PROFESSOR: Today, we're going to talk about Doppler effect. It is best known for sound. It probably reminds you of your high school days. If you move towards a whistle, you hear the pitch of the whistle higher than the whistle. And if you move away from the whistle, then you hear a lower pitch than the pitch of the whistle. Or if the whistle moves to you, you hear a higher pitch. And when the whistle moves away from you, you hear a lower pitch.

So you may think now, whether you move with a certain speed towards the whistle or whether the whistle moves with the same speed to you, that you will hear the same increase in pitch, but that is not true. There is actually a big difference. The situation is not symmetric.

If you move towards the whistle with the speed of sound, then the frequency that you hear is twice the frequency of the whistle. But if the whistle moves towards you with the speed of sound, you will hear nothing. Because all the sound stays with the whistle until the whistle passes you and then you hear a huge boom of sound. The frequency goes to infinity. You break through the sound barrier, so to speak, if you survive it. So there's a large asymmetry between them.

The speed of sound is about 340 meters per second at room temperature. It's about 770 miles per hour. That's the reason why commercial airplanes do not fly any faster relative to air then 770 miles per hour, because they are not designed to break through the sound barrier. That's the limiting factor for commercial airlines.

Let this be the transmitter of sound. And this is the receiver of sound. That could be you. And let us assume that you move with a certain velocity. I call this the plus direction. And you move with a velocity, vr. And let the transmitter move with a velocity, vt. And the transmitter produces a sound, f. But what you receive is a frequency, f prime. So f is the frequency. So if you go in velocity in this direction, that's positive, and the velocity in this direction then becomes negative.

Then f prime is f times the speed of sound minus vr divided by the speed of sound minus vt. And you should be able to derive that on your own. And it's very asymmetric. And you can see that immediately. Suppose I make vr the same as vs, so the upstairs becomes 0. So f prime goes to 0. That's obvious. If you walk away from the sound source with the speed of sound, the sound never reaches you. You walk as fast as the sound comes to you. So you hear nothing. So that's immediately obvious that f prime, then, has to become 0.

But if we now make the sound source go away from you with the speed of sound-- so again, the distance between the two of you grows. So now v of the transmitter, I make it minus the speed of sound. Then notice what happens. The frequency that I receive is simply one half the f. So f prime is now 1/2 f. So you see an enormous difference in the two. So you walk away from the source with the speed of sound. And you hear nothing. And if the sound moves away from you with the speed of sound, all you hear is half the frequency of the sound source.

I have a 4,000 hertz tuning fork there. And so f is 4,000 hertz. And I'm going to move my hand towards you as fast as I can, which is about one meter per second. In other words, I am the transmitter. So vt is approximately plus 1 meter per second. And then you should hear, then, f prime, which is about 0.3% higher than f, so it's about 4,012 hertz. And then I will move my hand away from you with about 1 meter per second, so I have to put a minus sign in there. And then I will find that your frequency that you will hear is about 3,988 hertz.

And the nice thing about this demonstration is that the difference between the motions is 24 hertz. And you can very easily hear that. So here is this tuning fork, 4,000 hertz.

[RINGS FORK]

PROFESSOR: Yeah? OK, there we go.

[RINGING CHANGES PITCH BACK AND FORTH]

PROFESSOR: I will do it a few times. And I will treat you too, because I gave them preferential treatment.

[CHANGES PITCH AGAIN]

PROFESSOR: Do you hear kee-kee-kee? Your turn. You hear it? You hear a higher pitch when I come to you and clearly a lower pitch when I go away-- a difference of about 24 hertz.

Suppose now that I rotate a sound source in a circle. I rotate it around like so. Angular velocity omega, radius R. This is where you are. And let the circumferential speed be v0. It goes around with constant speed. So v0 is omega R. And the sound source has frequency f. So it's clear that if it moves away from you, then f prime will be smaller than f. Here f prime will be f. And here, f prime will be larger than f.

And so as you listen to this sound, you will hear the sound change in a sinusoidal way, because what matters is not the speed of the sound-- the rate of speed of the source-- but the radial component in your direction. And the radial component here is the full v0 towards to-- here, away from you but the radial component here is 0.

So if you arbitrarily call this time equals 0 here, then the radial velocity-- the component in your direction-- is then v0 times the sine of omega t. And so if you record-- not only listen, but if you actually record-- as a function of time the frequency that you hear, then you'll hear something like this. And this, then, is the mean value, f, which is transmitted by the sound source.

And now you can ask yourself the question what do you learn from this? And it is amazing what you learn from this. You can close your eyes and just record this sound signal as a function of time. The first thing that you learn from it is the period, t, which is 2 pi divided by omega. So you know the period of rotation. You know omega. But you also record what f prime maximum is. And you also record f prime minimum.

And so with the Doppler shift equation that we have here, you can immediately calculate what v0 is. And so you know also v0. But since v is omega R, you also know R. So just imagine by recording that sound signal as a function of time, you can determine the period of rotation, this omega, the speed in circular orbit, and the radius, even.

So I want to demonstrate this to you just qualitatively. So I have here a whistle. I don't even know what frequencies it produces. Maybe it's a mixture of many frequencies. That's not so much the point.

[HIGH PITCHED WHISTLE]

PROFESSOR: I want you to hear that if I twirl it around, that now it's no longer a matter of high, low, high, low, high, low. But now it's a gradual change towards high pitch, low pitch, and then you see this in between. You hear that in between. But here we go.

