

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Physics Department

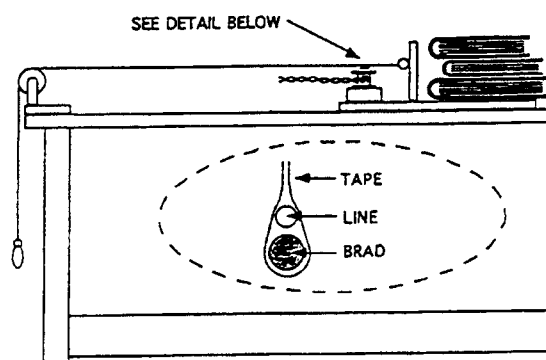
Physics: 8.03

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**Take-Home Experiment #3**

**VIBRATING STRING: RESONANCE IN A CONTINUOUS SYSTEM**

**Objective** The normal modes of continuous systems are usually displayed by sweeping the frequency at which the system is driven through the fixed resonant frequency of the mode. Variable frequency oscillators are common laboratory instruments, but are somewhat expensive. In this experiment the drive frequency is held fixed at an easily obtainable value. It is the resonant frequency of the normal mode which is swept by changing a parameter of the system.



**Experiment** Use the double sided masking tape to secure the pulley block at the edge of a table or desk as shown above. Tie one end of the clear nylon fishing line (280 yards weigh 1/4 pound) to the 10 ounce (nominal) lead weight. Tie the other end to the screw eye in the drive block leaving about 115 centimeters of line between the two knots. Position the drive block on the desk and hold it in place with something heavy (a pile of textbooks works well). Guide the line over the pulley so the lead weight hangs vertically. Use some Scotch tape to attach a wire brad to the under side of the line (see detail above) about one centimeter out from the knot at the drive block. Connect the AC power supply to the solenoid. Place the solenoid under the wire brad and shim it up until it is only a millimeter or two below the brad.

Start with the lead weight suspended just off the floor. You should have no more than 20 cm of line between the pulley and drive block. Plug in the power supply. Slowly draw back the drive block while watching the line. When the line starts to vibrate, adjust the position of the solenoid to get the maximum amplitude. Now adjust the position of the drive block to find the exact line length for this resonance. Measure the length between drive block knot and the top of the pulley.



Observe the mode pattern on the string. Is the mode linearly polarized (in other words, is the deflection of the string in one direction only)? Can you change this by changing the position of the solenoid relative to the wire brad? Does the vibrating line look any different under fluorescent (gas discharge) light than under a tungsten (filament) source? Why does the image of the line look brightest at the ends of its swing?

Measure how far you can displace the line length  $\Delta L$  on either side of its resonant length  $L_o$  before the amplitude of vibration falls to about half of its resonant value (obviously, this is not going to be a precision measurement). Disconnect the power supply from the solenoid and estimate how long it takes for the amplitude of vibration to drop by a half.

Finally, draw the block further back and see if you can find any other resonant modes. Measure the corresponding resonant line lengths.

### Questions

1. Which direction will the wire brad move when the electric current runs clockwise through the solenoid as viewed from above the string? Which direction will it move when the current runs counter clockwise? What is the frequency of the current in the solenoid? What is the frequency at which the line is driven?
2. What is the shortest length  $L_o$  at which you should find resonance behavior? At what other lengths should you find resonance? [Note: one yard = 0.914 meters, one pound = 0.453 kilograms, and one ounce = 0.0283 kilograms.]
3. How is an infinitesimal fractional change in the length of the line related to an infinitesimal fractional change in the resonant frequency? (Answer:  $dL/L = -d\nu/\nu$ )
4. For a simple driven damped harmonic oscillator, how far does one have to move away from the resonant frequency in order to see the amplitude of the response drop to one half of its resonant amplitude. Assume light damping and express your answer in terms of  $\nu_o$  and  $Q$ . (Answer:  $\Delta\nu = \pm\sqrt{3/4}\frac{\nu_o}{Q}$ )
5. Use the results of 3. and 4. together with your observations to estimate the  $Q$  of the first resonance.
6. How long should the vibration take to decay to half its amplitude after the drive is shut off? Compare this with your observation. (Answer:  $t_{1/2} = \frac{\ln 2}{\pi\nu_o} Q$ )

