PROFESSOR: I'm Walter Lewin. I will be your lecturer this term. Make sure you have a handout and make sure you read it. It tells you everything you want to know about the course.

This course is about waves and vibrations, about oscillations, periodic and not-so-periodic events. When you look around in the world, you see them everywhere. For one thing, your heartbeat. That's a periodic oscillation. At least I hope that for most of you it is periodic.

Your breathing is some kind of a periodic motion-- the blinking of your eyes. Your daily routines and your habits. Your eating, your sleeping, taking a shower, your classes, and occasionally doing some work-- all those are periodic actions.

When you drink-- I drink some orange juice. Now notice as I try to move the liquid down into my stomach that it's not a steady stream, but it is a periodic motion. Look at my throat. In fact, even if I don't want to swallow the liquid, but simply have a bottle with liquid and I turn it over, then we all know that the water doesn't come out like a steady stream, but it goes glug, glug, glug, glug. That's some kind of a periodic motion.

I have here a toy which I use to entertain my dinner guests. Particularly for physicists, it's interesting. And there's liquid here. The idea is to get the liquid there. And then the problem is, how can you do it in the fastest possible way?

Well, if you turn it over, you see that phenomenon that I just mentioned, which is that glug, glug, glug. It's not a steady stream. It's almost pathetic, the way that it runs from one side to the other. It will take minutes before it's there. But it can be done in 17 seconds, and during the five-minute intermission that we have, you may give it a try, and I hope you won't break it, and see whether any of you can think of a way that you can transfer the liquid in 17 seconds.

You have breakfast in the morning, and you casually put your breakfast plate on the table. What do you hear? Some kind of a periodic motion.

And two things can happen to this plate. It can move azimuthally. I call this "azimuthally" because I'm an astronomer-- but could also wobble. In fact, something can wobble without moving azimuthally, and something can move azimuthally without wobbling. In this case, it does both.

And a fabulous example of that is what's called the Euler's disk, which is a metal disk-- you will see shortly there-- and this metal disk we're going to wobble in a similar way that I wobbled the plates, and then we'll follow its motion azimuthally and the wobbling frequency. And what is interesting, as you will see, that the azimuthal motion, which has a certain period-- that period gets longer in time. But the wobble motion, the frequency goes up.

So I'll start it here, and then I'll show it to you in a way that is more appealing, and you can follow that. It is an amazing toy. To work out the physics is very, very difficult. I was told that Professor Wilczek at MIT once gave a one-hour lecture exclusively on the explanation of this Euler's disk.

So try to see the azimuthal motion. It will become clearer as it slows down further. You may be able to hear the wobble motion. I'll hold my microphone close up.

[WHIRRING NOISE]

PROFESSOR: Can you hear it? Very high frequency already. Did you hear it?

It's quite amazing, isn't it, when you look at this.

So the wobbling frequency increases quite rapidly. Look how the azimuthal motion slows down, and how the frequency of the wobble goes up.

[LOUDER WHIRRING]

PROFESSOR: Ah! And now it comes to a stop. That's a very difficult piece of physics, right there.

If you take a tennis ball-- this is a SuperBall-- and you bounce it-- whoa. It's called "student involvement." Thank you.

Then you also get some kind of a periodic motion whereby, again, the frequency increases, just like in the case of the Euler disk and the breakfast plate.

Here's an object that is floating in a liquid, in water. And if I push that a little farther in and let it go, it wobbles, and there is a very unique frequency that you will be able to calculate in 8.03-- a very unique period of uncomplete oscillation as this object goes up and down.

Even wind, steady wind, can generate periodic or almost periodic motions, which all of you have experienced. You walk outside, it's windy, and your hair goes like this. Your hair doesn't go flap, like this. Always has this tendency, just like a flag does the same thing.

If I generate wind here, and I have here some aluminum, now you will see that this wind doesn't make the aluminum just go straight out, but it wobbles. There's a certain period to that.

After work, if you want to have some fun, what is more fun than riding your own rocking horse? That's a periodic motion.

Falling in love can be a periodic event. Now, don't do it too often, because as most of you know-- quite exhausting.

The motion of electrons, atoms, molecules-- periodic and oscillatory. The motion of the moon, the planets, and the stars-- periodic, oscillatory.

Sound is a beautiful example. I produce sound by oscillating my vocal cords. I produce, thereby, pressure waves. My vocal cords push on the air, suck on the air, push on the air, which produces a pressure wave. And that pressure wave propagates out in the lecture hall, reaches your eardrum, your eardrum starts to move back and forth, and your brains tell you that you hear a sound.

I have here a tuning fork, which is designed so that if I give a hit, that the prongs move 256 times per second. We call that 256 hertz. A hertz is 1 oscillation per second.