[PITCH SLOWLY GOES FROM HIGH TO LOW AND BACK]

PROFESSOR: Can you hear it? So it's clearly high at times and it's low at other times. But it is now a sinusoidal change-- cosinusoidal change in time. So keep in mind, which is going to be important in what follows today, that from this you can determine the period, the speed, and the radius.

Now electromagnetic radiation also shows Doppler shift. That means if you move towards a source of electromagnetic radiation, you will record a higher frequency than was transmitted. And if the source of radiation comes to you, you will also record a higher frequency. And if you move away from each other, you will record a lower frequency.

And the Doppler shift equation for electromagnetic radiation is not so easy to derive. You need special relativity for that. But I will give you the result. I want you to appreciate that there is no such thing as a velocity of the receiver and a separate velocity of the transmitter, now, because in special relativity, the only thing that matters is the relative velocity between the two. It is an illegal question even to ask who is moving towards whom and who is moving away from whom? So there's only one velocity in special relativity.

So let this be the transmitter of electromagnetic radiation and this be the receiver. And this is the relative velocity, v. And this angle is theta. I'm not trying to tell you that it is this object that is moving in this direction. It could be this one that is moving in this direction. This is the relative velocity between the two and the line of sight.

So the component of the velocity, which we call the radial component, which plays an important role there, too-- so this component is v cosine theta. And with this picture in mind, if this one radiates the wavelength lambda which frequency f, and this one receives a wavelength lambda prime with frequency f prime, then lambda prime is c divided by f prime, and lambda is c divided by f. And I will give you the results of the Doppler shift equation in terms of lambda. And then you can always use that relationship to do it in terms of f.

And the complete relationship is that lambda prime, the one that you will receive is lambda times 1 minus beta cosine theta divided by the square root of 1 minus beta squared. And beta is v over c, c being the speed of light. So you see here that if theta is smaller than 90 degrees, but larger than 0, that means you are approaching each other-- this stands for approaching. That means that lambda prime is then smaller than lambda. So you record a smaller wavelength when it's coming to you.

And we have a name for that. And the name for that is blue shift. And the reason why that is called blue shift is obvious-- because blue light has a shorter wavelength than red light. So even though this Doppler effect may not even be in the visible part of the spectrum, we still call that blue shift. So when the relative velocity is approaching, you go towards shorter wavelengths, and we call that blue shift.

Now if theta is between 180 degrees and 90 degrees-- so that means you are receding from each other-- then lambda prime is larger than lambda, and of course, you've guessed it, that is called red shift. And astronomers talk all the time about red shift and blue shift.

If beta squared is much, much smaller than 1, you can forget about the downstairs if you like that. If you take beta-- say, 10% of the speed of light, which is substantial, right? Which is 30,000 kilometers per second. So if beta is 0.1, then beta squared is approximately 0.01. And so downstairs-- 1 minus 100. You could, through a reasonable approximation, ignore that. But that depends, of course, on the accuracy that you want. In that case, you would get lambda prime is approximately lambda times 1 minus beta times cosine theta. And if now you want to know it in terms of frequency, then f prime is roughly f times 1 plus beta cosine theta. And I'll leave you with proving that this is indeed, then, a good approximation.

Doppler shifts are being used in sports to measure the speed of tennis balls and baseballs when they're being pitched. Radar is sent to the object and it reflects off the object. And so you can measure this way the speed. Weather radar works the same way. You can measure the speed with which weather approaches you or goes away from you. The police can check your speed of your car. It sends radar to you. It bounces off your car. And then it can sense your speed.

And in my problem set seven, I think I have a problem based on that. If you're approaching the police radially-- so if theta is 0, then of course you would simply get 1 minus beta here and 1 plus beta. So theta can be 0, of course. It doesn't have to be but it can be.

You can imagine how important the role was that this played in astronomy. Because this allows you now to calculate the velocity of stars relative to our solar system, relative to us. Because almost all stars in their spectrum show absorption lines. And if you know the absorption line frequency or the absorption line wavelengths as measured in our own laboratory, then you can measure the velocity of that star. And I will give a particular example so that you see these at work.

There is a star-- I picked only one star-- which is called Delta Leporis. The reason why I give you the name-- not to remember it, but you know it's a real star. And the star, in its absorption spectrum, it shows a very clear dark line, which is the result of calcium in the atmosphere. Perhaps you remember that I did a demonstration here of sodium absorption that I have here, creating an atmosphere of sodium vapor, which created an absorption line in the spectrum.

Well, in the same sense, you will see calcium absorption line in the spectrum of the star. In the laboratory, calcium-- it's actually called the Calcium K Line. It has nothing to do with potassium. So in the laboratory, we know that wavelength very well. It is 3933.664 angstrom. I like to work in angstroms. Some of you prefer nanometers. 1 angstrom is 10 to the minus 10 meters. Just because we always worked, when I was a student, in angstroms, but, of course, if you prefer nanometers, that's fine.

And when you observe the wavelength in the spectrum, you will see that it is lambda plus 1.298 angstroms. The accuracies are stunning. You can determine wavelengths. And so you clearly see that lambda prime is larger than lambda. So that means there is red shift. And when there is red shift, it means that we are receding from each other. And we can calculate the radial velocity. All you know is a radial velocity in this case. And so with that information, you get that beta cosine theta-- if you use that relationship and the Doppler shift equation, you get minus 3.3 times 10 to the minus 4. And so you multiply that by c. And then you get that v cosine theta is then very close to minus 99 kilometers per second.

And so we are moving away from each other radially-- I have no information in this direction-- radially, with about 100 kilometers per second. It is illegal to ask who is moving-- they or we-- it is only a matter of relative motion, which is all that matters in special relativity.

