PROFESSOR: So last time, we had driven damped oscillations relations and we did steady state solutions. And the steady state solutions have no adjustable constants. And that is strange. Because at time t equals 0, I can put the object at a certain position and I can give it a certain velocity.

And so I have two free choices. And yet, those free choices did not show up in our steady state solution. In other words, the system had lost its memory of what happened at time t equals 0. And therefore, I discussed with you that you have to wait to go into steady state. And that is what I will address today.

There must be something missing what we did, so that the time t equals 0 adjustables do show up. So let's return to our spring system. So we have here-- spring, spring constant k, mass m. And this object can be driven. You can that with the force directly on it here. Or you can shake on the left side.

Omega 0 squared equals k/m. I will call it that way. And gamma equals b/m. So that's the damping.

Now, let us assume that I don't drive it, that I just give it a kick, time t equals 0. I let it do its own thing. We discussed that two lectures ago.

So we get an undriven situation, not driven. And then the solution-- you should remember, has this exponential decay in it-- is at x equals some value x that follows from the initial conditions times e to the minus gamma over 2 times t times cosine omega prime t plus alpha. And these, the x and the alpha, can be found if you know, at t equals 0, what x is and what x dot is.

Omega prime is a frequency which is the square root of omega 0 squared minus gamma squared over 4, just a hair under omega 0. But that depends, of course, on gamma. So this is the situation when we are not driving.

Let's now take a situation that we do drive. So we drive, for instance, with a force that we apply directly to this object, which is one way of driving it. And that force is

F0 times cosine omega t.

This omega is my omega. I force that omega on that system. I don't give a damn what omega prime is. I determine what omega is. And that's tough luck for this object.

And we know that, in the steady state solution, only this omega survives. That one will go. And the solution that we have seen, that we derived last time, is some amplitude A times the cosine of omega t minus delta. That is my omega. That is non-negotiable. This is the steady state.

And we derived that the tangent of delta was omega gamma divided by omega 0 squared minus omega squared. And we found, for A, this monstrous solution. Remember, upstairs we had F0 divided by m. And downstairs, we had the square root, omega 0 squared minus omega squared squared plus omega gamma squared.

And we spent the entire lecture on dealing with this A. We changed omega. And we evaluated omega equals 0, omega at resonance, omega at very high values. There is no adjustable constant in this solution. But there was one in that solution.

Now, let us look at the differential equations that we were solving. And so first we go to the undriven system. Undriven means, at t equals 0, you give it a kick. And you'll just let it do its own thing.

The differential equation that we then had x double dot plus gamma x dot plus omega 0 squared x equals 0. That's the one we had. And we solved it. And we found that solution there.

Then, we were driving it. So now, we drive it. What now was the differential equation? Well, we had an x double dot gamma x dot plus omega 0 squared times x. And now we had, here, this driving term.

In the case that we directly put the force on the object, then we had here F0 divided by m times cosine omega t. Because the m comes in because you divide by m. You have Newton Second has ma.

But in the case that you shake the left side with your hand, with this F as 0 cosine omega t. That's another way of effectively driving the object. We work that out. Then, we have here eta 0 times omega 0 squared times cosine omega t.

But in any case, you see here a driving term. And we solve our equations. And if you take this case, then this is the solution. If you take this case, then this takes the place of that.

Suppose now, I take this solution, and I substitute that solution in this differential equation. Then I get the result is 0. Because look, forget this. So I put it in this part. I get a 0 because it fits this differential equation. So what is wrong with adding 0?

Therefore, if I add these two solutions, it must be a solution to this differential equation. Because you just add 0. And if you take 1803, then they use very nice terms. They say, yes, of course. If you have this special solution, which is this one, you have to add the homogeneous solution. And this word homogeneous means that you put a 0 here.

And so the general solution is really the sum of the two. By adding this one, you effectively add 0. So you add nothing, but you get something in return. And what you get in return are your two adjustable constants. So now, you can deal with the situation that, at t equals 0, you know exactly where that object is and what its velocity is.

So I will write down now the general solution, which is the one that governs a major part of the lecture today. So x now, function of time, is the steady state solution, a cosine omega t minus delta.

This is my omega. This is my will That's Walter Lewin's omega plus x times e to the minus gamma over 2 times t times cosine omega prime t plus alpha.

And this is the will of the oscillator. And this is my will. They are two different omegas.

And now, you can see what happens. You see that this term will never die out. This will last forever and ever and ever. But this one is going to die with a 1/e decay time of 2 over gamma.

And so if 2 over gamma happens to be 10 hours, then you have to wait 10 hours for this one to be down by a factor of e. But if t over gamma happens to be 1 millisecond, then all you have to do is wait 1 millisecond for that term to go down by a factor of e.

So this is the one that will die out. That's why we have to wait. And so this is called the transient. And it will die out faster, the higher gamma is. And this is called, then, the steady state solution, which ultimately survives.