[TONE]

PROFESSOR: I don't know if you can hear that. Pressure waves are generated. We will use them in 8.03. They travel through the air, reach your eardrum, and your eardrum to shake.

This is a higher frequency-- 440 hertz.

[HIGHER TONE]

PROFESSOR: Most human beings can hear in the range from 20 hertz to 20 kilohertz. Now, animals who can go way beyond 20 kilohertz.

And to be nice to you for the first time, this first lecture, I would like to test your hearing, and that will be free of charge. I'm not so much interested in knowing what your lowest frequency is, but what your highest frequency is. So I'm going to generate here sounds. I will start with 100 hertz, and then we'll go up higher and higher, and then we'll see where your hearing stops. So let's start with 100 hertz.

[LOW TONE]

PROFESSOR: I'm not going to ask you who can hear it, because clearly all of you can.

Let's now go to a kilohertz, 1000 hertz.

[VERY HIGH TONE]

PROFESSOR: Piece of cake, right?

[TONE GETS HIGHER]

PROFESSOR: 2000, no problem.

I have to change now my scale. 4000.

[VERY HIGH-PITCHED TONE]

PROFESSOR: Ah, I didn't say that this is going to be a pleasant test. OK. 5000. Here's where the violins come in.

6000. Anyone in my audience who cannot hear 6000?

7000. Anyone in my audience who cannot hear 7000? I cannot hear 7000. I hear nothing.

With age, you lose the ability to hear high pitches. You will experience that in your lifetime. You won't escape that. Now, some people lose more than others, but I cannot hear above 6000 hertz. I hear nothing.

OK. 10,000. 12,000. 14,000. Now I want to see hands if you cannot hear it any longer. Who cannot hear 14,000? Don't be ashamed of it. It's not your fault.

14,000. All right. We're slowly going up. 15,000. Who cannot hear it? Raise your hand. Ah, Professor Mavalvala you're also getting old, Nergis.

16,000. Who cannot hear 16,000? Of course, the ones who have already raised their hands, you don't have to raise your hands again. Who cannot 16,000? Who cannot? 17,000? 18,000?

OK, so now we're going to change it. Now I want you to raise your hand if you can hear it. So I'll first now go to 20,000. 19,000, I'm going to 19,000. Oh, sorry-- I was only off by a factor of 10.

19,000. Who can hear it? Fantastic. 20,000. 21. Ah. You see how the cutoff? Very sharp. 22. Very good. 22. 23. 25. 27!

Some of you have amazing ears, because I turned it off at 21,000.

All right. Key, absolutely key in this course, will be simple harmonic oscillations, because they are extremely common in nature.

A simple harmonic oscillation-- and you've seen this, of course, in 801-- can be written as follows. x equals x0 cosine omega t plus phi. You can write a sign here if you want to.

x0 is the amplitude. It's the largest displacement from equilibrium. Omega is the angular frequency, angular frequency omega, which we express in terms of radians per second. The period t, 2 pi divided by omega, is then expressed in terms of seconds, and the frequency s, which is 1 over t, is what we call hertz-- the number of cycles per second. Do not confuse omega with s. There is a factor of 2 pi difference.

If I have a uniform circular motion, and I project that uniform circular motion onto any line in the blackboard, then I get a simple harmonic motion. So I take, for simplicity, just this horizontal line, I could pick any other line-- let's call this the x-direction, and let this point be x0. And I take an object which is rotating around-- here is the object that's going around-- uniform circular motion-- if I project this onto the x-axis, and this angle is theta, then this position here is x0 cosine theta. And if I make theta a function of time-- theta equals omega t-- this omega is what we call-- not angular frequency, but we call it angular velocity.

It's very awkward in physics that we have the same symbol for angular velocity and for angular frequency. In this case, they happen to be the same numerically, because it's a uniform circular motion. That's an accident. So now you see that x0 then becomes cosine omega t, because the two are the same.

I do not have to call the position at t equals 0 here. I can choose t equals 0 anywhere along the circumference, and that introduces then phase angle phi. We call that the initial condition. So x0 is the amplitude, omega is the angular frequency, and phi has to be adjusted so that at time t equals 0, you get the right angle at the right position.

An easy example of a simple harmonic motion is a spring system. If I have here a spring, and this is in a relaxed position, the spring constant is k, the mass is m, and x equals 0 here. And I bring it further out, I bring it to a position x, then there is a spring force that wants to drive it back to equilibrium. It's a restoring force. That's the spring force. Let's arbitrarily call this direction plus.

The spring force we call minus kx, minus, because if x is positive, then this force is in the opposite direction.

If the mass of the spring can be ignored, if it is negativity small compared to the mass of the object, I can write down Newton's Second Law, F equals ma. You may remember that from your good old days. And so ma is mx double dot, is now minus kx. It's really a vector notation, but since it's a one-dimensional problem, the minus sign takes care of the directions.