Now Doppler shift measurement of stars and galaxies have had a spectacular impact on the way that we think about the world, about ourselves, and about the universe. And I will discuss now only two examples, which, in my view, are the most impressive of all. The consequences are very far reaching in both cases.

In the first, what I want to discuss with you is the discovery of black hole binaries-- how that came about as a result of Doppler shift measurements. There are many stars which are binary systems. That means there are two stars going about their common center of mass. Our sun is not. Our sun is a single star. If you look at a star which is a binary system, you may only see one star in the sky, because you couldn't spatially resolve them. But you will know that it is a double star, because you see the absorption lines of the spectrum move.

Suppose you see only one of the two stars. Suppose one is invisible because it's too faint, but the other is very bright. Then you will see the absorption lines do this. They move back and forth. And you can tell what the period is-- the orbital period of the stars. If both stars produce enough light, then if one star comes to you-- that gives you blue shift-- the other star must go away from you. Because they go around each other like that. So now you see one set of lines, which goes like this, and the other set of lines from the other star goes like this. And so you now see the motion of the absorption lines of both stars.

In a situation like that, that you see the spectral lines moving back and forth, you know it's a binary system. And so we know of many stars whereby we only really see a spectrum of one, because the companion is too faint. We still know that it is a binary system because of the motion of the absorption lines in the spectrum. So what you can derive now from that is the period of the orbit. And you can find the radii of the orbits if you see both pairs of lines moving. And therefore, you can find the orbital speeds in exactly the same way that we did with sound-- period, speed, and radius.

So now I want to work this out in quite a bit more detail, which then, ultimately, as you will see, led to the discovery of black holes in binary systems. So let's make a picture of a simple binary system whereby the orbits are circular. That is a little special, but it will get the point across. So this is the orbital of one star. And this is the orbital of the other star. So it's a circular orbit. And you are here. This is the center of mass. And let this object, this star, have a mass, m2. And let its speed in orbit be v2. And let this separation from the center of mass be r2.

So the other star must be exactly here, m1. And that has a speed, v1. And let its distance to the center of mass be r1. Circular orbits, the center of mass, means that m1 r1 is m2 r2. If you don't remember, go back to your 8.01 notes. And now you are observing here on earth the optical spectra. You see the absorption lines in Doppler shift. You see them moving like this. And so you will find from star number one-- so this is star number one-- you get the period of the orbit. You get its velocity in orbit. And you get r1, just like we got from the sound. There is no difference.

Now you have star number two. You get the period in orbit. Nice. You can check now that you get the right answer. You have v2 and you have r2. All of that comes out of the Doppler shift measurements. But there is more. You know Newton's Law of Gravity, which leads to Kepler's third law, which is that T squared equals 4 pi squared times r1 plus r2 to the power of 3 divided by m1 plus m2 times G, which is the gravitational constant.

Now look. You know r1 and you know r2 two from your Doppler shift measurements. So you know this. You know the period from your Doppler shift measurement, so you know this. So what you don't know is what is m1 plus m2. But you know that m1 r1 is m2 r2. And so we have two equations with two unknowns. And you'll find m1 and m2. Think about this for a minute. Out of the Doppler shift of this binary system, you get their orbital radii, you get their velocities, you get their period in orbit, but you even get the masses of the individual objects.

Now when you observe from earth, you are probably not in the plane of the orbit. Notice what I did very cleverly. I put you in the plane of the orbit. I said you were here, which is the plane of the blackboard. In reality, you will probably not be in that plane. You will not see the orbital period edge-on. This is also edge-on.

But it will be tilted a little. And if is tilted, then the radial velocity, which is the only one you will measure, will be lower than v2. In fact, you can easily see that. Suppose the orbit was like this. So you are on Earth, and they go around each other like this. Just like this. Then there is no radial velocity at all. So the inclination of the orbit is very important. But I will not further address this today, as it doesn't affect the basic principle behind the Doppler shift. But you can imagine, of course, that this is a key issue for astronomers to get a handle on the inclination.

In our galaxy, there are a few hundred-- I will lower this and get it back up later again-- there are a few hundred very special binaries whereby one star is more or less like the sun-- pretty common, pretty boring. But there is another one, very small in size-- a neutron star or a black hole, as you will see very shortly-- which is very close.

So it's a very-- we call it a closed binary, not because they're close to us, but two objects are close together. So this one could be a neutron star, or, as you will see very shortly, a black hole. And they go around a common center of mass. Say-- and the blackboard is the plane-- they go around like this.

And there is a point here between the two where the gravitational pull in one direction towards this star is the same as the gravitational pull in this direction. It has a name. We call it the inner Lagrangian point. There is also such a point somewhere between the earth and the moon. It's very close to the moon, but there is such a point. And so if the inner Lagrangian point lies here, so that the force in this direction on the test mass is the same as the force in that direction, if it lies under the surface of that star, then the matter that is on this side here wants to flow towards the neutron star. It's energetically more favorable, because the force in this direction is larger.

Now since they go around each other, it cannot fall radially. And so what will happen is that this matter from this star is going to spiral in through a disk, and finally finds its way onto the neutron star. So it shouldn't surprise you, therefore, that this star is called the donor. It provides the mass. And this small object here-- could be a neutron star-- is called the accretor. And this disk is called the accretion disc.

Let us take a closer look at this neutron star. I'm going to make a blow up here. This is the neutron star, which has mass M. And it has radius R. And I take a little bit of matter, a little test mass, from a very large distance and I let it fall onto the neutron star. And now the question is with what speed will it reach the surface of the neutron star?

