Physics 8.03
Vibrations and Waves

Lecture 10

Fourier Analysis

Last time:

- Wave equation in 2-D

$$\frac{\partial^2}{\partial x^2}\xi(x,y,t)+\frac{\partial^2}{\partial y^2}\xi(x,y,t)=\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\xi(x,y,t)$$

$$\Rightarrow k^2=k_x^2+k_y^2=\left(\frac{n_x\pi}{L_x}\right)^2+\left(\frac{n_y\pi}{L_y}\right)^2$$

- Arbitrary motion ➔ Superposition of normal modes

$$y(x,t)=\sum_{m=1}^{\infty}A_m\sin\left(\frac{m\pi}{L}x\right)\cos(\omega_m t+\beta)$$

- Orthogonal functions ➔ Fourier coefficients

$$A_m=\frac{2}{L}\int_0^L y(x,t=0)\sin\left(\frac{m\pi x}{L}\right)dx$$

- Fourier analysis continued
- Time evolution added

Fourier expansion recipe

- Start with superposition of all possible modes
- Determine the simplest basis functions using
  - Boundary conditions
  - Symmetry
  - Initial condition
- Determine the Fourier coefficients, A_n, at $t = 0$ using initial deformation $y(x, \; t = 0)$ and orthogonal functions
- Add the time-dependence