And so I can massage this a little further, and I can write this as x double dot plus k over m times x equals 0. And what is the solution to this differential equation? This is a differential equation, x double dot and x-- this is the solution. The simple harmonic motion, provided that omega is the square root of k over m.

So I advise you to take this function, substitute it in here, and you will see that out pops-- yes, you can satisfy this equation, provided that omega is the square root of k over m.

Notice, which is not so intuitive, that this angular frequency omega, and therefore also the period of oscillation, 2 pi divided by omega, is independent of x0. So it's independent of how far I move it away from equilibrium. If I move it far out, it will take the same amount of time for one oscillation than if I move it out a teeny-weeny little bit. Not so intuitive.

So omega is independent of my initial conditions. It's independent on how I start the experiment off. It's independent of phi. It's independent of what I call t equals 0. Nature doesn't give a damn what I call t equals 0. Nature has one answer for the frequency, and that's only determined by k and by m. Not by my initial conditions. Not so intuitive.

If I take the same spring, and if I hang the spring vertically-- there's the spring-- due to gravity, the object will come to a halt, equilibrium, a little lower, obviously. If now I displace it from this equilibrium position and let it oscillate, I get exactly the same frequency. Maybe that's not so intuitive, either. And you can work it out for yourself. It's an 8.01 problem.

What that means is that you can define this as x equals 0, ignore gravity completely, and set up your differential equation as if there was no gravity, and this is x equals 0. So you offset it over a distance x from that equilibrium position. You only allow for a spring force minus kx, and everything works. And of course you should be able to prove that that is correct.

If a spring oscillates in a simple harmonic fashion, and we have such a spring here-- Marcos, if you could do me a favor and get it up here-- then I should be able to demonstrate that a uniform circular motion projected on the wall-- we call it shadow projection-- should be able to-- thank you, Marcos-- should be able to have the same motion as my spring, provided, of course, that we very carefully make the period of oscillation of the spring exactly the same as the time for this object to go around. We then shadow project it on there, and then I will even try release this one-- it's very difficult-- at the same time that this one is here, and what you will see then-- you will see the uniform circular motion projected becomes a simple harmonic motion, and you'll see the spring simple harmonic.

And so we'll try to do that in shadow projection. We'll make it a little darker. And for that, I need some light here. OK. Someone already turned it off.

So here you see the spring, and there you see this object, which is rotating in a circle, but you think it's a simple harmonic motion. And that's of course my objective.

So now that is difficult. I will have to block you for a few seconds. I will try now to release this at the same time, and also at the same amplitude.

Ugh! Boy. That wasn't my best day, was it? No. No. Oh, this is perhaps the best I can do today.

So they don't go exactly next to each other, but you see, they have the same period, and they both represent simple harmonic oscillation. The spring, because we just calculated that, and the projection of the uniform circular motion.

So if we return to the spring-- maybe we should remove this. If we return to the spring, then we have a period for the spring system which is 2 pi times the square root of m over k. And I want to bring this to a test, to a quantitative test, how accurate this is. I'm going to double the mass that I'm going to hang on that spring, I'm going to measure the period, and then I want the mass, which is twice as high-- I want that period to be the square root of 2 times higher, because that's what this equation predicts.

Now whenever you want to do a measurement in physics whereby you want to compare numbers, so you have a certain goal in mind, a measurement without the uncertainty in the measurement is completely meaningless. You must know the accuracy of your measurement.

So n1 is 500 plus or minus 0.2 grams, and n2 is 1000 plus or minus 0.2 grams. That's the best we can do. That's an extremely small error. This is an error of only 0.04%, and this is only half as large.

Now comes the question. If I measure the period of oscillation with the 500 grams hanging on the spring, how accurately can I do that? On a good day, I can do it to 0.1 second accuracy. I have to start it, I have to stop it, and if I do that 10 times, obviously you'll get different answers, and they vary by about 1/10 of a second on a good day. On a bad day, 2/10 of a second.

I don't know whether today is a good day or whether it's a bad day, but let's say it's in between. So let's say I can do it to 0.15 seconds, which I cannot guarantee you, but I'll try. So I can measure the period to plus or minus 0.15 seconds-- this is n1. However, I can get a very accurate measurement for the time, for the period, if I make 10 oscillations. Because if I make 10 oscillations, the error in t goes down by a factor of 10, because the 0.15, that's not going to change.

So I'm going to oscillate it 10 times, and then we're going to make a prediction about what we should measure for the higher mass. So let's first measure the period of, this is the spring, and here is the 500 grams plus or minus 0.2. I'm going to oscillate it-- we already notice that it's independent of amplitude-- and then I'm going to start it when it is down-- that's the easiest for me-- and I will count to 10, you will count to 10, and then we'll stop it.