That's an 8.01 question that all of you should be able to do in no time whatsoever. Gravitational potential energy is converted to kinetic energy. And when the object reaches here, the amount of gravitational potential energy that is released is mMG divided by this R. And that is all converted to 1/2 mv squared. This is the speed with which it hits the neutron star. And so the little m cancels. It doesn't make any difference whether this is a large mass or a small mass. It will reach the neutron star with the same speed. And you will see that that speed is then the square root of 2MG divided by R. You've seen this in 8.01.

You may also have seen this in terms of what we call escape velocity. If you ask yourself the question, if you were on the surface of the neutron star, what is the speed that you need to make it out to infinity-- in other words, to break away from the gravitational pull of the neutron star-- that is exactly that same speed, of course. It has to do with the conservation of energy. So you can also think of this as escape velocity.

Now comes the amazing thing. If you take a neutron star, the mass of a neutron star is very roughly 1 1/2 times the mass of the sun. This is the symbol for sun. And the radius for a neutron star is about 10 kilometers. If you substitute those numbers in there, you will get a phenomenal speed. You get 200,000 kilometers per second, which is 70% of the speed of light.

So whenever any matter falls onto a neutron star from a large distance, it hits the neutron star with 70% of the speed of light. This is kinetic energy-- hits the surface, and it's all converted to heat. And so, if enough kilograms per second fall onto the neutron star, the surface of the neutron star gets very hot, and we know there are cases where it can get as hot as about 10 million degrees. And at 10 million degrees, this neutron star will radiate almost always energy in the x-rays-- very little in the optical, almost all in the x-rays. And so it becomes a strong x-ray source.

To make you appreciate the energy released when something hits a neutron star-- if you take a marshmallow, and you throw a marshmallow from a large distance on a neutron star, when it hits the surface the energy that it is released-- that means the explosion that is caused by the impact-- is comparable to the energy that was released when the atomic bomb was thrown on Hiroshima at the end of the Second World War. So that enormous amount of energy that caused this incredible explosion is comparable to the energy that is released when a marshmallow hits the surface of a neutron star.

Now when you look through an optical telescope from the ground, and you look at this system, you will only see the light from the donor, because there's very little light coming from the neutron star because it has an enormously small surface. Most of the energy comes out in x-rays, anyhow. So you only see the light from the donor.

Astronomers have learned over the years-- this has nothing to do with Doppler shift-- that when you look at the spectrum of a star, they, in general, can tell how massive that star is. In other words, a star which is 10 times more massive than our sun has a very different spectrum than the spectrum from the sun. And so they look at the spectrum and they say, yeah, this is probably five times more massive than the sun, and this is probably 20 times more massive than the sun.

So astronomers can tell by simply looking at the spectrum of the donor what the mass of the donor is. So let's call this star number one. So they look at this star in optical light from the ground and they say, we know roughly m1. Now they see the Doppler shift of this one. So they see an absorption line in the spectrum goes nicely back and forth, and they say, aha, we now know the orbital period. And we know the velocity, v1. And we know the radius, r1. We know all that.

But, of course, astronomers know, I hope, they also know-- where is Kepler's law. I didn't erase anything. Did I hide it? Ah. They also know Kepler's law. Now look at Kepler's law. So you know r1. You know m1. You know T. But what you don't know is r2. It's what you would like to know. That is the mass. That's the radius of the orbit of the accretor. So you don't know this one. And you don't know that one.

But you do know that m1 times r2 1 is m2 times r2. So you have two equations with two unknowns. And you can solve for m2 and you can solve for r2. And so now you will find the mass of the accretor-- immensely powerful.

So this has been a game which has been going on for many years, whereby, actually astronomers-- this is what my specialty has been for the past-- I can't even remember-- since 1966, for the past decades. So in x-ray astronomy, we have made a living out of trying to determine the mass of these accretors. And in many cases do you find that the mass of the accretor is very close to 1.4 solar mass. We are sure that in that case, we're dealing with a neutron star. It's not an accident that we find 1.4 times the mass of the sun.

Chandrasekhar demonstrated-- it's a quantum mechanical calculation-- in 1930 that if you take a white dwarf and you dump matter onto the white dwarf and it reaches a mass of 1.4 solar masses, that the white dwarf can no longer support itself, but collapses. And so that's the upper limit of the white dwarf, and that's called a Chandrasekhar limit. He got for that a Nobel Prize in 1983.

A white dwarf, by the way, all by itself is a bizarre star. It has about the radius of the earth-- about 10,000 kilometers-- and it has about the mass of the sun. So they're already weird stars all by themselves. But he demonstrated that the upper limit on the mass of a white dwarf is 1.4 solar masses. So we were not surprised that many of these accretors had a mass of 1.4 solar masses. In other words, they probably were white dwarf in their past, and they were pushed over the Chandrasekhar limit.

Later in time, came general relativity-- Einstein's theory of general relativity. Out of that theory, it follows that if you keep dumping mass onto a neutron star, the moment that you cross the 3 solar mass line, the neutron star can no longer support itself and collapses into a black hole.

And now comes the question, what is a black hole? A black hole is an object from which light cannot escape. I have to erase something, and I think I'm going to erase the left part of the blackboard. It is an object from which light cannot escape. I have to be more specific than that. A black hole is an object from which the escape velocity is a larger than the speed of light. We know what the escape velocity is because this-- remember, that's why I paid attention to it and mentioned this is also the escape velocity.