Suppose I told you that, at t equals 0. X equals 0, and x dot also equals 0. And I was so nasty to say, oh, by the way, why don't you solve for x and alpha. It would be a very nice thing to do. But it would take you 15 minutes of grinding, not so fast.

Because remember, if you have an x dot, you have to take the time derivative of this entire function. You have a t here. You have a t there. And you have to substitute in there, time t equals 0. And then, you have to make that equal 0.

And it takes you 15 minutes. And out pops, indeed, a value for x and a value for alpha. You haven't learned much physics when you do it. But you've learned some algebra.

So I decided not to spend my time on doing that. But in principle, you must agree with me now, that if I specify the initial conditions, I don't have to call this 0. I could call this x0.

I could do anything I want to. I can give x dot and value I want to. Then, I get unique values for x and for alpha. And that is ultimately, then, what the solution is. I must add the two.

The bottom line-- and that's really where the physics is. And that has to do with problem 2-5, that you have this week on your plate, is then the following. If I make a plot of x as a function of time then, the solution is really the sum of these two. This is the steady state solution, which has an amplitude capital A, which is non-negotiable, has nothing to do with the initial conditions.

It has its own omega, Walter Lewin's omega. So let's assume that this is the period t. And so out of that pops a nice cosinusoidal-sinusoidal curve. Let me put it in here. And this never changes. This goes on forever and ever and ever. And this here, this time t is 2 pi divided by omega, my omega.

However, there is also this one. This one dies out. So I will now put in some kind of a exponential decay. Something like this.

It has its own frequency, omega prime. I can choose anything. I can make omega larger, omega prime larger than omega. I can make it smaller.

So I just pick one. And let's suppose that the zero crossings are here and here and here and here and here and here and here. And so for instance, the curve, then, that has to be added could be something like this. And this time this t prime is 2 pi divided by omega prime.

And so you see that the sum of the two, which I will not try to sketch, is the solution. But it is the pink one that's going to die out. That's why, sometimes, you have to be patient. Except if it is 1 millisecond 2 over gamma, you don't have to be very patient.

Now, let us look at the situation that-- and I can arrange that. And I'm going to arrange that. That omega and omega prime are close together. I can choose that.

The system cannot choose omega prime. The system is stuck. This is omega prime.

The system has no choice. But I have a choice. I can make omega any value I want to. So I can make it very close to omega prime.

What do you think is going to happen now? When I turn the system on, all of the sudden the driver-- and my omega is very close to that omega prime. It's true that the transient will die out.

But let's say we take a system with a pretty high Q. So it doesn't die out so fast. What do you expect you're going to see?

Excellent, you're going see beats. Because now you have two harmonic oscillations which have to be added. But the frequencies are a little different. And if this one survives long enough, there comes a time that they are in phase. There comes a time they're of phase.

And when they're out of phase, you see very low amplitude. And so you're going to see a beat phenomenon. And that is exactly what I want you to see.

For that, we need a system, preferably with a high queue. And then the driving frequency has to be close to the omega prime. And for that, I have chosen this system here.

This is an air track. And we can make the damping very low, unpleasantly low. Believe me. A very low value of gamma, and when I turn on the air flow, so here is a spring, spring constant k.

Here is one spring constant k. And here is the mass. And there's very little friction. And now, I'm going to drive it just a little bit off resonance, a little bit below the resonance frequency. And what you're going to see now is the sum of these two.

But since gamma is so low, it will take a long time for this transient to die out. And that is exactly what I want you to see. In addition, you're going to see beats. And as long as it is beating, you know that you haven't reached the steady state solution yet.

But if you're patient, and I'm patient, we probably will see it go into steady state. But it may take several minutes. So are we ready for that? I'm going to drive it here. And I started the driving now.

So relax and look at the amplitude of this object and see what happens. Hey! Hey, the amplitude is going down. Hey, hey hey! I call that a beat. You see that? Did you see the amplitude go down?

The two frequencies were beating against each other. And now, it's picking up again. It's nowhere near in steady state. Very low gamma, very high queue, there we go again. Amplitude is way down. It picks up again. Just be patient.

Let's be patient and see whether we have the privilege of seeing it go into steady state. It's a very high queue system. Since I am just below resonance, the driver and the car will be in phase when I'm below resonance, in steady state solution.

So this delta will be very close to 0, below resonance, above resonance 180 degrees out of phase. But I'm just below resonance. So when we go into steady state, we also will see that we are very close to a delta of 0.

Now, let's see what the amplitude is now and whether the amplitude is changing. Well, we're getting there. It pays off to be patient.

Later today, I will do an experiment whereby 2 over gamma is 2 milliseconds. So all you have to do is wait 4 milliseconds. So I'll make up for the fact that now we have to wait so long.