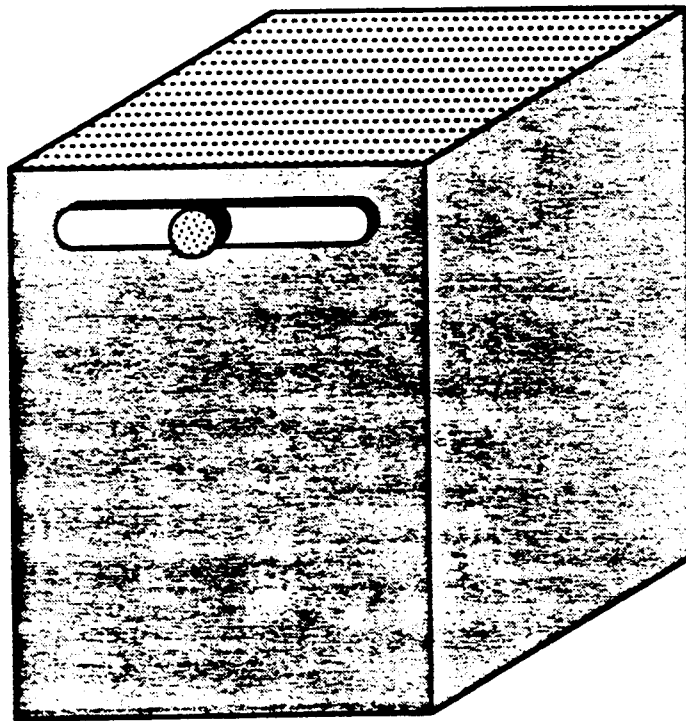
### 3A - CAPILLARY WAVES

**Objective** Wave motion on the surface of a liquid is a complex but practical topic. We will investigate just one aspect of the problem, short wavelength oscillations on the surface of a deep fluid. They are called capillary waves, or sometimes just ripples. Under these conditions the restoring force originates in the surface tension and the dispersion relation is given by

$$\omega(k) = (\sigma k^3 / \rho)^{1/2}$$

where  $\sigma$  is the surface tension of the liquid-vapor interface and  $\rho$  is the liquid density. In this experiment you will drive a water-air interface at a fixed frequency and measure the wavelength of the resulting standing wave. From this you can determine the surface tension of water.

**Experiment** Obtain a half gallon milk or juice carton that has an approximately rectangular cross section. The walls should be quite flexible. Cut off the carton cleanly at a height of about 10 centimeters above its base. Wash the inside and rinse thoroughly to remove any possible soap film. Cut one of the popsicle sticks so that it is about 3 centimeters shorter than the width of the side of the carton. Use the double sided tape to mount it about one centimeter below the open edge as shown below. Use another piece of double sided tape to attach the small magnet to the center of the wooden stick.



Fill the carton with water. Place it under a broad light source in such a way that the water surface acts as a mirror. Connect the solenoid to the 12 volt transformer and bring it near the magnet. Observe the various patterns created on the surface. You may notice that the nature of the standing waves seems to change markedly if the drive is too strong. It is the weaker drive that we need here. Do not expect to see a set of perfectly uniform and parallel standing waves. Rather, aim to set up a pattern for which you will be able to count the number of wave crests across the width of the surface in a reliable fashion. You may find that the pattern can be simplified somewhat by taping another popsicle stick as a stiffener across the outside of the facing carton edge and overfilling the carton slightly so that the surface of the film is above the edge of the carton (stabilized by surface tension).

What happens to the direction of force on the magnet when the direction of current flow in the solenoid is reversed? What is the frequency of the capillary waves being excited? How does this differ from the frequency at which you drove the string in a previous experiment?

On one complete cycle (wavelength) of a plane wave on the surface, how many locations are there that would reflect light from a distant source to your eye? Count the number of wavelengths from one side to the other in your standing wave patterns, measure the carton width, and thus determine the wavelength of the capillary waves at this frequency.

Use the dispersion relation above to derive a relation between the wavelength  $\lambda$  and the frequency  $\nu$ :

$$\lambda = \left[ \frac{2\pi\sigma}{\rho\nu^2} \right]^{1/3}.$$

Use your measurements to determine a value for the surface tension of water. The accepted value is  $\sigma = 73$  dynes/cm at a temperature of  $20^\circ$  C.

You can use the magnet and solenoid to drive capillary waves in other containers. Try a plastic cup with a circular opening (no need for the popsicle stick here). Can you get a change from the weak to strong drive patterns in this geometry?

### 3B - POLARIZATION OF LIGHT

**Objective** In this experiment you will investigate the various polarization states of light with the help of polarizers and retardation plates.

#### Experiments

**Polarizers** The polarizers in your kit are made from a Polaroid material. You can read about Polaroid materials in the text book. The material is shipped with a thin plastic protective sheet on each side. Check to see that these have been removed from your Polaroids. The easy axis of the material, along which the electric field has the least attenuation, should be parallel to the short edge of your rectangular pieces. How can you test to see that this is indeed the orientation for your Polaroids? As you do the following experiments think about how the outcome would change if the easy axis of the Polaroids were at some other angle to the edge.