So here is a black hole. There is a radius around that black hole which has a name, a wonderful name. It's called the event horizon. And all I have to do is make this c, and I can calculate what that radius is. And that means if you're then inside that radius, then the escape velocity would be larger than c. So you can't get out. And so the radius of a black hole is then-- the radius of the event horizon. I wouldn't want to call it the radius of the black hole. But the radius of the event horizon, R, is then 2MG divided by c squared.

If you are closer to the black hole, the escape velocity would be larger than c, which is not possible. To give you a little bit of feeling for what this means-- if you take the earth and you take the mass of the earth, which you can look up in your book, and you substitute that in there, you will find that the radius of the earth, to become a black hole, is only 1 centimeter. So you substitute in here the mass of the earth-- c squared, you know; G, you can look up-- until you find that the radius is about 1 centimeter.

So if you could squeeze the earth with its present radius of 6,400 kilometers-- if you could squeeze it to 1 centimeter, then you would have created a black hole. But, of course, that may be problematic to do that.

If you take the sun-- which by definition has 1 solar mass, and so you put in here the mass of the sun, which you can look up in your book-- then you will find that the radius of the event horizon is 3 kilometers. If you take a 5 solar mass black hole, then you'll find that the radius of the event horizon is 15 kilometers. Notice it scales linearly with M. That's what you see here. M is linearly upstairs. So if you go from 1 solar mass to 5 solar mass, the radius of the event horizon scales linearly.

In 1971, astronomers Tom Bolton and Murdin and Webster came with a daring statement that one of these systems was a black hole binary. They looked at the spectrum of the donor, and they concluded that it probably was a 20 or 30 mass solar star. And then they made the Doppler shift measurements of the absorption line in the spectrum of the donor. And so they went exactly through the reasoning that I showed you earlier, which I think has been erased now. And they came up with the fact that the accretor had a mass of about 15 solar masses.

And so they came to the conclusion that-- since it was an x-ray source, they knew that the accretor was very small, otherwise you couldn't get these high temperatures in the x-rays-- and so they made the daring statement that the accretor was probably a black hole. This had an enormous impact on the astronomical community. And a lot of people emotionally couldn't handle it. And there were lots of publications for years to come arguing that it was all nonsense, that black holes didn't exist, the data was misinterpreted, they came up with all kinds of other weird explanations.

But the black hole still stands. And most sane astronomers-- that doesn't mean all astronomers are sane-- but most sane astronomers do believe that Cygnus X-1 is a black hole. And we now know of about two dozen of these systems whereby the accretor is a black hole. And I decided to make Cygnus X-1 part of your problem set. So you have some time to wrestle with it.

I want you to see a picture of Cygnus X-1. It is not very dramatic, but I want to see it for emotional reasons. You will just see a star. You can't see the black hole. You just see a star, and that's all. But at least you can tell your parents that you've seen a star that is in orbit with a black hole. So if we can get the first slide, the only slide I think I have. And we should be able to show you a star. It's a negative. In astronomy, we almost always work with negatives.

So the stars themselves are black and the sky is white. And it is this star here that is the donor. It's quite bright. You cannot see it with naked eye. For those of you who are familiar with magnitudes of stars, it's a ninth magnitude star. And it is in orbit with a black hole. It has a mass of about 20 to 30 solar masses. And the black hole has about a mass of about 15 solar masses.

And all of this is the result of Doppler shift measurements-- profound, far reaching consequences. So I think this is a wonderful moment to have a break. And I know that most of you came here not to listen to my lecture but to take the mini-quiz. And so, let's do that now then. So if you can help handing these out. For those of you who come here only for the mini-quiz, I have some advice. You may have noticed that the question that I ask on a mini-quiz on Tuesday almost always goes back to the previous lecture on Thursday. Think about that. OK. So if you can help me handing this out here.

The biggest impact that Doppler shift measurements of the electromagnetic radiation had on the perception of ourselves and of the universe came in the 1920s. An industry had already been developed by astronomers to measure the radial velocity of stars. I gave you one-- the Delta Leporis is a typical one. We were receding 100 kilometers per second. Velocities were typically 100 kilometers or 200 kilometers per second. Some cases receding, in some cases, we were approaching each other.

Slipher, in the early '20s, noticed that there are some nebulae in the sky which have phenomenal velocities-- up to 1,500 kilometers per second. And in all cases, it was red shift, so they were receding. And it was soon recognized afterwards that these nebulae were not at all in our galaxy, which was originally thought, but that they were galaxies on their own, so that they contain some 10 billion stars.

And so if you take a spectrum of a complete galaxy then, of course, what you end up with is the average red shift of all the stars that have absorption lines in their spectra. And so that gives you, then, a handle on the radial velocity of that galaxy.

And Edwin Hubble and Humason found, surprisingly, a correlation between the distance of galaxies and their velocities. Now to measure the velocity is easy. That's a Doppler shift measurement. That's relatively trivial. All you have to measure-- lambda prime and compare it with lambda. A distance measurement is very, very difficult in astronomy-- very problematic. It's a lecture all by itself.

And so in the days of Hubble, his distance estimates were very different from what they are today. His distance estimates to the nearby galaxies were seven times smaller than what we think today. But that doesn't matter. He did find the linear relationship between velocity and distance. And that was a spectacular discovery. And this is known now as Hubble's law.

So the radially receding velocity of galaxies is some constant-- which was later called after him, Hubble's constant-- times the distance, d, to the galaxies. So v is the radial velocity. It's receding. We don't give it a minus sign anymore. That v is always receding. The present most reliable value for H is approximately 70 kilometers per second per megaparsec. I gave it to you in this form because that's the way you will always see it in the literature, but I owe you an explanation, of course.

