is the same as 1 over the frequency. And 1 over the frequency is v divided by lambda.

So therefore, the power is simply the energy that I'm generating and v divided by lambda is the same. It's 1 over the period of one oscillation. Energy divided by period-- the time that it takes to make one oscillation-- is per definition power. And so then you get the result here. You get a lambda squared here and you get a v there.

AUDIENCE: There should be 2 there because it's a standing wave.

PROFESSOR: Excuse me?

AUDIENCE: This is a standing wave right?

PROFESSOR: This is a traveling wave. I am-- it's good that you asked that. I am generating a traveling wave. I have to do work for every wavelength that I produce, this is the amount of work that I have to do. Do we agree? There is energy in it. Potential energy because I have to give it that shape. And there is kinetic energy because it moves. If I generate this amount of energy-- what is your name?

AUDIENCE: James.

PROFESSOR: James, if I generate this amount of energy, and I divide that by the time that it takes me to generate that energy, I get the power-- the average power that I have to put in. It's really average power. And so I have to divide this by the period of my oscillation-- the time that it takes to make one oscillation. And that is lambda divided by v, that is the period. And so I have to multiply this by v divided by lambda. So this is really what we call frequency. And so now you get the power. And so you can drive-- you can move energy from one point to another without any mass moving. All right.

This is an ideal moment for a break and for a fatherly talk. I've had some very interesting email exchanges with quite a few of you. And as a result of that, I have decided to lower the 10% of the mini quizzes to 5% and to raise the homework from 10% percent to 15%. I think that'll make probably most of you happier but not all. Some of them thought it was fine, but I think most of you will be happier. And I'll make that change this afternoon on the website so you will see that reflected when you look tomorrow morning.

I will also be more careful that the people here in the front row don't have five minutes and the people in the back maybe only one minute. And so therefore, I will make sure that each one of you, from now on, has the same amount of time. And so what I'm asking you now is to-- not to start until everyone has. And then I will blow the whistle, and then I will give you five minutes. So if you start handing this out.

[WHISTLE] OK.

PROFESSOR: I'm now going to change the boundary conditions which is something that you will need very much for your problem set. Here is a string-- mu1, v1. And here is a string-- mu2, v2. And obviously, they are at the same tension-- one string. And so v1 is the square root of T divided by mu1, and v2 is the square root of T divided by mu 2. I will call this point here, for simplicity, x equals 0-- that is the junction. And this is the y direction, and this is the x direction.

And I'm going to have a wave coming in from the left, which I call the incident wave-- incident. I will give that an i, moving in this direction. And then there will be a reflection, which I call r, something may come back when it goes into this direction-- both in medium 1. This is medium 1 and this is medium 2. And then something is being transmitted into the second medium-- I will give that subscript tr. And that is going into medium number 2. And that's what I want to evaluate now.

Now I have boundary conditions at x equals 0. I told you earlier, all of 803 hangs together-- the wave equations and boundary conditions. And that's true. The boundary conditions is that at x equals 0, y1 must be y2, unless the string breaks. So the y right here must be the same as the y right there. Otherwise there would be a break of the string. But not only that. dy1 dx must also be dy2 dx. If that were not the case, that would be a kink in the rope at the junction. And a kink would mean that there is a tangent here. And that there's a tangent here, which would give a net force down, but the junction itself has 0 mass.

And so that would give an infinite acceleration. So you can never have a kink in a string, not even when it is connected like this with another string. So it must be something that is always fluid here-- could be like that, or could be like that, but it cannot be like this. And so that's then the condition that the derivative-- the spatial derivative from the left-- must be the same as on the right side. So let us start with an incident wave-- comes from the left. And so y1, yi, i, that's incident has some amplitude A of i-- that is the amplitude of the incident 1 times the sine.

And I'm going to write this now. I can write it in many different ways but I'm going to write this now as omega 1t minus k1x. That's clearly a traveling wave that moves in the plus direction because the signs are different. And now I have a reflected wave-- amplitude A reflected times the sine-- obviously the same frequency. Now I got plus k1 times x. Notice the difference in sign. This one is going this way, this one is coming back.

The amplitudes are different, but the k's are the same because the wavelength in medium 1 is the same. The k is 2 pi over lambda. And that wavelength is not going to change for this wave as it is for this one. And so now I get to transmit it-- one-- this amplitude transmit it-- times the sine of the same omega. Because if this junction shakes up and down with frequency omega, that's the same for both media. So omega is a given. You can't change omega. You can't say omega is different from the right side than it is for the left side.

In other words, omega 1 is v1 times k1, but it is also the v2 times k2-- it is the same omega. And so I get here minus k2 times x. So this goes into the second medium. The minus sign indicates that it's going in this direction, but the k2 is different because the speed is different-- because v2 is different from v1. And so with the same frequency, I get a different value for k. So if now I go to my boundary conditions, y1 equals y2, which you see here, then I get there that A of i. So x equals 0. At any moment in time, I must meet that condition. So A of i plus A of r must be A tr. If not, then the string would break.