So let's give it an oscillation. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 14.96. So let's write this down. We have 14.96.

Do you want to see whether this is a good day or whether this is a bad day? I can measure it again. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 14.98. So this is not a bad day, but it's luck that it comes out so close, of course.

So now we can make a prediction that 10 times Tm2 must be 1.414, which is the square root of 2, times 14.97.

OK. So I take the 1.414, I multiply that by 14.97, and I find 21.17. And of course this has to be multiplied also by 1.4, so that becomes plus or minus 0.2 seconds.

That is a prediction. It is predicted. And now comes the observation. This is a thrilling moment for you, because what is at stake is the integrity of physics. And this is going to be measured plus or minus 0.15, right? Every time I make a measurement, plus or minus 0.15. I'm nervous.

So I'm going to add 500 grams. Here it will indeed increase. I'm going to oscillate it. 1, 2, 3, 4, 5, 6, 7, 8, 9.

Oh boy. Oh boy. What have I done? What have I done? What is it, 20 point? 52.

We have a problem. Physics is not working. Any one of you have an idea whether there's something wrong with the equation, or whether there's something wrong with Walter Lewin? Any idea?

Come on. Give it a try. The worst case, your suggestion is not correct. Yeah?

AUDIENCE: You can't [INAUDIBLE].

PROFESSOR: Ah! You accuse me, right? What's your name? Questioning my 0.15. You say, man, you couldn't even do better than 0.4 seconds, maybe. And then of course the two would be consistent with each other. Thank you. Very nice of you. Yeah?

AUDIENCE: [INAUDIBLE] friction in your [INAUDIBLE]?

PROFESSOR: Friction? OK. Now, that's a very good suggestion-- in other words, you begin to think like a physicist. Now, you also thought like a physicist, because indeed, if my uncertainty is higher than 0.15, you could be right.

Friction-- in this case, we will deal with friction late in the course-- has such a negatively small effect that it couldn't be measured to either one of these two. In any case, it's almost the same for both, because the shape has not changed much. It's a very good suggestion. Friction doesn't come near the proper explanation, but you tried, and that's good. One more try.

AUDIENCE: Mass of the spring?

PROFESSOR: Mass of the spring. We have said earlier-- you may not have heard it, but I did say it, we can replay the tape, I can prove it to you-- I said, if the mass of the spring is negligible, then that is the equation.

Now what do we do when the mass cannot be ignored? That's not so easy. But I requested some mandatory reading-- and I'm sure all of you have done that before this lecture. And the mandatory reading was French page 60 to 61, among others, and French says that if the mass of the spring itself is capital M, and if capital M divided by 3 is substantially less than the mass at the end of the spring, then a very, very good approximation is that the period of oscillation is then this, and he actually derives it. Period is higher.

So we can bring this to a test now. In other words, the mass of the spring, we have weighed that. In our case, it's 175.6 plus or minus 0.2 grams. And so M divided by 3 is 58.5 plus or minus 0.07 grams.

That's a very small error, by the way. It's 0.1% error.

And so we can now do the following test. We can now take the ratio of these two and eliminate thereby k, so we can write now 10 T and 2 divided by 10 T and 1 is now the square root of M2 plus M divided by 3 divided by M1 plus M divided by 3.

And that number is easy to calculate, because you know M2, you know M1, you know these numbers, and I have calculated it for you, and it is 1.377.

And the uncertainty is so small compared to my timing uncertainty that I don't even have to allow for any uncertainty in that number. Because remember, the uncertainties in the masses was in the order of 0.1%. Compare that with the uncertainty in the observations of the time, which were closer to 1%.

So we can bring this now to a test, and all I must do now is multiply-- if I want to find now 10, T, and 2, then I take 1.377, and I multiply it by T and 1, times 10T and 1. So I take this number.

And now I'm really getting nervous. It's not joking. 14.97 multiplied by 1.377-- that is 20.61. And the uncertainty would be the same uncertainty as in there, which is a 1% uncertainty, so that it's 0.2 seconds.

This number you can now compare with this number. On the button, within the error of measurements, they now agree. This is what we observed, and this is what we predict if we apply the proper relation and take the mass of the spring into account.

So you see that physics works, except that this equation was too simple to be used for our observations. Notice, by the way, that this 1.414, in our case, is lower.

All right. In 8.03, we will often, though not always, use complex notation. And the reason why we do that is that it can, at times, simplify your life, and you're completely free to choose when you want to use it and when you don't want to use it. You can be the judge.

So let's talk a little bit about complex numbers. I start with a circle, and this is the complex plane. The blackboard is a complex plane. That's quite a promotion for the blackboard. And here, I call this axis the real axis, so all the real numbers lie on this axis. And let this be plus 1, let this be minus 1, and I call this axis the imaginary axis. So this one is plus j, and this one is minus j, and j is the square root of minus 1. We don't call it i in general, because i stands for currents, so we pick j.