A megaparsec is simply a distance in astronomy. It is the distance that light travels in 3.26 million years. So it is also 3.26 million light years. And if you know what the speed of light is-- 300,000 kilometers per second-- then you will find that it is about 3.1 times 10 to the 19 kilometers. Notice that the dimension of Hubble is 1 over seconds. It's 1 over time, because kilometers is distance and megaparsecs is distance. So it is 1 divided by seconds.

Hubble himself found a value which is 500 kilometers per second per megaparsec. The reason was that he had a different estimate for the distance. He thought that the distance to the galaxies that were in his sample were seven times smaller than what we think today.

Suppose you trust Hubble's law at face value as it is. And you and I look at the particular galaxy in the sky, and we measure that lambda prime is 1.02 times lambda. In other words, it's red shift, and the radial velocity is there for 2% of the speed of light. So you can measure that beta cosine theta is then minus 0.02. I carry that minus sign because I gave you that Doppler shift measurement. In the form that I want you to use, we don't even think of that minus sign anymore, because it's always receding. But I carry the minus sign. And v cosine theta, then, is this number times the speed of light. And that becomes, then, minus 6,000 kilometers per second receding.

So the radial velocity-- receding speed-- is 6,000 kilometers per second. This is about 2% of the speed of light. If you trust Hubble's law, you can now use Hubble's law and calculate what d is. So d is now our 6,000 kilometers per second. And you divide that by 70. And so you get the distance, now, in megaparsecs, which is 85 megaparsecs. And that corresponds to about 280 million light years. 280 million light years-- I will abbreviate that lyr. All right. It means that the light you're order processing on the ground was emitted by that galaxy 280 million years ago. In astronomy, we look back in time.

I'd like you to see now the spectra of three galaxies which are at vastly different distances from us. Let me first turn on the slide projector, otherwise I may end up in the darkness and not know what I'm doing. That's right. I'll show you the spectra of three galaxies.

This is a galaxy which is the closest of the three. That should be rather obvious, because it's also the largest one. And the spectrum of the galaxy is what you see in the middle here.

These are measurements of known lines with known wavelengths in the laboratory. So this is just a calibration here, and this is a calibration here. But this is the spectrum of that galaxy. And what you see here are two dark lines. Those are absorption lines-- the average of the stars in that galaxy. They have shifted relative to the wavelengths in our laboratory.

By the way, one of those lines is the same line that I used for Delta Leporis, which is the Calcium K absorption line. The other one is the Calcium H absorption line. H has nothing to do with hydrogen. And it had been shifted over the distance that you see indicated by this small arrow. And if you measure now the velocity, you'll find 1,150 kilometers per second. And if you use today's value of Hubble's law, this object is 52 million light years away from us.

And now you go to this object, which is further away. According to Hubble's law, it has a higher speed, the receding speed. And you see the spectrum here. This is the spectrum. And here you see those two absorption lines. They have shifted all the way. And out of that then follows the radial velocity, which in this case is 7% of the speed of light-- 22,000 kilometers per second-- which puts this object at 1 billion light years away from us.

And when I go to this one here in the corner-- it's hard to see which one it is-- the absorption lines have shifted even further. You see the absorption lines here. And that object is going with 20% of the speed of light, receding. Then it would be roughly at a distance of 2.8 billion light years.

The most modern work on Hubble's law was done by a group under the leadership of Wendy Freedman in California. Wendy used data from the Hubble Space Telescope. And she measured the Doppler shift and the distance to a few hundred objects. Distance is always the can of worms. And she came up with presently the most reliable value for Hubble's constant. Let me get the next slide, if I succeed.

This is Wendy Freedman's work. Forget what you see here. Just look at what you see here. It's distance. So the farthest object that she was able to get a reliable distance for is 400 megaparsecs. That's a stunning distance, by the way. That's about 1.3 billion light years. And she was able to measure radial velocities up to about 10% of the speed of light-- velocities of 30,000 kilometers per second. And so she can draw this line, which is the linear relationship between velocity and distance, and out of that line comes Hubble's law. And she concludes that it is very close to 72 kilometers per second per megaparsecs plus or minus a few. I used 70 just to round it off.

What is interesting is that Hubble, in his entire data, the velocities were all less than 1,100 kilometers per second. Put that in scale here. These already is 2,000 kilometers per second. So Hubble had only data in this teeny-weeny little portion here, where Wendy doesn't even have any data. She doesn't even use them. And the reason why she doesn't use them, she says it's too close. It's not really representative for the-- what we call the Hubble flow. So she stays away from the local stuff.

But nevertheless, based on that very local stuff-- which is not even put in here-- Hubble came with his linear relationship, which was a monumental statement in the history of mankind, because it puts the universe at a very different level than what people thought before. The universe is expanding.

So let's now ask ourselves the question what does this all mean? The first thing that may come to mind, since all objects are moving away from us, you may think that you are very special. We all like to think that we are very special. So you may even think that you are at the center of the universe, because everything moves away from you. And persuasive is to think that there was a time in the past where there was a huge explosion, which we refer to as the big bang where all of this happens.

And there were galaxies with large speeds in the explosion. They are now the ones that are farthest away from us. So they have the highest speed. And some have the lowest speed. So that's a picture that presents itself. It's a little bit naive, though, and partially wrong. However, the idea of the big bang theory, that there was such an explosion a long time ago, that still holds. And that is considered the birth of our universe.

