Physics 8.03
Vibrations and Waves

Lecture 3

HARMONICALLY DRIVEN DAMPED HARMONIC OSCILLATOR

Last time:
Damped harmonic oscillator

- Equation of Motion

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

- Three solutions that depend on size of damping

$$x(t) = \begin{cases} = Ae^{-\gamma t/2}\cos(\omega t + \phi) & \omega_0 > \frac{\gamma}{2} \quad \text{Underdamped} \\ = (A + Bt)e^{-\gamma t/2} & \omega_0 = \frac{\gamma}{2} \quad \text{Critically damped} \\ = A_1 e^{-\Gamma_1 t} + A_2 e^{-\Gamma_2 t} & \omega_0 < \frac{\gamma}{2} \quad \text{Overdamped} \end{cases}$$

- Damping slows down natural frequency of oscillator (or makes it stop oscillating altogether)

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

DRIVEN HARMONIC MOTION

- Add driving force term to equation of motion
- Effect
  - Oscillator loses it's own identity and oscillates at the frequency at which it is driven (not it's own natural frequency)
- Mathematical solution
  - Amplitude of oscillation depends on driving frequency
  - Phase of oscillation (relative to driving force) also depends on driving frequency
  - When driving frequency = natural frequency

RESONANCE!

- Examples ➔ shattering a wine glass with sound

Next time: Transient behavior

- What happens when driving force is first turned on? Transients
- We started with a second order diff. eqn. so we should get two constants of integration. Where are they?
- Complete solution to the diff. eqn. includes the homogeneous solution (we got that today) AND a particular solution (that describes the transient behavior of the driven oscillator)