I now pick a position here which now represents a complex number. Call this angle theta. And I project this-- this is position z, complex number. This is the real part of that complex number and this is the imaginary part of that complex number. So you can see that indeed z can be written, since this length is 1, as the cosine of theta plus g times the sine of theta, so that at this part, which is real, and this is the sine of theta, because this is 1, I have to multiply that by j.

And this now, according to Euler-- great mathematician, Euler, after whom this disk was also mentioned-- already in 1748, he proved that this is the same as e to the power j theta.

This equality is mind-boggling, and when I saw this equality for the first time, I didn't believe it, number one, and I could hardly sleep at night, because I couldn't prove it. See, I hadn't had any Taylor expansion yet, so I couldn't prove it. My teacher in high school said, this is the case. And I said, why? He said, this is the way it is.

But we now can prove it. You can do the Taylor expansion of the cosine theta, Taylor expansion of the sine theta, and the Taylor expansion of e to the power j theta, and it's exactly correct. Not an approximation.

So why would we ever want to use this? Well, if you make this thing go around, going back to my uniform circular motion here-- if I make that point go around, and I only look at the real part, I have a simple harmonic motion. And so if I change theta into omega t, then I get that z is equal to cosine of omega t plus j times the sine omega t, the real part of which is a simple harmonic motion. And of course I'm not stuck to an amplitude of 1. I can easily make the amplitude 8 times larger. And of course there's nothing wrong, depending upon my initial conditions, to have here a face angle phi.

And this, then, is a times e to the power g omega t plus pi, according to Euler. So what that means is that if you use this as your trial function to solve a differential equation, and you can manipulate this very easily-- you can take first derivatives, second derivatives of exponentials extremely easy-- and then when you're done, you take the real parts of z, and out pops x as a function of time, and you've done.

As I said, it's up to you when you want to use it. Next lecture I will give you an example whereby it's clearly the way to go. I wouldn't even know how to do it in any other way. But often, you do have a choice. So we are interested, then, in the real part of that, which is then our acceptable solution.

So if we have a complex number z equals a plus jb, then we should always be able to write that as an amplitude times e to the power j theta. And then the amplitude a is the square root of a squared plus b squared, and tangent of theta is b over a. That follows immediately from that figure.

And so in problem set 1, you will get some chance to practice. We'll give you a few interesting cases. And a classic case that all of you in your lifetime have to be able to do once is the very non-intuitive problem j to the power j.

When I solved for the first time j to the power j, I said to myself, well, what on earth can be more complex than j to the power j? But it's real. It is not complex. And you will wrestle with this. There's an infinite number of solutions-- not one, all of them are correct. And I will help you a little, because the first time, I want to be nice to you. But only the first time.

I could also write g as e to the power g times pi over 2. Do you agree? Because it simply means that the angle theta is pi over 2. Here. So I end up here. But that's j. I'm not saying it is a very nice way of expressing j, but it is j.

But not only is this j, I could also rotate an integer number times 360 degrees whereby n, 0, 1, 2, 3, rotate clockwise or counterclockwise, and it's again j. Because if I rotate 90 degrees, it's j. But if I rotate another 360 degrees, it's again j, or if I rotate back 360 degrees. And so you see that this is also a way to write j. And that will help you, believe me.

I will always have a five-minute break during this 85-minute lecture so that you can stretch your legs. If you can manage to make it back and forth to the bathroom, that's fine, but that's your problem. I will start exactly after five minutes.

However, every Tuesday, during part of these five minutes, we will have a mini quiz. It's really mini. This small. And we will collect it after the lecture, and you will even get some credit for that. That's only on Tuesdays, but not today.

Before we go into this five-minute break today, I want you to see something so what you have something to think about. Believe me, it's healthy, MIT student, to sleep, but it's also healthy sometimes to not sleep. Sleepless nights and worry, just the way that I had sleepless nights in high school about Euler's equation. It's healthy. The reason why that's healthy is because once you see the solution, you say, ah, of course! And you never forget it. If someone tells you from the start, you say, yeah, of course. And you forget it, and the next day, you don't remember.

So what I want you to see is a remarkable example of an oscillation that can be produced not by wind, as we have seen, but by heat and by cooling. I have here a nice pipe, and there is a grid here-- I can touch it, I'm touching it now-- that's all there is. It's an open pipe, and there's a grid here. And when I heat that pipe and cool it, somehow it generates 110 hertz oscillation and pressure wave, which you will be able to hear. And I'll give you until the end of December, maybe mid-December, to come up with a solution why it's doing that.

[WHIRRING]

PROFESSOR: I'm heating the grid now.