I can now take the derivative of my function, dy1 dx. And so when I do that, I get the first one is going to be an A of i. And then the derivative against x gives me a minus k1 here. And then I get a cosine of omega 1t because x was 0 remember. So I get a cosine omega 1t but each term will have a cosine omega 1t. So I will dump all the cosine omega 1t's. So I go to my next one which gives me now plus A of r times k1. And that now-- this is so, this is the left side. This is my dy1 dx. And my dy2 dx in my second medium is now this one. Gives me a minus k 2 times A transmitted times minus k2, and I dump the cosine.

So this is the second equation. And you could solve these two equations easily except you have three equations-- two equations sorry-- with three unknowns. You have Ai, Ar, and A transmitted. And of course, you cannot solve two equations with three unknowns, but what you can find-- that is the only goal I have-- is the ratio of the amplitude of what comes in over what comes back. That's my whole goal. If you calculate for this one, k1 over k2, and you substitute that in here, then you can replace the k's by the ratios of v's. You see that? So you take k1 divided by k2, and you get rid of your k's.

So when you do that, you get A of i minus A of r times v2-- equals A transmitted times v1. So that by simply combining these two. And so now all you have to do is calculate the ratios of those amplitudes for this equation-- one equation with three unknowns, and the second equation with 3 unknowns. And what comes out of that-- this is also worked out in French-- what comes out of that is that the reflected amplitude over the incident amplitude is v2 minus v1, divided by v1 plus v2. And A transmitted divided by amplitude of the incident 1 is 2v2 divided by v1 minus v2.

Sometimes I call this with shorthand notation R, reflectivity. It's the ratio of amplitudes. And sometimes I call this shorthand notation, Tr, which is transmitivity the ratio of the amplitude that penetrates the second medium divided by the one that came in. But that's just my shorthand notation. We can now do some interesting examples. And you will begin to understand what is happening here. Take an example, for instance, that mu 2 is infinitely large. That's a wall. So in other words, that second string is a wall. If that second string is a wall, you better believe it that that point cannot move. The incoming wave couldn't possibly move the wall. So this is the same as what we earlier called-- it's a fixed end.

In other words v2 is 0. Right? Because if mu2 is , infinity then v2 is 0. So now I go to those equations and I say what is now R? v2 is 0. Well if v2 is 0, I get minus 1. Hey, hey. Minus 1-- that means when a mountain rolls in, what comes back?

AUDIENCE: A valley.

PROFESSOR: A valley. And when a valley rolls in, what comes back? A mountain. And we've seen that. We demonstrated it last time-- a fixed end. And what do you think the Tr is going to be? What do you think it's going to be?

AUDIENCE: 0.

PROFESSOR: 0. Nothing go through. And look at it. If v2 goes to zero, Tr is 0. So aren't we happy? Take now an example that v1 is smaller than v2, in other words, mu 1 is larger than mu 2. When we have a case like that, notice that R is always larger than 0. If v1 is smaller than v2-- this is a speed now. This is always larger than 0. What that means-- just a second, I'll give you a chance. What that means is that a mountain comes back as a mountain.

Now the amplitude of the mountain will be changed, but it comes back as a mountain because that's the plus sign. And notice that the transmitivity is always larger than 0. It never gets a minus sign. Of course it cannot get a minus sign. That you can explain to your kid brother because if a mountain comes in here, that point is going to move up, no matter what. And if this point is going to move up, what goes into the second medium is a mountain. And if this point goes down, because it is a valley, that's what goes into the second medium is a valley. So it is completely logical that here you never can get a minus sign, but here you can. There was a question.

AUDIENCE: Shouldn't the denominator be [INAUDIBLE]?

PROFESSOR: Thank you very much. My goodness. Oooh. Yes, v1 plus v2. Thank you very much. Thank you very much. So now we take an example . which is absolutely thrilling-- v 1 equals v2. That means mu 1 equals mu 2. That's another way of saying there is no junction. It's just one rope-- all the same material. Well if there's one rope with all the same material, do you think of anything will reflect? Of course not. So I predict, without even looking at the result, that R is 0 and that the transmitivity is going to plus 1. Let's check that. When v1 is v2, this is zero. And when v1 is v2-- you just corrected me at the right time, by the way, when v1 is v2, you see that this goes to plus 1. With a minus sign, it would've gone to infinity. I would've caught it. But you got it earlier.

OK so now let's do a specific example whereby we give some numbers because that's what I plan to demonstrate. So the example that I have in mind is that v1 is 2v2. So v1 is 2v2. In other words, mu 1 is 1/2 mu 2. If I put that in that equation, I find that R equals minus 1/3, and I find that the [? transmitivity ?] is plus 2/3. That's just a matter of sticking those numbers in. And that means then the following. That if this is my incident wave-- so this is now the incident 1. And incident 1, it has an amplitude 1. And is moving in this direction.