Let's take a look at this now. I think it looks terrific. It looks terrific. In phase, I don't see much beating anymore. It looks like the amplitude is constant. I think we are now-- we've killed this one. And I think this one has survived.

If you increase the damping, this will happen, of course, earlier, that you go into the steady state solution. Looks great. I don't see any change anymore in the amplitude. So that's the A that you have there, capital A. And they're nicely in phase.

It's a very high queue system. So the change to go from delta 0 to delta pi/2 at resonance takes place over an extremely narrow range of frequency. So it is still in phase. All right.

If we are driving this system with a force, say directly on the object, F0 cosine omega t, then in steady state there is energy dissipation. Because there's friction.

And where there is friction, there is heat.

And that means energy. That means I have to do work to provide that energy. It's a steady state situation.

So as the thing is never changing, it's A, just going on forever and ever. While that happens, I must put in energy, which comes out in the form of heat.

So let us return to the good old days of 801. And I want to remind you that work is the dot product between a force and a displacement, dx. It's a dot product. It's a scalar work.

A little bit of work is done by this force, if it moves over a distance dx. If the two are perpendicular to each other, then no work is done. Satellite into a circular orbit around the earth. No work is done.

And so now I can calculate what the power is because the power is dw/dt, how many joules per second I have to put it into the system. And so if I take the derivative, time derivative, then I get f dotted with v because the dx/dt is simply the velocity.

Now, if I have a one dimensional system. And what I mean by that is that the force is either in this direction or in this direction. And the velocity is either in this direction or this direction. That's what I mean by one dimensional system.

Then I can delete the dot. And then the signs will automatically take care of the direction, minus v is then this, plus v is that, and the same for force. So I can kill the dots.

So now, I have to know what the velocity is in the steady state solution. Well, that's easy. Because I go to the steady state solution here, and I calculate what x dot is. So I'll put that here, x dot, which is v.

So the derivative of cosine omega t is minus omega times the sine. So I get minus omega times A times the sine, omega t minus delta. And that's the velocity.

But I know what the force is. It is F0 cosine omega t. So there we go, F0 cosine omega t. But I'm going to put this in also now. So I get omega A. And then, I put in the cosine omega t. And now I'm going to put in this one, the sine of omega t minus delta.

So take a deep breath. F0 cosine omega t is the force. See that there? And the velocity has been derived from the first derivative of the steady state solution, gives me a minus omega a sine omega t minus delta.

Now this, what we have here between brackets, can be written as the sine of omega t cosine delta minus the cosine omega t times the sine of delta. So what you see here between brackets is the same as what you see there.

Am I really interested in knowing, every moment in time, what exactly the power is? Not really. Most of the time, I'm really interested in knowing what the average power is that is required to keep the thing going. Average over 1 oscillation or over hundreds of oscillations-- that's really what I'm interested in, not necessarily the instantaneous power.

In other words, most of the time, my interest is really in what this is. That is the time averaged value. And think of it as being one oscillation. That is fine. But you could think of it also as days.

Now, I need some experts in the audience. I see here a cosine omega t. And I see here a sine omega t. What is the time average value of that product? The time average value would mean a sine omega t and a cosine omega t, if I average it over one cycle or two cycles or three cycles. Come on. High school, yeah. What is the time average of sine omega t times cosine omega t, time averaged over one period?

AUDIENCE: 0.

PROFESSOR: 0. This one times this one time average gives me a 0. If you don't believe it, go back to high school. Ask you high school teacher. He will agree with me.

Here, I see a cosine omega t. And I see a cosine omega t, there. What is the time average of cosine squared omega t? Ah, you guys are waking up. It's 1/2. So therefore, for this product, sorry, for this product I can write 1/2. And so now we get-- notice there is a minus here.

And so this minus picks up this minus. So it becomes a plus. So I get F0. I get the half that is here. Then, I get omega. I get my A. And I get the sine of delta. Are we happy with that? You see, it collapses into something that is relatively simple.

What is sine delta? Well, I remember what tangent delta is. If I know tangent delta, can I then calculate sine delta? And the answer is, yes, of course.

If this is delta, and if this is omega gamma, and this is omega 0 squared minus omega squared. Then it only takes Pythagoras to calculate what this is. And so I know that the sine of delta must be omega gamma divided by the square root of omega 0 squared minus omega squared squared plus omega gamma squared. So yes, I do know what the sine of delta is.

So now I can come to a close by substituting in here what the sign of delta is. And I can substitute in here what A is. Here is A.

You know what is nice. Look at the downstairs, here. It's the same as the downstairs there. So if I multiply them, the square root goes away.

So if I can write down now what P average is. Let's go slowly. So we get F0 divided by 2. I get an omega.

I will go to the A, very shortly. I will first pick omega gamma here, which is my sine delta. So that makes this a square. And I get a gamma.