[LOW TONE]

PROFESSOR: 110 hertz, roughly. If you want to play with this, don't break it. Try to transfer the liquid in 17 seconds. I will start, I will resume this lecture, exactly 5 minutes from now.

If you turn this into a tornado, you rotate it, then you open up a funnel of air, and so it is never the problem that the liquid cannot go through. There's always pressure equilibrium, and I don't remember how long it takes, but I thought it was 17 seconds, but if you want to, we can time it. May even be less.

I now want to address the issue of simple harmonic oscillation of a pendulum. As you will remember from 8.01, if you have a pendulum, length l, mass m, and if the mass of the string is negligibly small compared to the mass that is hanging here, then the period of oscillation is 2 pi times the square root of l over g, g, in the Boston area, being, to a high degree of accuracy, 9.80 meters per second squared.

If you simply take l approximately 1 meter, then you can see that you get a period of about 2 seconds. And if you make the length about 25 centimeters, that is, 4 times shorter, than you would expect this period which is 2 times shorter, which is about 1 second. And without any pretense of accuracy, just eyeballing, not really testing-- if I just eyeball this to be about a meter, and if I oscillate this back and forth, it's about two seconds for one oscillation-- one, two, one, two, one, two. If I, however, make it 25 centimeters, four times shorter, then it is very close to 1 second. No interference here. One, one, one, one, one, one.

Remarkable, when you look at this equation, is, that just like in the case of the spring, it is independent of the amplitude. In other words, whether I have a large amplitude or a small amplitude, it would take the same amount of time to go back and forth.

Well, not quite for a pendulum. When we derive this period, you remember that you have to assume what we call small angle approximation. You will see that again and again with 8.03, called small angle approximations. With small angle approximations, the final theta is always the same as theta in radians.

Now if you ask me how small is small, that's a matter of taste. In 26100, we have the mother of all pendulums-- 5.18 meters long. Quite impressive. So we have a pendulum with L is 5.1, plus or minus 0.05 meters. We cannot measure it any better than 5 centimeters, because it has to be under stretch when we measure it. And then you have to go all the way to the ceiling and all the way down-- Marcos does that, risking his life, and he claims that the best he can do is 5 centimeters.

We have 31 pounds hanging under there. We tried during the summer, believe me. We tried, with technicians of MIT, to have that pendulum here. And one day it looked good, but finally, they said no, we can't do it, we can't install it here, it's a safety issue. So unfortunately, we don't have the mother of all pendulums here.

In 26100, when I lectured on Newtonian mechanics, I demonstrated that the period that this pendulum produces is extremely close within the error of measurement which you predict. In other words, the mass of the string is indeed negligibly small compared to the mass of the object. We even weighed the string ones. I don't remember what it was, but it was such a small fraction of m that indeed it could be ignored.

And so the prediction then is, if you simply put this l in there, T predicted, purely on the basis of that simple equation, equals 4.57 plus or minus 0.02 seconds. And this 0.02 is the result of this 0.05. There's a 1% error in here. 5 out of 518 is 1%. And so the error in t is half a percent, because it's the square root, and so you get a half a percent error. And I rounded that off. So that is the prediction.

And then I made two measurements, one a 5 degree angle and one a 10 degree angle, and I did that 10 times. So 10t at 5 degrees and 10t at 10 degrees.

Now this was in 1999. Those were my good days. They were my good times, right? Past is always the good.

And so I then claimed that I could do this to an accuracy of 0.1 seconds. I had a lot of courage in those days. And I measured the 5 degree, and what did I find? Unbelievable, truly unbelievable, purely lucky-- I found exactly that number, which of course is an accident, because my accuracy was no better than 0.1 seconds. And then I did it at a 10 degree angle, and then I found this.

And so I demonstrated that indeed, 5 and 10 degrees, I still considered small angles for that approximation, and it is within the uncertainty of my measurement, what you expect.

Then I wanted to demonstrate-- which is not so intuitive-- that the period which is independent of mass, which is not the case for the spring. So now if you change the mass and you don't change l, you expect no change in period.

And that's what I really want to show you here, but I can't. Therefore, I've decided to show you what I did in 1999, if you can show the two-minute version of my video lectures, and you can judge for yourself to what extent the mass does not influence the--

One of the most remarkable things I just mentioned to you is that the period of the oscillations is independent of the mass of the object. That would mean if I join the bob, and I swing down with the bob, that you should get that same period. Or should you not? I'm asking you a question before we do this awful experiment.

Would the period come out to be the same or not? Some of you think it's the same. Have you thought about it, that I'm a little bit taller than this object, and that therefore maybe effectively the length of the string has become a little less if I sit up like this? And if the length of the string is a little less, the period would be a little shorter. Yeah? Be prepared for that.