And so now comes the question, when did this big bang occur? Well, we could make a simple assumption, perhaps not quite accurate, that the velocity which with all these galaxies are moving never changed in time. We can do that, right, as a start. So, no matter which galaxy you take, the velocity now is the same that it was in the past. So then we have 8.01 which says the distance that that galaxy travels is the velocity with which it traveled-- the radial velocity-- times the time that it traveled-- which is then the age of the universe-- how many seconds the universe lasted. So this is simply 8.01.

But we also know Hubble's law. In other words, we can replace this v here, Hd, by this one. So we get d equals H times d times t universe. And so you'll find that the age of the universe is 1 over H.

And remember I mentioned that H has the dimension 1 divided by seconds. So this has the dimension of seconds. And if you use the value for H of 70 kilometers per second per megaparsec, then you will find that the age of the universe-- you can check that for yourself-- is 4.3 times 10 to the 17 seconds. So tu becomes about 4.3 times 10 to the 17 seconds, which is about 14 billion years. So you would have to conclude, then, that the big bang explosion took place about 14 billion years ago.

Now you can easily imagine that, due to gravitational attraction, that the speeds in the past were probably higher. You would think that this expansion probably decelerated. And so on that basis, you could conclude that it's probably less than 14 billion years. We have very good reasons to believe that the oldest stars in our own galaxy are 10 to the 10 years old-- that means 10 billion years old. So the big bang must have been somewhere in 10 to the 14 and 10 to the 10. And most astronomers now would say, yeah, it's probably something like 12 billion years ago that the big bang took place.

And the issue of the deceleration is, of course, at the heart of research in cosmology. It is now believed that in the very early universe, there was acceleration of the expansion. It is called inflation. It was followed by a deceleration, which is what you would expect due to gravity. And it is now also believed, very mysteriously, that there is, again at a later time, again acceleration-- not very well understood.

We have MIT experts here, Professor Alan Guth is a world authority on this. And Bertschinger and Scott Burles do also research in this. Alan Guth is the godfather of the inflationary universe. I will get back to that.

So the question now that I would like to raise is how far can we look back in time? But all I can tell you is that the largest red shift that has been measured to date-- lambda prime divided by lambda-- is 7.5. That is enormous. And so now comes the question what is the speed and what is the distance? And I cannot answer that. The reason is that the equation that I have given you, the Doppler shift equation, was only derived using special relativity. It did not take general relativity into account. And general relativity introduces also red shift, which has nothing to do with the Doppler red shift.

And so I will leave it up to my colleagues, Professor Guth and Scott Burles and Ed Bertschinger, to tell you what perhaps the speed is of this object and what the distance is. And they may not even give you one answer. They probably give you more than one answer, depending upon certain assumptions.

General relativity changes the picture quite dramatically. There is no doubt, however, they will all agree. That this object is very far away, probably more than 10 billion light years. So you're looking back in time more than 10 billion years. So you're almost getting to the point that you're beginning to see the actual burst of the universe, the big bang itself.

What is also interesting is that, remember I mentioned the calcium absorption line-- the K-line-- was roughly at 3,933 angstroms. If you multiply that wavelength-- this is lambda-- if you multiply that by 7.5, it ends up at 29,500 angstroms, which is outside the optical spectrum, impossible to observe from the ground. And so how can astronomers ever find red shifts, which are as large as that ratio 7.5?

And the way they do that-- they look at a line in the spectrum, which cannot be observed from the ground at all-- it is a line emitted by hydrogen. It's called the Lyman alpha line. It is really of hydrogen. And the wavelength is 1,216 angstroms, which is in the UV, which you cannot observe from the earth. The atmosphere absorbs the UV.

But if you multiply this by 7.5, then you get it all the way down to about 9,100 angstroms, which you can observe from the ground. And so it is remarkable, when you think of it, that lines which are invisible due to Doppler shift are shifted all the way to the extreme red part, to the infrared part of the spectrum, where they can just barely be seen.

Now comes the question, are we at the center of the universe? Well, not likely. Suppose you have a raisin bread. You have made dough, you put the raisins in there, and you put it in the oven. And this raisin bread is going to expand. Each raisin will see all other reasons move away from it radially. Think about it. Because if the double expands uniformly in all directions, everything moves away from your radially.

And one raisin will see the nearest raisin going only with very low velocity. But when it's farther out in the cake, it goes with a higher speed. So that raisin can actually be the discoverer of the expansion of its own universe. And it can measure the distances to the other raisins and the velocities. And it will come up with Hubble's law.

But not only will one raisin come up with Hubble's law, but any other raisin that you pick will come up with the same Hubble's law. And it will also think, perhaps, that it's at the center of that cake, which is, of course, not the case. So anything that expands uniformly in all directions will fit the idea of Hubble's law, and also will make you think erroneously that you may be at the center of all that.

There is a very nice and even better analogy that I can draw. So I will abandon the raisin cake now. And I will now discuss with you the universe of flatlanders. Flatlanders are people who do not have three dimensions but only two dimensions. And so they can only move in a plane. They do not know, but we do, in this flatland world that they really live on the surface of a balloon. But they don't know that. And so the lights in their flatland world can only travel along the surface of the balloon. But they don't know that they live on the surface of a balloon.

And now, this balloon can expand. And the galaxies, which are on the surface of that balloon-- here they are. Here are the galaxies. Some are actually very nice. Here, there's a spiral galaxy, you see that? They're nice galaxies.

So when you blow up this balloon, each galaxy will see other galaxies move radially away from each other, because light travels along the surface. And the farther a galaxy is away, the higher the velocity. And so let us expand this balloon and see what this world-- oh, I first have to connect it to the source of the expansion. OK.

