Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 10

Due Friday, December 3, 2004 at 4 PM

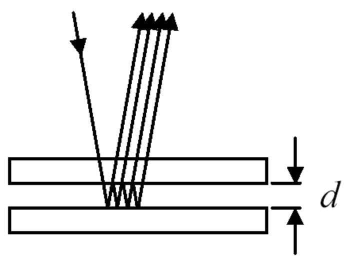
Reading Assignment

Bekefi & Barrett pages 519-522, 525 & 526, 532-548.

Problem 10.1 – Thin film interference

Do problem 8.1 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

White light is incident normally on an air film of thickness d formed between two glass plates. What must be the smallest film thickness d if only blue light of wavelength 4000 $\AA$ ($= 4 \times 10^{-7}$ m) is to be reflected strongly?

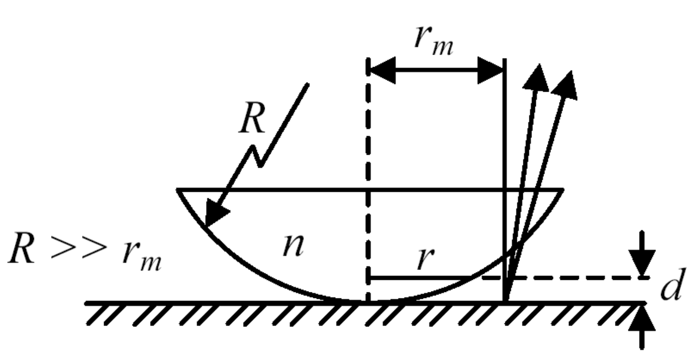


Problem 10.2 – Newton rings

Do problem 8.4 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

This problem was on the 8.03 Final Exam in the spring of 2004.

A plano-convex piece of glass (index of refraction n) rests on a plane parallel piece of glass as shown. The radius of the spherical surface is R and it is much greater than r_m. Light of wavelength λ is incident normally and reflected at the spherical glass-air interface and at the air-glass interface of the glass plate. The two reflected beams then interfere to produce a series of alternately bright and dark concentric circles when viewed from above.



(a) Find the radial distance, r, from the point of contact at which the separation between the spherical surface and the plate upon which it rests is d, i.e., find the relation between r and d.

(b) Derive an expression for the radial distances, r_m, at which bright rings will be observed.

(c) Same as (b) for dark rings. Let $R = 2$ m and $\lambda = 640$ nm.

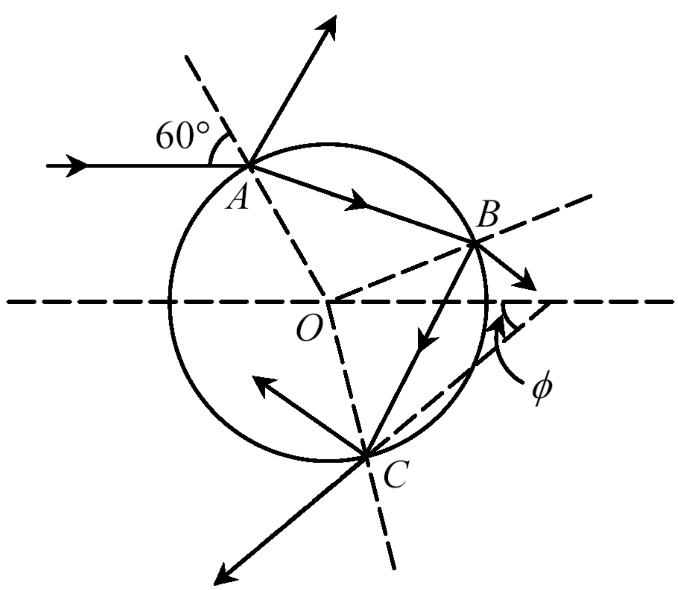
(d) What then is the spacing (difference in radii) between the first 2 dark rings and what between dark ring #25 and #26?

Problem 10.3 – Take-home experiment #6 – Thin film interference

The experiments are very nice, and easy to do, and they will give you a good insight into this common and intriguing interference phenomenon (also observed in soap bubbles, and oil spills on the road). **I strongly encourage you to do the experiments as you will see Problem 10.1 and 10.2 in action!** You do not have to write up your findings to get full credit for this Problem Set. However, in preparing for the Final Exam, I will assume that you have done these experiments.

Problem 10.4 – Rainbows

A very narrow beam of unpolarized red light of intensity I_0 is incident (at A) on a spherical water drop (see figure). The angle of incidence is 60° . At A, some of the light is reflected and some enters the water drop. The refracted light reaches the surface of the drop at B where some of the light is reflected back into the water, and some emerges into the air. The light that is reflected back into the water reaches the surface of the drop at C where some of the light is reflected back into the drop, and some emerges into the air. The index of refraction, n, of water for the red light is 1.331.



(a) What is the intensity, and what the degree of polarization of the light that refracts into the drop at A?

(b) What is the intensity, and what the degree of polarization of the light that reflects at B?

(c) What is the intensity, and what the degree of polarization of the light that emerges into the air at C?

(d) Let the angle of incidence at A be θ_1 and the angle of refraction θ_2. Express ϕ (see figure) as a function of only θ_1 and θ_2.

(e) Calculate the angle ϕ in case θ_1 is $60°$. Do this for the red light and also for blue light (the index of refraction for blue light is 1.343). The speed of blue light in water is about 1% slower than that of red light.

(f) For a given wavelength, there is one and only one value of θ_1 for which ϕ is a maximum (ϕ_{max}). Prove that this is the case when $(\cos\theta_1)^2 = \frac{(n^2-1)}{3}$. Here n is the index of refraction.

(g) Using the equation under (f), calculate the values of θ_1 for both the red and the blue light that give rise to maximum values for ϕ. Using your result under (d), calculate the maximum values for ϕ (each wavelength will have its own set of values for θ_1 and associated ϕ_{max}).

(h) In a world, far-far away, rain comes down as small drops of glass (with index of refraction of about 1.5). The living souls there talk about a "glass bow". What is the maximum value of ϕ for these glass bows? Compare this with our rainbows.

Formation of Rainbows: The $60°$ angle of incidence (see the figure), is very close to the values you found in (g). Thus the value of ϕ as shown in the figure is also very close to your ϕ_{max} values in (g). The fact that ϕ_{max} is different for the red light than for the blue, is key in the formation of the rainbow. The geometry shown in the figure will play a central role in the lecture on rainbows on December 7. A rainbow will be made**.** It is advisable to bring an umbrella.

Problem 10.5 – Superposition of N oscillators

Do problem 8.5 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

We desire to superpose the oscillations of several simple harmonic oscillators having the same frequency ω and amplitude A, but differing from one another by constant phase increments α; that is,

$$E(t) = A\cos\omega t + A\cos(\omega t+\alpha) + A\cos(\omega t+2\alpha) + A\cos(\omega t+3\alpha) + \cdots$$

(a) Using graphical phasor addition find $E(t)$; that is, writing $E(t) = A_0\cos(\omega t+\phi)$, find A_0 and ϕ for the case when there are five oscillators with $A = 3$ units and $\alpha = \pi/9$ rad.

(b) Study the polygon you obtained in part (a) and, using purely geometrical considerations, show that for N oscillators

$$E(t) = (NA)\frac{\sin(N\alpha/2)}{N\sin(\alpha/2)}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right].$$

(c) Sketch the amplitude of $E(t)$ as a function of α.
(The above calculation is the basis of finding radiation from antenna arrays and diffraction gratings.)