And now, I turn to the A, which is an F0 over m. So I get a square here. And I get an m here.

And now, I get this downstairs times this downstairs. The square root disappears. And so I get omega 0 squared minus omega squared squared plus omega gamma squared.

Almost end of story. I'm going to rewrite it a little. And Tony French, in his book, rewrites it in, again, a different way. He loves to work with Q's. He puts the Q's in there.

I'm going to divide upstairs and downstairs by omega squared. And when I do that. I get F0 squared. I get a gamma, here. I get 2m, here. And then, I get here omega 0 squared divided by omega minus omega squared plus gamma squared.

That's what I get. That's not the only way you can write it. But that's one way you can write it. Let me check that. I have F0 squared. I've have gamma, 2m here. And we have this downstairs.

So the time has come now to try to see through this equation. You remember last lecture, we spent the whole lecture not to look at this dumb equation, but to see through it. And we were able to see through it, see all its idiosyncrasies.

Let's look at the idiosyncrasies of this average power over 1 cycle or a multiple of cycles. Well, let us first make gamma infinitely large. Don't look even at the equation, gamma infinitely large.

That means there is an infinite amount of friction. The system never gets going. There's no way that it will ever move.

So clearly, the average power must go to 0. And indeed, you see that. The gamma is downstairs. The gamma goes to infinity. The power is 0.

Let's make the mass infinitely large. If the mass is infinitely large, you have an infinite high inertia. Nothing will ever get going. No force will ever get the mass going.

Well, that means you expect that the power goes to 0. And indeed, you see here. If m goes to infinity, the power goes to 0.

Let's say, the force goes to 0. We are not even driving it. Well, if we're not even driving it, I hope you agree with me. You don't have to put in any work, right.

Nothing gets going. So clearly, you expect then that the power will go to 0. And indeed, if F0 goes to 0, you see that the power is 0.

Suppose you make omega 0. So that means there will never be any velocity. You never pick up any velocity. It takes infinitely long. Omega is 0. So clearly, if nothing ever moves, the power will go to 0.

Now, look. If the omega makes 0, and this is 0. This one goes to infinity. And therefore, the power goes to 0.

So you need no equations for that, just common sense, to immediately concluded that this has to be the case. Suppose you go to infinity with omega, very, very fast. Well, if you go infinitely fast, because of the inertia of the system, it can never react. It can never get going. So I will predict that then the power must go to 0.

And if you put omega infinity in here, this goes to 0. This goes to infinity. And so the power goes to 0.

So all this is complete common sense. All of this, you could have predicted without that equation. But isn't it nice that the equation supports my intuition?

So now comes the question if omega goes to omega 0-- and that is the reason why I wrote it in this way. Notice, when omega goes to 0, when omega goes to omega 0, this one goes to 0.

And therefore, that is the frequency at which the average power is the maximum ever. It can never be any higher. Because it's independent of gamma, right, my omega.

So I have an omega equals omega 0. We reach the maximum value of power. We can never go any higher. It's exactly at omega 0.

And this is 0. You lose one gamma. And so you find that this value then becomes F0 squared divided by 2m gamma.

And you can write that, rewrite that a little bit. I do that because Tony French likes

Q's. And so I write it with a Q in there. Remember that Q is omega 0 divided by gamma.

So I can rewrite this as Q times F0 squared, upstairs. So I get Q in there, which is always nice, divided by 2m omega 0. So this is the same thing.

So if I now plot, make a curve for you of P average, not P maximum, but P average, as a function of frequency-- so here is omega. And here is P average. And here is omega 0. It goes through a maximum, exactly at omega 0.

It starts at 0, you see? And it ends at 0. And it sweeps up to a maximum. And then, it goes down again. And for reasonable values of Q, these curves look extremely symmetric. And so this value here is then P average max, which is that value, this value.

If we look at the width of this curve, at half maximum of the power-- so this is one half times P maximum. Then you can show-- and it is not so difficult, algebraically. But I will not attempt it. You can show that the width at half maximum is very close to gamma. Remember, gamma and omega have the same units, 1 divided by seconds.

In other words, if you go to half maximum, this point here is omega 0 minus gamma over 2. And this point here omega 0 plus gamma over 2. So you see immediately, which of course makes sense, that if gamma is very small, that the peak gets very narrow. And if gamma is very high, the peak gets very broad. That's intuitively quite pleasing, high Q systems, very narrow peaks.

And that's the way that Tony French likes to plot his data. I will show you that on the overhead here-- this is just a picture from your book. And what Tony does here, he plots not omega here. But he plots omega divided by omega 0. So that means the resonance is at one.

And he doesn't plot the average value for P here. But he plots it into strange units, into the unit F0 squared divided by 2m omega 0. So now, he effectively can compare the vertical axis with the Q value. Because he likes the fact that it is Q times higher than something. And he has plotted this in terms of that something.