Using two polarizers, observe the transmission and extinction as one is rotated relative to the other throughout the full 360 degrees. For which end of the optical spectrum is the extinction the strongest? Check the consistency of the alignment of the easy axis with respect to an edge for your three polarizers by observing the orientation corresponding to maximum extinction. It is conceivable that some of them were not cut square to the easy axis.

Use one polarizer to search for sources of polarized light. First try primary light sources: light bulbs, fluorescent lamps, discharges, and flames. Examine the polarization of the blue sky, particularly in a direction directly overhead. What about clouds in the same position in the sky? Look for polarization in the light reflected from various objects: water, metallic surfaces, glass or plastic. The light reflected from book covers shows interesting polarization effects. Try looking at a digital watch and the display on a notebook computer through the Polaroid.

Insert a third polarizer between crossed polarizers and rotate it. Be sure you understand why you see transmitted light even though the first and last polarizer are crossed. In which direction is the easy axis of the center polarizer when the transmission is a maximum? Can you achieve a similar effect by having the third polarizer outside the two crossed ones?

**Quarter Wave Plate** The half and quarter wave plates in your kit are made of a birefringent plastic. The fast and slow axes should be parallel to the edges of the rectangular samples. The difference in the optical path lengths for the two orthogonal directions of incident polarization is 140 nanometers for the "quarter wave" material. For what color will this actually be a quarter wave plate? What fractional wave material will this be at either end of the visible spectrum, 400 and 700 nanometers?

Place the quarter wave plate between two polarizers. Hold the plate at various angles,  $\theta$ , with respect to the first (polarizer) while rotating the second (analyzer)

through a full 360 degrees. For what values of  $\theta$  can you achieve complete extinction? For what values of  $\theta$  is the transmitted intensity least sensitive to the orientation of the analyzer? How should one set  $\theta$  so that the polarizer and quarter wave plate produce circular polarization? linear polarization? elliptical polarization? Cross the polarizer and analyzer then rotate the quarter wave plate between them. At what angle  $\theta$  is the transmission a maximum? Can you explain why this is so?

You should have noticed that the color of a white object being viewed through the stack changed slightly during the rotations carried out above. Why is that? Set up the polarizer and analyzer to produce circularly polarized light. Place various color filters in front of the stack and use the analyzer to determine how close the polarization is to circular. The apparent effect will be subtle since the eye tends to have a logarithmic response to intensity. It responds over many orders of magnitude, but has difficulty distinguishing small changes in absolute intensity.

Try the following coin trick. Place a silver coin on a piece of black paper. Hold the quarter wave plate and a Polaroid together with their edges rotated by  $45^\circ$ . Place the stack on the coin, plate side up. Now you see it. Place the stack on the coin, plate side down. Now you don't. Why is this? Can you think of a way of using a Polaroid and a quarter wave plate to make an anti-glare filter to use with computer screens? If you can, don't rush to patent it; someone else got there first.

**Half Wave Plate Test** the supposition that a half wave plate rotates the plane of polarization of linearly polarized light through an angle which is twice the angle one of its axes makes with the incident polarization. Try this with and without the color filters. The material used for these plates has a retardation of 280 nanometers.

**Make Your Own Wave Plates** It turns out that clear cellophane tape (it has to be glossy, but not all clear glossy tape is cellophane) and saran wrap develop birefringence during their manufacture. Put a piece of clear cellophane tape on a microscope slide and test it. The retardation should be of the order of a quarter or half wave. Try some of the saran wrap from your kit. Its retardation is less. It may take several layers to build up a quarter wave of retardation. Beware of Scotch Magic Tape. It is great stuff, but not for this application. The substrate is not very birefringent, and it depolarizes the light by scattering.

**Stress Birefringence** Plastics which may not naturally be birefringent can develop birefringence when stressed. The total path length difference through the sample can be 1000 nanometers or more, thus the effect on linearly polarized incident light can be much more wavelength dependent than you saw in the above experiments. This can lead to some very colorful displays. Set up two crossed polarizers separated by an inch or two. Taping them to the opposite covers of a thick textbook is one possibility. Now place various objects between them and look at the transmitted light. Try some of the following: drafting tools

such as triangles and French curves, hard plastic boxes such as CD and audio tape cases, a clear plastic tape dispenser, a clear plastic picture frame, eyeglass lenses, and stretched or crumpled plastic or cellophane sheet.