So these poor flatlanders have no idea what is happening to them. So they are living on the surface of this balloon. They cannot see the third dimension, so they have no way of knowing. And this balloon grows and grows and grows. And so the distance between the galaxies gets larger and larger and larger. And each one of those galaxies may have astronomers. And each one of them will discover Hubble's law. And they will find the same constant for Hubble's law when they make the measurement at the same moment in time.

Now, they may not call it Hubble's law. The name of that astronomer may be different, of course. But that is a detail. So as their universe expands-- and we're going to work with that a little now-- we'll see what kind of conclusions they draw.

There goes the universe. We'll leave it there. Oh god.

[LAUGHTER]

PROFESSOR: Here is the balloon. And the balloon has a radius, R, which they do not know, but you and I do. And this is today. Tomorrow, the balloon is there-- or next month, or next year. And so the radius of that balloon next year is R plus dR. So this radius is R plus dR. And here is a galaxy, which tomorrow is here. And here is a galaxy, which tomorrow is there. And the distance between these galaxies I call S. But tomorrow the distance is S plus dS.

So in this universe, it is clear that S plus dS divided by R plus dR-- so that is this triangle-- must be the same as S divided by R. And so I can work that out. So I get SR plus RdS is SR plus SdR. Now I can look at the evolution in time. So I can give that radial expansion certain value. So I divide by dt.

What is the meaning of S? That is the distance between the galaxies as you measure them. So we call that d in our universe. What is the meaning of dS dt? That is the receding velocity with which they move away from each other. Light travels along the surface of the balloon. So this, dS dt, is that v.

So what do we find? We find that v equals d times 1 over R dR dt. And this dR dt is what I would call the expansion rate of this balloon, expansion rate of the universe.

And so the flatlanders measure v. They measured d. And at a particular moment in time-- there is a value of dR dt, which they don't know. There is a value of R, which they don't know. But at a particular moment in time, this is a constant. And so they call this Hubble's constant. It has indeed the units of 1 over seconds, because dR is in meters. And R is in meters. So it has indeed the units of 1 divided by seconds.

They have no way of knowing what it means. But we do. But we can look in the third dimension. So we know that R in the past was smaller. They don't know that, but we do. So even if dR dt never changed-- that the universe expanded with a constant rate-- we know that Hubble's constant in their past must have been different. Because if R is smaller in the past, Hubble's constant must have been larger in the past.

That is the reason why we claim that Hubble's constant in our universe is 70. We always put a little 0 there. So we always say H0 is, in Wendy's case, 72. Even if you look at the slides, she had that little 0 there. That means as we measure it now, we make no claim about Hubble's constant, how it was 5 billion years ago, or how it will be 5 billion years from now. We make no statement about that.

Now if the expansion rate of the balloon decreases in time, then there comes a time maybe that the balloon will come to a halt. That means when dR dt becomes 0, Hubble's constant will then be 0. All galaxies on the balloon will stand still. And then gravity will take over and the balloon will start to shrink. And Hubble's constant will then become negative. The sign will change. All red shifts will change to blue shifts. Everything will approach each other. And ultimately, the whole universe may collapse onto itself, which we generally refer to in the literature as the Big Crunch as opposed to the Big Bang.

We used to talk about a closed universe up to a few years ago. A closed universe would be a universe that would collapse onto itself. And so there would be the Big Crunch. We compare that to a few years ago with an open universe, which would never collapse onto itself. It would always be expanding. And we even introduced the idea of what we call a flat universe, which is just in between the two. That means the universe would be expanding forever and ever and ever. And it would come to a halt, but you would have to wait infinitely long. It's like having a tennis ball thrown up from the earth. It would just make it to infinity with zero speed. That's basically that idea.

But a lot has changed with the discovery a few years ago of what we now refer to as dark energy. And so we now believe that the universe will expand forever. And that there will not be a Big Crunch. You should not carry the analogy between the flatland balloon universe too far. But it is very interesting. It gives you some interesting insights. And it certainly gives you food for thought.

For one thing, it suggests that our 3D universe perhaps is curved in the fourth dimension. Just like the 2D universe of the flatlanders is curved in the third dimension-- a very fascinating thought. And if you want to learn more about this, I would advise you very strongly to take a course in cosmology. If you're so lucky that Alan Guth is teaching it, then don't miss that opportunity. He gives wonderful courses on the early universe. But also you may want to learn a little bit more of general relativity, which of course plays a very key role in cosmology.

Doppler shift measurements of electromagnetic radiations from galaxies have played a key role in the way that we think about ourselves and that we think about the universe. And it was in 1979 that Professor Alan Guth-- who is still at MIT, he was also then at MIT-- made an exciting prediction that the universe may have gone through an expansion phase very early on after the Big Bang-- an exponential growth in expansion-- and that the universe doubled in size in about 10 to the minus 34 seconds. The universe doubled in size. The early universe-- it's called the inflationary universe-- in 10 to the minus 34 seconds.

How long this phase lasted is not very well known. But it may have lasted 10 to the minus 24 seconds. So after about 10 to the minus 24 seconds, our universe had increased in size by a factor of 2 to the power of 10 billion. Unimaginable. Beyond comprehension. And this idea of the inflationary universe, which came, first from Alan Guth, has been the driver behind NASA's cosmology program.

There's an observatory about 1 1/2 million kilometers from earth. It's called the Wilkinson Microwave Anisotropy Probe. It was launched in June 2001. And that is now confirming many of the predictions that follow from Alan Guth's inflationary universe. There is no doubt in my mind that Alan will get the Nobel Prize for this. It's not a matter of if. It's only a matter of when. You will see the day, and I hope I will see that day too. Thank you and see you next Tuesday.