On the other hand I'm also prepared-- well, I'm not quite prepared for it. I will try to hold my body as horizontal as I possibly can in order to be at the same level as the bob. I will start when I come to a halt here. There we go.

Now! You count. This hurts. Ugh!

[BARELY AUDIBLE COUNTING FROM AUDIENCE]

PROFESSOR: I want to hear you loud!

Oh-- ah--

AUDIENCE: [INAUDIBLE]

PROFESSOR: Thank you!

AUDIENCE: Six, seven, eight, nine, ten!

[CHEERS]

PROFESSOR: 10t is Walter Lewin. 45.6 plus or minus 0.1 seconds. Physics works, I'm telling you!

All right. So I think it was convincing, at least for the freshmen, that indeed, the period of a pendulum is independent of the mass, provided that you can ignore the mass of the string itself, which is the case for that pendulum.

Many pendulums-- some we will see in 8.03-- are more complex, more complicated, than simply a mass of string with an object at the end. And those pendulums we call a physical pendulum. For instance, I could have this pair of compasses, and just let it oscillate like this. That is not just a simple pendulum. Or I could have a ruler like this, put a hole through here, and have a pin, and have it oscillate like this. But I could also have it oscillate here, at different periods. If I oscillate it right in the middle, then it doesn't oscillate at all.

Now comes the question, how do we deal with that? And most of you must have seen that in 8.01, but I do want to address that in quite some detail.

So a physical pendulum, then, looks like this. This is an object, and I drill a hole in here-- point p-- and I put a pin in the wall, and it can, without friction, oscillate back and forth. And let the center of mass be here, position O, and this separation between d, p, and o is v, and o is the center of mass. And you can choose p anywhere you want to. There's no restriction on p.

So you see that this pendulum is offset over an angle theta, and it will start to oscillate back and forth. And the question is, what is the period?

So clearly, we may put the entire gravitational force at point o, in the center of mass. So this is the force acting at that point. Now comes the question, are there any other forces acting on this object? Or is this the only one? Because when you study it, you're going to take all forces into account.

Who is happy that we have taken all forces into account? Raise your hand. Most of you are getting scared, right?

Who says no, there has to be at least one other force? And which force is that?

Yeah, I'm not even so sure-- where is the active location? Yeah.

So there must be somehow a force at p to hold it up. Otherwise it would just start to accelerate down. Now, I'm not even sure that it is a straight up. I doubt that. It may simply be in a direction, I don't want to think about that. But surely there has to be a force up.

Now remember, F equals ma. When you deal with rotation of objects-- and this is going to be rotational-- then the equation changes into tau, the torque, its the moment of inertia, times alpha, whereby alpha is theta double dot, the angular acceleration.

And so if I pick p as my point of origin, then the torque, due to this force, does not contribute to my torque equation, because the torque is r cross f-- it's a cross product between the position vector and the force, and this is the position vector to the center of mass, and the position vector from p to p is 0. So if we deal with the torque relative to point p, that force is of no consequence.

So I'm going to take p as my origin. And so now is the question, what is the torque relative to point p? Well, it's r cross F. R is this distance, which is b F is mg. But I have a cross product, so I have to take the sine of this angle into account.

So that is the magnitude of the torque, and the magnitude of that torque, then, according to my rotational equivalent of F equals ma, equals the moment of inertia for rotation about that point p times theta double dot.

However, it is a restoring torque. The torque-- and you can do that with your right hand, whatever way you have learned how to do that-- the torque is in the blackboard, perpendicular to the blackboard, in the blackboard. r cross f is in the blackboard. I have rotated it counterclockwise, which is a vector out of the blackboard. So one is like this and the other is like this. That is like saying, the torque is restoring.

The same reason why we wrote down F equals minus kx, which is the spring, is why we now write this equals minus this. Take into account the direction of the vectors.

And so this is a differential equation that you would have to solve.

And if now we go to small angle approximation, then the sine of theta goes to theta if theta is in radians, and so I can rewrite down this theta double dot plus dmg divided by the moment of inertia about point p times theta equals 0.

And now, small angle approximation, we have a differential equation which is, again, a piece of cake. Simple harmonic oscillation. And so the simple harmonic oscillation solution must be that theta is some maximum angle theta 0 times the cosine, omega t plus phi, so by this omega is the square root of this number. Just like we earlier had the square root of k over m, omega now must be this. Omega is the square root of bmg divided by i of p. That means t period of oscillation is the moment of inertia about point p divided by d and t.

I want to repeat what I said earlier. This omega is called "angular frequency." The angular frequency is a given. That's the angular frequency. Do not confuse that with theta dot, which we also call omega, which is called angular velocity. And the angular velocity in this case is a strong function of time. When the object comes to a halt, the angular velocity is 0, because theta dot is 0. It is unfortunate that we give them the same symbol. So this is independent of time, but theta dot does depend on time, and theta dot is the angular velocity. And in the case of the uniform circular motion, the two omegas are the same.