And so if you take the curve for Q equals 10, which has the peak here in power. You see, indeed, that he finds that very close, on his scale, to 10. Notice, also, the nice symmetry.

And you see, for lower values of Q, which are curves here, that indeed, the peak gets broader. The width of this peak is 1/Q. Because you remember, this axis is omega divided by omega 0. So if the width is gamma, on that peak, it is now gamma divided by omega 0 in this plot. And gamma divided by omega 0 is 1/Q.

So here, the width, in this presentation, is directly inversely proportional to Q. So if Q is 10, then the width there is 1/10. He then shows you another plot, whereby he does what I did there. He plots it as a function of omega, not as a function of omega divided by omega zero.

And then, he emphasizes the fact that the width here is that gamma that I mentioned. At half the maximum power, you get here the width of gamma. And this is a picture that I chose verbatim, from your book.

This is the best moment for the break. That means the mini quiz. I realize it's a bit early. We're only 40 minutes into the lecture. But it's a natural point. You will see what comes afterwards. It's better that we make the break now.

So therefore, I need some help from people who are willing to hand out the mini quiz. It would be nice if I can find the mini quizzes. I had them here. But someone took them.

Oh, no. They're still there, a nice conspiracy. Afterwards, after the break, we will collect them this time in some boxes. So that's it's a little bit more organized.

And so I'm returning to an RLC circuit, which we discussed earlier, good old days of 802. I'm going to drive it now, not with a battery, but with an alternating power supply. V0 cosine omega t-- yeah, put it in here. Thank you.

So here is the circuit resistor R, self-inductance L, capacitor C. And I have to write down now the differential equation. I will adopt a positive current in this direction. That will be my positive current. The charge here, on this right plate, I will call q.

And therefore, by that definition, I is then dq/dt, sign sensitive. I call the potential difference over this capacitor, in going from the right side to the left, side-- I call that V of c. That then is q divided by c. All of that is sign sensitive.

I go around this circuit. And I want to calculate the closed loop integral of E dot dl. And that closed loop integral of E dot dl is not 0, which many books tell you, even many professors tell you. It is not 0. But it is minus d phi/dt.

This is Faraday's Law. And this runs our economy. Because of the magnetic flux change in closed loops, we can generate induced EMFs, which run our economy. Look at the lights.

Luckily, this is not 0. This phi is the magnetic flux that goes through a surface, any surface that you can attach to this closed loop. So I've done this before. So I can do it a little faster.

I go from here to here. So that is IR. There is no electric field inside this ideal self inductor because super-conducting wire cannot be any E field. So that is 0, going from here to there.

When I go over the capacitor, I get my V c. And here, depending upon the face, if I assume this plus and this minus-- but you can reverse that. Then, when I walk into this direction, I would get minus V0 cosine omega t. But if you feel like reversing that, I have no problem with that.

That's just a matter of 180 degree phase. It's no different physics. And this now equals minus d phi/dt.

The only thing where you apply Faraday's Law is you should always integrate in the direction that you have your current assumes. Then it is minus L di/dt. If you do it in the opposite direction, then it is plus L di/dt.

I have learned a certain discipline in my life. It took me many years. So you have a long way to go. And I always go in the direction of I. So I never have to think. So this as minus L di/dt. This now covers the minus d phi/dt.

So now, what I do, I bring the L in, and I take one more time derivative. And so I get L times I double dot plus R times I dot plus V c. But I take the time derivative. So the q dot becomes I. So I get I divided by C.

And that now becomes the time derivative of this function. But it goes to the right side, which makes the minus sign a plus. But when I take the derivative of the cosine omega t, I get a minus omega. So I get here minus V0 times omega times the sine of omega t.

This is the differential equation that has to be solved. And I will divide this out by L. I will divide everything by L. I will put the C a little higher.

And so with R over L gamma and with omega 0 squared equals 1 over LC, this becomes then I double dot plus gamma times I dot plus omega 0 squared times I. That now equals minus V0 divided by L times omega times the sine of omega t.

Here you see the differential equation. And that differential equation looks amazingly similar to this one. And so you should be able to solve that.

In fact, you wouldn't even want to solve it. You can write down, immediately, the answer. You're going to get an I, which is an I0, which takes the place of that A there.

This is a steady state solution now. I only go for steady state solution, times the sine of omega t minus delta. No adjustable constants, it's a steady state solution that I have, steady state. And I will leave you to find me I0. And you can work out what delta is. That as part of your problem set, anyhow. But with the knowledge that you have here, you could write it down in a matter of seconds

So without my telling you what I0 0 is, at least working it out algebraically, we can talk 802. And then, we can make all kinds of predictions, without even looking at the equation. So that's interesting. So everything that I'm going to tell you now I do without knowing what I0 is. And it better work out that way.

