Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 3

Due Friday, October 1, 2004 at 4 PM

Reading Assignment

French pages 119-135, Bekefi & Barrett pages 98-109. **Brush up on Cramer's Rule!**.

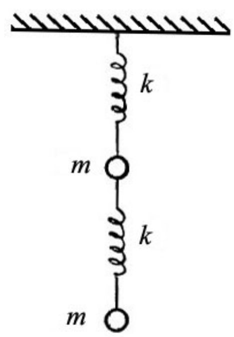
Problem 3.1 – Take home experiment #2 – Coupled oscillators, resonance, and normal modes

Describe briefly your findings during each of the various stages of this experiment. In doing so, answer the questions and make sketches.

Problem 3.2 – Coupled oscillators using two springs

Do Problem 5-10 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

Two equal masses are connected as shown with two identical massless springs of spring constant k. Considering only motion in the vertical direction, show that the angular frequencies of the two normal modes are given by $\omega^2 = (3 \pm \sqrt{5})k/2m$ and hence that the ratio of the normal mode frequencies is $(\sqrt{5}+1)/(\sqrt{5}-1)$. Find the ratio of amplitudes of the two masses in each separate mode. (*Note:* You need not consider the gravitational forces acting on the masses, because they are independent of the displacements and hence do not contribute to the restoring forces that cause the oscillations. The gravitational forces merely cause a shift in the equilibrium positions of the masses, and you do not have to find what those shifts are.)



Problem 3.3 – Coupled spring and pendulum

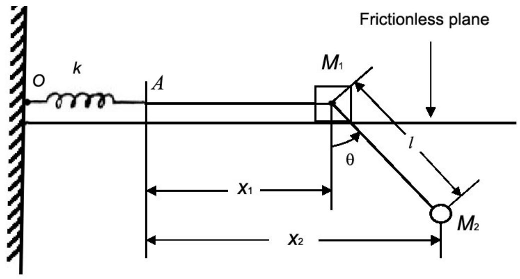
Do Problem 5-11 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

The sketch shows a mass M_1 on a frictionless plane connected to support O by a spring of stiffness k. Mass M_2 is supported by a string of length l from M_1. OA is the length of the relaxed spring. x_1 and x_2 are the positions of M_1 and M_2, respectively, relative to point A. The figure is not to scale; x_1 is much smaller than OA.

(a) Write down the differential equations of motion for each mass.

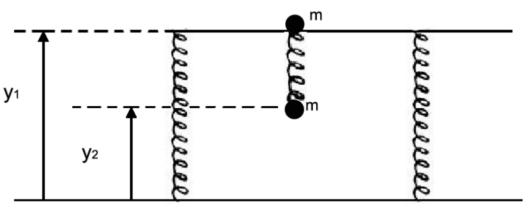
(b) For $M_1 = M_2 = M$, calculate the normal mode frequencies (use the small angle approximation for the pendulum).

(c) What are the associated ratios of the amplitude of the two masses?



Problem 3.4 – Coupled oscillators using three springs

Two springs, each of constant k, support a rigid, massless platform to which a mass m is firmly attached. The position of this mass is $y_1(t)$. A second mass m hangs at the end of another spring (of constant k) from the center of the platform, as shown in the sketch. The position of this second



mass is $y_2(t)$. Assume that the two longer springs move together with the same frequency and in the same plane.

(a) Write the differential equations of motion for each of the masses.

(b) Solve the equations to find the normal mode frequencies and find suitable expressions for $y_1(t)$ and $y_2(t)$.

(c) Sketch the configuration of the system for each of the two normal modes. Label the sketches to indicate which configuration corresponds to the normal mode with low frequency ω_1, and which configuration corresponds to the mode with high frequency ω_2.

Problem 3.5 – Driven coupled oscillator - This problem was on the Final last term

Use the figure under problem 3.3 but now the left end of the spring is not attached. Instead, we oscillate it harmonically in horizontal direction. Its position $X(t)$ is given by $X_0\ cos(\omega t)$.

(a) Write down the differential equation of motion for each mass.

(b) Let both masses be M. What now are the amplitudes (steady state) of the two masses as a function of $k,\ M,\ X_0,\ l\ and\ \omega$? (*Hint: Use Cramer's rule!*)

(c) Make careful plots of the amplitude of the two masses as a function of ω. Observe the phases. Be quantitative at $\omega = 0$ and also indicate frequencies that are "special".

(d) There is one frequency for which one mass stands still and the other oscillates. What is that frequency? You should recognize this answer as "very familiar". What's going on here?