So now we have all the ingredients in hand to calculate for absurd-looking objects what the period of oscillation is, provided that we are able to calculate the moment of inertia about the point of rotation, and of course we have to know v and the mass of the object.

You have a wonderful example in your problem set. I will solve that equation, will calculate this t, for a hoop-- this is the hoop. All the mass is at the circumference, so it should be very easy to calculate the moment of inertia. And we have a hole in here, and so we're going to oscillate it right at the rim. And so our geometry is easy, but we should be able to bring this equation to a rigid test, provided that we take into account the uncertainty of our measurement.

And so let me put in here this circle, which is this hoop. So all the mass, very good approximation, is at the conference, and the oscillation is about an axis perpendicular to the blackboard, point p. This is the center of mass o, and I'm going to offset this hoop. So this is when it is at equilibrium, and this is offset over an angle theta. So point o is now here. Call it o prime. And so in analogy with what we did there, we have here the force mg, and the derivation is identical. We don't have to go over that again. And the radius is r. And m is given, if you need it, and r is given. I'll show you what these numbers are later.

So all I have to do now is go to this equation and calculate the moment of inertia for rotation of an axis like this through point p. Who remembers how to do that?

8.01. Come on. In the worst case, it's wrong. I see one hand there. Who remembers?

Let me ask you this. Suppose it were rotating through an axis right through the center of mass. It's difficult, because there's nothing to hold onto. Would you know then what the moment of inertia is?

What is it then? Yeah? You say yes, but now you're quiet.

AUDIENCE: It's mr.

PROFESSOR: The moment of inertia is never mr. It's dimensionally wrong. But you tried, which is better than not trying.

AUDIENCE: mr squared?

PROFESSOR: Yeah. mr squared is what the moment of inertia would be if the axes were straight through o. I'm slowly working you up now. Now we've moved the axis from o to p. What happens now?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Well, what do we call that theorem? Parallel axis theorem. Now we have to add the mass times the distance between the center of mass and that point squared. That's the parallel axis theorem. And so the moment of inertia about point p is mr squared. For rotation about this point, we take the same axis, we move it to p, and we have to add m distance squared, so we have to add plus mr squared, so we get two mr squared.

And then we have b. What is b? What is the distance from p to the center of mass? That's r. So we call now was a prediction that t is 2 pi times the square root of 2 mr squared, divided by rmg. m goes. m always goes with pendulums. You never have to worry about m if you do it right. m always goes not with springs, but with pendulums. 1r goes, and so you get 2 pi times the square root of 2r over g.

Before we bring this to a test, isn't this is a remarkable answer? What does that make you think of? Excuse me?

AUDIENCE: Mass [INAUDIBLE]

PROFESSOR: It makes you think of a single pendulum whereby the length is 2r, which is by no means obvious, is it? In other words, if I had a pendulum here, and I would hang here an object m, that would have the same period, because it has a length 2r. So t is 2 pi times the length divided by g.

By no means obvious. Absolutely not clear why that is, but that's the way it is.

So now comes the acid test. And so we don't have to measure the mass, but we did measure as accurately as we can the radius. That's really all we have to do. And the measurement of the radius is a little uncertain, because it's not a perfect circle. So we measured it at various places, and we find that r equals 40.0 plus or minus 0.5 centimeters. So that's a 1% uncertainty.

And so we make a prediction now, t. We get the square root of r, so the 1% uncertainty becomes half a percent, because of the square root. You take 2r, you divide it by g, and you'll find that the prediction-- this is a prediction-- is that t is 1.795 plus or minus, that is half a percent, 0.01 seconds. That's because of the square root. So this becomes half a percent error.

And now we do the observation, and you guessed it, of course. We're going to do 10t. And if this is a good day, 0.1. But we'll give myself a little bit extra leeway today, 0.15. I'm fairly sure I should be able to do that. And so we bring this now to a test.

If you're ready for this-- oh, still on. I always like to start the timer when the object comes to a halt. That is a better criterion than when it goes through equilibrium. And I will not look at the-- even if I did look at it, there's no way I can stop that when I want to.

So we'll give it an offset. I first want it to swing in a way that is not wobbling. Because I make a very strong prediction-- I want to get that number, 17.95, so I better make sure that it's oscillating happily. No, this is not happy. I don't want any wobbling like this. Maybe a little more.

OK. I think this looks good. If you're ready, I'm ready.

One, two, three, four, five, six, seven, eight, nine, now. Yeah. 17.80. Ah, man! 0.15! So wasn't such a bad day after all for me.

OK. See you Tuesday. And work on your problem sets.

8.03 Physics III: Vibrations and Waves
Fall 2004