Suppose I make omega go to 0. Remember 802? Remember the word reactants? That a capacitor has a certain reactants, which is 1 over omega c, which has units of ohms.

If omega goes to 0, this reactant goes to infinity. No current can ever flow. So I predict that I0 will go to 0.

Suppose my omega goes to omega 0. Now, your memory may fail you here, on 802. But we have a wonderful demonstration, in 802, that at resonance, 1 over omega c, which is the reactant of the capacitor, minus omega L, which is the reactant of the conductor, is 0.

That determines, actually, the resonance. And when this is the case, perhaps you remember that the system doesn't even know there is a capacitor. And it doesn't even know there is a self inductor. The two, at all moments in time, exactly cancelled each other.

And therefore, Ohm's Law holds. There is no L. There is no C . There is only the power supply and the resistor. And so since Ohm's Law says V equals IR, you must get I0 equals V0 divided by R. That is what you must get at resonance. It's non-negotiable.

Now, if omega goes to infinity, omega L says, aha, over my dead body. No current ever-- imagine a very fast-changing signal. That's what the whole self inductance is about. It doesn't want any changes. It's conservative, like you and me.

And so the self inductance says, sorry. The reactants is infinitely high, no current. And so I0 goes to 0. And so I make these predictions.

And that's always nice, that you can use some knowledge to make predictions, without even having ever looked at this differential equation. Any of these predictions I made did not come out of my knowledge of that differential equation.

So if we make now a plot of the current I0, that is not the current. But that is the maximum possible current. It is that I0 that you see here. Without having solved it, I can look now what it's going to do.

Exactly at omega 0, it will go to a maximum. It will start at 0, go to a maximum. And then, it will fall off at 0. And this value here is V0 divided by R.

This is not power. I have now plotted current. Power, of course, would go as I square R. That is the heat that you dissipate in the resistor. So that will go as I squared. I've plotted here I as a function of, not t, but this is I as a function of omega.

You know, when you catch an error that I make, you get partial credit for this course. So please, when you see me make a mistake, scream. So this is omega. Here is 0.

And this is what I want to demonstrate, now. I'm not going to show you I0 only. But what I'm going to do is the following. I'm going to show you what I is, as a function of omega. Here being the resonance.

Let us suppose I pick this omega. Well then, this is my solution. So yes, the amplitude is I0. But here is the sine omega t minus delta. So all you will see then is this. Shh-- plus 0 minus plus 0.

If I do it here, then it goes shh. And the demonstration that we have prepared for you is one whereby, we will sweep omega from 0 to a value, which I remember in terms of Hertz is about 2000 Hertz. And we'll do that in 1/6 of a second.

And so what you see is-- you see this as an envelope, which is the I0 envelope. But you will see this. Shh-- And then it speeds back.

And so you see two things. You will see the sine omega t term, shh. But as omega changes, you will see it go through resonance. And then, you'll see it go over resonance.

So I will give you the values that we have chosen. R equals 50 ohms. L equals 50 millihenry. And we choose C 0.5 microfarad, which is substantially higher than what we did before. And we choose C so high because we want a low Q system.

Omega 0 is now 6.3 times 10 to the third, using radians per second. So F0 is about 1,000 Hertz. And we're going to sweep it from 0 over 1,000, which is resonance, to about 2,000, and then, we sweep it back. And so the Q of this system, which is omega 0 divided by gamma, is about 6.3.

And what the V0 is of that circuit is not so important. But it is fourfold. But that's not so important.

And what are we going to show you-- we measured the potential difference over a very small resistor, which is somewhere in that circuit. 1.7 ohms, I believe. And so that potential difference over that resistor is IR. And R is a constant. And that's what we're going to show you. So we're going to show you something that is directly linear proportional with I.

It's not power It's I. If you want to know power, you have to square it. I squared R is the power. And we're going to sweep it one sixth of a second this way and one sixth of a second back.

Why did I only take into account the steady state solution? Why don't I have to also include the transient solution, which with this experiment, took us five minutes to finally arrive at the steady state solution? Why am I leaving it out?

AUDIENCE: [INAUDIBLE].

PROFESSOR: I can't hear you. Where's the sound coming from?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes. What is 2 over gamma, which is the decay time? 1/e decay time, what is 2 over gamma? I didn't write down what gamma is. Gamma is 1,000. Gamma is R/L, right? Gamma is R/L is 1,000. So 2 over gamma, which is the 1/e decay time, is 2 milliseconds.

Or another way of putting it is that, in about two oscillations, I am down by a factor e. Remember, Q/pi oscillations will reduce the amplitude by a factor of e. So in two oscillations, already the transient phenomenon is effectively killed.

And so I don't have to take it into account. You won't even notice it. So I told you earlier, when the decay time here was very long. I'll show you another experiment where the decay time is extremely short.