That the returning one has an amplitude which is 3 times smaller and it is flipped over, but it has the same length because it has the same speed. So suppose the returning one were here, then you would see the returning one with only 1/3 of its amplitude. That's about here. So you would see something like this and it will be moving in this direction. And this amplitude is 1/3.

And now, what goes into that medium number 2 is a pulse with 2/3 the amplitude. So this is the 2/3 mark. So the amplitude is only 2/3 but since the speed is lower because-- notice the speed is lower. Therefore it shrinks. And so not only will be the amplitude be 2/3 of the incident 1, but this length will only be half. And so you will see this. And this is now 2/3. And this is moving with velocity v2. This is moving with velocity v1. And this is moving with velocity v2. So that's the meaning of R and Tr. There was a question here.

AUDIENCE: [INAUDIBLE]

PROFESSOR: mu1, v1 is 2v2. The new one is 1/2 mu2. Is that wrong?

AUDIENCE: [INAUDIBLE]

PROFESSOR: You have a good point. Let's just remove it.

[LAUGHTER]

PROFESSOR: OK now? Thank you. So this now I want to demonstrate. And the way we're going to demonstrate is we're going to use this machine. And this machine allows me-- this has a medium here whereby the speed is twice as high as the speed here. I'm going to generate here a mountain. And the mountain will go into that medium. And the first thing I want you to notice is that as it enters this medium, where the speed is lower-- the speed is higher here than there. I want you to see two things. That first of all the mountain goes through as a mountain-- whatever goes through must always have the same polarity. If a mountain comes in, a mountain goes through.

So look at two things. The mountain remains a mountain and it will shrink. And then later we will look at the reflection. OK? So here--oh we have to make it completely dark, don't we for this? I think that's more romantic, yeah that's what they like. OK, so I'm going to generate here a mountain as fast as I can. The speed here is larger than the speed over there. And you will see that the pole shrinks, but the mountain remains a mountain. You ready for that? Did you see that the mountain remains a mountain? And did you see that it shrinks?

I need some light because I think I-- I broke something again.

[LAUGHTER] .

PROFESSOR: No problem-- fixed. OK now we want to do something else. Now I want you to see that when I drive in a mountain, that a mountain goes through, but the valley comes back. And the valley that comes back has the same length as the incident 1-- that is the minus 1/3 remember? So the valley is very shallow but the mountain will come back as a valley when it hits this point here. So we have already seen that the mountain goes through as a mountain. You have already seen that the poles get shorter. And now you're going to see that when it hits this point, that the reflection will be minus 1/3 so the mountain will come back as a valley.

You ready for this? And there it comes. Could you see it? That it came back as a valley? So I'll try again-- you see the speed is very high. That's the problem. So it's very hard to see. I'll try once more. Yeah, yeah. I could see that the mountain came back as a valley. In fact, what would happen if I drive a mountain in from this end. What will come back? A mountain or a valley? So if I do it from that end, the mountain returns as a valley. If I do from this end, the mountain must return as a mountain. Shall we take a look at that?

AUDIENCE: You broke it again.

PROFESSOR: And it comes back as a mountain. I'll do it once more.

[LAUGHTER]

PROFESSOR: I think I broke that one too. Yeah.

[LAUGHTER]

PROFESSOR: Boy. I only have one problem. And that problem is going to be your problem. And that is the following. I would like to explore now the possibility that mu2 is going to be 0. That means v2 has become infinitely high. If mu2 is 0, there is nothing here. It means there is tension on the rope, but there's nothing here. It's empty. We call that open end.

So imagine in your head that's something holding it tight. The end can move freely, that's no problem, but there is no mass there. Remember it was like the massless string and the rod. So now in this case, which is really an open end, there is nothing in medium 2. I can now go to my equation and ask what R is. And I will be very pleased when mu2 is 0 and v2 goes to infinity. When v2 goes to infinity, this is plus 1-- mountain comes back as a mountain. And all of you predicted that last time-- an open end a mountain comes back as a mountain. Whoopee. Physics works. What do you think is going to be the [? transmitivity? ?]

AUDIENCE: 0.

PROFESSOR: 0 yeah. That's what you think. Put in there v2 equals infinity. Now that takes you by surprise-- plus 2. Holy smoke.

[LAUGHTER]

PROFESSOR: What is going on here? If this were true, we would have solved the energy crisis of the world. Because a pulse comes in and the whole pulse comes back. Everything that went in comes back-- mountain comes back as a mountain. But there's something in addition that goes into nothing. I want you to think about this and you may not be able to sleep tonight.

[LAUGHTER]

PROFESSOR: And if you can't, it is not my fault. It's the fault of physics. See you next time.

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