While I'm at it-- the experiment is set up, there. And you will see it here. I'm going to make this 100 ohms. I'm going to double it, to show you that this point will exactly go down by a factor of 2. Because remember, the peak is V0 divided by R. And I'm not changing V0.

And so by when I double R, you will see it come down to here. And you will see it get broader. And then, I will go to 150 ohms. That's easy to do for us.

So we go to 150 ohms. So at 100 ohms, the Q is 3.15. And at 150 ohms, the Q is 3 times smaller. It's about 2.1.

So at 150 ohms, it will be one third of this height, somewhere here. And the peak will be broader. And all of that you get today, very easily, by changing the resistance. Again, this is not power. Power is I squared times R. This is simply the curve.

All right. I will give you the correct light setting. Because we have to make it a little darker. And then we-- And there it is.

Notice the vertical oscillations go so fast that your eyes cannot even follow them. But that's the sine omega t. And then when it's sweeps over resonance, you see very dramatically this value, which V0 divided by R.

Now, notice on the scale here that Marcos has set the I0 at resonance at 3 scale units. So it is nice. When we go from our 50 ohms to 100 ohms, it should go down by a factor of 2. And you can check that.

And when we go 150 ohms, it should go down from 3 units to one unit. So we can quantitatively check this. And you will see that the curve gets broader because it has a lower Q. So now, I will make it 100 ohm. You see it's down by a factor of 2, from here to here.

And you may have noticed that it also gets broader. And now, I will go 150 ohms. And you see it's down by a factor of three. And again, it is broader.

So this is an amazing way how, with RLC circuits, you can do wonderful things. Because you can manipulate omega 0 very easily by changing L and C. And you can manipulate the driving frequency also very easily.

So it is clear that systems respond strongly when they are exposed to their resonance frequency. We've seen that for pendulums. We've seen that for springs. We've seen it for a wine glass. And we've seen this now for an RLC circuit. So these systems, at resonance, absorb a large amount of energy per unit time, out of the driver.

I have here two tuning forks, which have an extremely high Q. In my attempt to measure it, I concluded it's way larger than even 1,000. And they have exactly the same frequency, both 256 Hertz, this one and this one. Both 256 Hertz, that's the way they are designed, to a high degree of accuracy, to better than a fraction of one Hertz.

But the Q's are so high that if you were to plot-- if you drive these tuning forks, and you were to plot here this average power, as a function of omega. Then you would get something like this. It means you have to drive it exactly at the right frequency. Otherwise, it will not go into resonance.

Well, we know how to get this to going. You just bang it. That means you dump a whole spectrum on it. It picks out the frequency that it likes. And now, I'm going to show you something remarkable.

When this one generates 256 pressure waves, this one feels those pressure waves. And it loves it because it's just at the right frequency. And so it starts to oscillate.

And so when I stop this one, you will hear this one. And that's called resonance absorption. Let's do that first.

Now, you must understand that the sound waves go from here to there. Not much power reaches that point. So when I stop this one, you hear sound. But it's not overwhelming. So you have to be very quiet.

You hear it? And I can do the same by hitting this one. And then, this one will start to resonate.

Now, if the driving frequency is off by a fraction of a Hertz, one Hertz is enough, one Hertz difference. Because the Q is so high, then this system will not be able to get this one going.

And I can make this frequency a little lower than 256 Hertz by putting this weight on here. And we've measured the frequency at this loaded way. It's roughly 255 Hertz.

And you're somewhere here because it's so narrow. The resonance absorption peak is very short. By the way, this is power. Because what reaches here is joules per second.

That's what gets it going, energy per second. So it's really a power transfer. So now I have changed the frequency. And there we go, nothing.

Just change this by one Hertz. And you hear nothing, dead. So now, you see. You get some respect for high Q's.

If you want to get resonance absorption in a high Q system, you've go to be dead on that frequency. So if you, for instance, banged all the keys on the piano, and this one would be nearby. It would only start to resonate, if one of those strings would produce exactly the 256 Hertz. Otherwise, it would not. It knows everything. It's only sensitive to that resonance frequency.

You probably, in high school, have learned a little bit about atomic physics. And you probably know that electrons have discrete energy levels in discrete orbits in atoms.

And you can excite the atom. You can bring an electron in a higher orbit, discrete orbit, which costs you a discrete amount of energy. And when the atom recombines, when the electron falls back, you get that energy back, exactly the same amount that you had to put in.

And that energy that you get back comes out, most of the time, in the form of what we call electromagnetic radiation. Now, I know that in 803 we're going to deal with electromagnetic radiation in the future. But it's enough for now that you know that light, infrared, UV, gamma rays, x-rays, all of that, radio emission, all of that is electromagnetic radiation.

And so here I have the energy level, energy increasing. And here is an electron, in orbit. That's the energy of that electron. A higher energy state is when I bring this electron here. I cannot do anything in between. Quantum mechanics says it's one or the other.

And this difference is delta E, in energy. This E stands now for energy. Then when the electron falls back from here to here, it emits electromagnetic radiation with this energy. But if I radiate onto this atom electromagnetic radiation with exactly that energy, then this electron can go from here to there. And that is called resonance absorption.

Now, let us stick, for now, to visible light. The higher the energy, the bluer the light is. Or as modern physicists would say, the higher the frequency. And the lower the energy, the redder the light, the lower the frequency. So our visible light that we can see with our eyes goes all the way from the red, low energy, to the violet, high energy.

Let's go to the sun. The sun radiates in the visible spectrum all the way from the red to the violet. But in the solar atmosphere are elements. And when these elements see just the right energy from that spectrum to which they are exposed, they love to take out of that spectrum just the right energy that gets them into an excited state, which is called Resonance Absorption.

And so that energy is removed from the spectrum. So when you look at the solar spectrum, there are bands in the spectrum where the colors are missing, absorption in the spectrum, dark bands in the spectrum. They were discovered in 1802 by William Wollaston. And in 1814, Fraunhofer had catalogued 475 of these lines. And they're now referred to as Fraunhofer absorption lines.

Even though they did not understand the physics, this is a quantum mechanics picture that came from Niels Bohr, 20th century. Even though they did not understand what happens, they had noticed that these black lines in the solar spectrum coincided with emission lines in the spectrum that they can generate in the laboratory by heating up the various elements. And so without understanding why, they were able to say, ah, I see magnesium in the sun. I see aluminum in the sun, in the solar atmosphere. And so that opens a whole new industry of spectroscopy, which allowed astronomers to determine the chemical composition of the atmospheres of stars.

And it was in 1868 that Joseph Lockyer found at least one dark line, which did not coincide with any emission line in the laboratory. There was no element on Earth that he could say, that must be the cause of that dark line. And so he called it helium because the Greek word for sun is helios. So helium is an element that was first discovered on the sun, before it was later found on earth.

I want to show you resonance absorption on the scale of an atom. And the way I'm going to do that-- the setup is here-- is as follows. We have a carbon arc-- think of that as being the sun-- which produces a spectrum, a beautiful, continuous spectrum. I will show you that spectrum, all the way from the red to the violet.

And then, we have here a burner. And we're going to put table salt here on the grid, which dissociates the table salt, gives me sodium gas. That's what I want. Because sodium, when you heat it, can produce an emission line in the yellow. But if it can produce that emission line, when the electron goes from here to there.

It's the 11th electron, by the way. It's the most outer electron of sodium, 11 protons, 11 electrons. So if it can produce an emission line, when it goes from here to here, it can also have resonance absorption. Namely, when it sees that yellow line, the energy that corresponds with the yellow light, it sucks it up. And it produces then a dark line.

Because when it absorbs out of here, this yellow line, it re-emits it almost immediately. But the re-emission will be in all directions. And so what is left over here is very little of that yellow. And so a dark line appears in the spectrum. And that then is an absorption line.

It's power, what I'm going to show you. Because light intensity, which I will show you, there on that screen, is how many joules-per-second. So it is resonance absorption of power.

Now, there is a catch. And the catch is that probably only you, of here, will be able to see it. And others could come down.

You're going to see the spectrum, here first, of the sun, which is my carbon arc. Then I will put in the sodium. And you will see an unbelievable, unimaginable, beautiful, sharp line, like a razor, in the yellow. But you've got to be close.

So let's first, Marcos, if you manage to open the gas. He knows exactly where that gas valve is. Then I will ignite it. We won't put it yet in the beam.

OK. So we're going to make it completely dark. So here are the salt crystals. And we're going to show you the spectrum there. We tried, actually, to make you see it here on the TV. But that didn't work out well.

OK. So I'm going to turn on the sun. There is the sun. OK, now we make it completely dark. And I'll give you a minute or so for your eyes to adjust.

So you see a spectrum here. And you see a spectrum there. How we do that is our problem. And you will know how we do that in a month or so, when you will learn about gratings. You will get a grating, actually, from us.

There is a grating here, a wonderful piece of physics, which decomposes the light in colors. It works way better than a prism. And you get one, on the right side. And you get the mirror image, on the left side.

Look here. And let your eyes adjust. And then, comes the moment of truth. I'm going to put in here now the sodium. Unbelievable, I see a line here, sharp as a razor blade. Can you see it, Nichole? Isn't it incredible?

Come closer. All of you, come closer. Look at that line. Come on. Come out of your seats.

Look at that line. And I will move the sodium out. And now, I move the sodium out.

And now, I move it in again. And there it is. There it is. You see that? Isn't that amazing?

And now I move it out. And now I move it in. Isn't that's a fantastic line? Look at that line.

Look at that line, resonance absorption of an extremely high Q system on an atomic scale. I hope you can sleep tonight. See you Thursday.