

Class 24: Outline

Hour 1:

Inductance & LR Circuits

Hour 2:

Energy in Inductors

**Last Time:
Faraday's Law
Mutual Inductance**

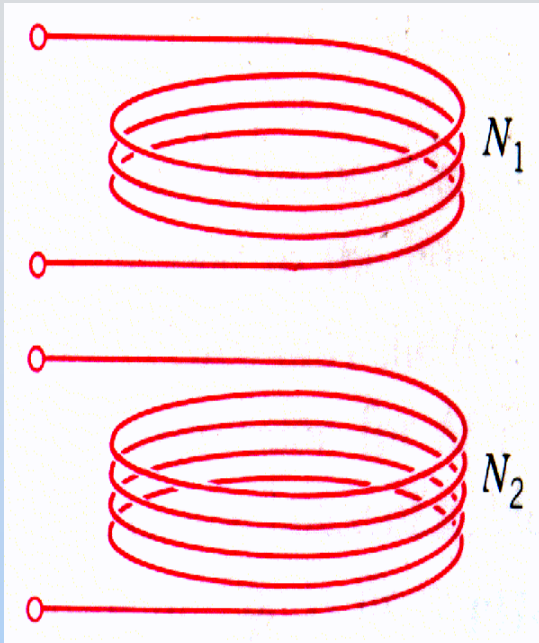
Faraday's Law of Induction

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Changing magnetic flux *induces* an EMF

Lenz: Induction ***opposes*** change

Mutual Inductance



A current I_2 in coil 2, induces some magnetic flux Φ_{12} in coil 1. We define the flux in terms of a “mutual inductance” M_{12} :

$$N_1 \Phi_{12} \equiv M_{12} I_2$$

$$\rightarrow M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

$$M_{12} = M_{21} = M$$

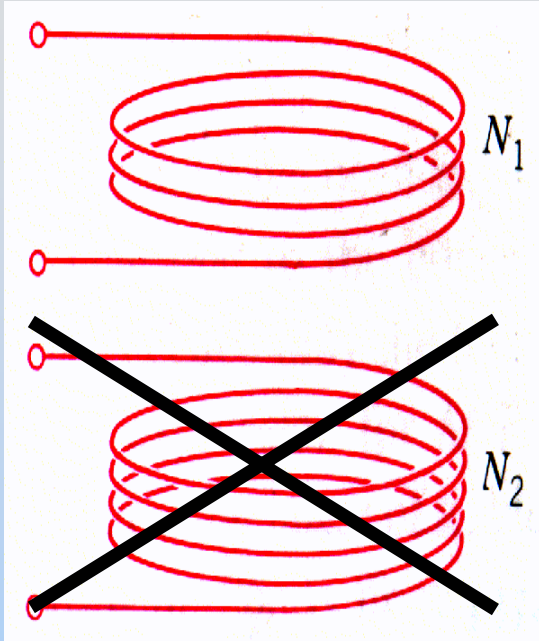
$$\mathcal{E}_{12} \equiv -M \frac{dI_2}{dt}$$

You need AC currents!

Demonstration: Remote Speaker

This Time: Self Inductance

Self Inductance



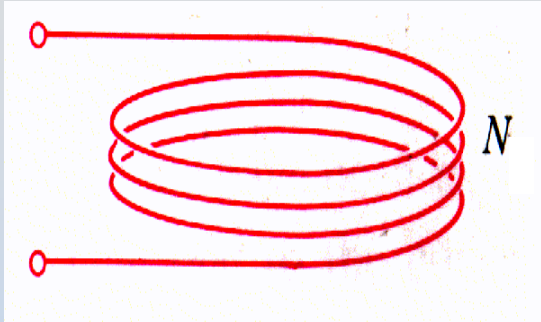
What if we forget about coil 2 and ask about putting current into coil 1? There is “self flux”:

$$N_1 \Phi_{11} \equiv M_{11} I_1 \equiv LI$$

$$\rightarrow L = \frac{N\Phi}{I}$$

$$\mathcal{E} \equiv -L \frac{dI}{dt}$$

Calculating Self Inductance



$$L = \frac{N\Phi}{I}$$

Unit: Henry

$$1 \text{ H} = 1 \frac{\text{V} \cdot \text{s}}{\text{A}}$$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

Group Problem: Solenoid

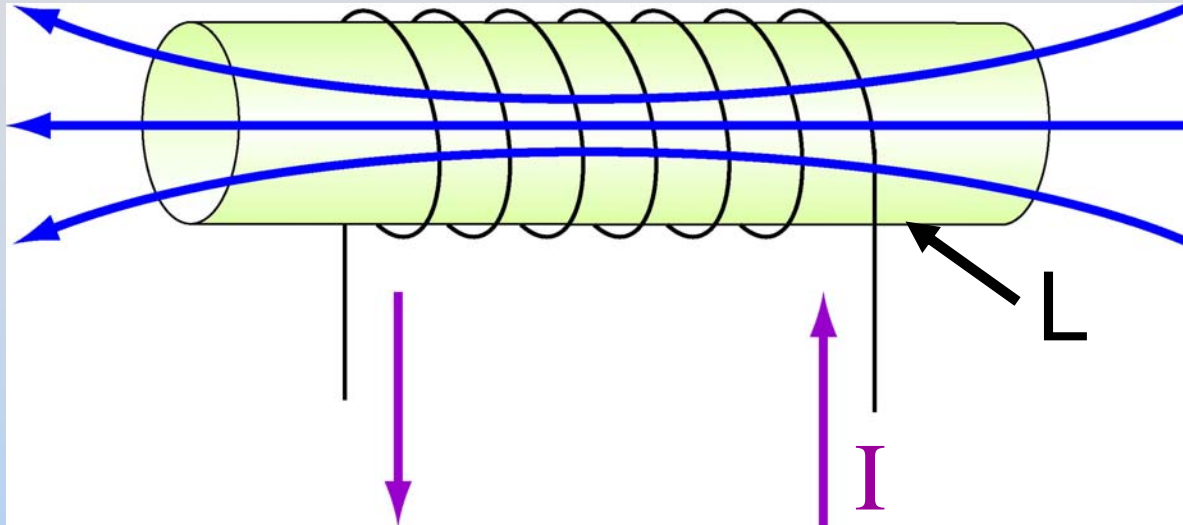
Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ , radius R)

REMEMBER

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

$$L = N\Phi/I$$

Inductor Behavior

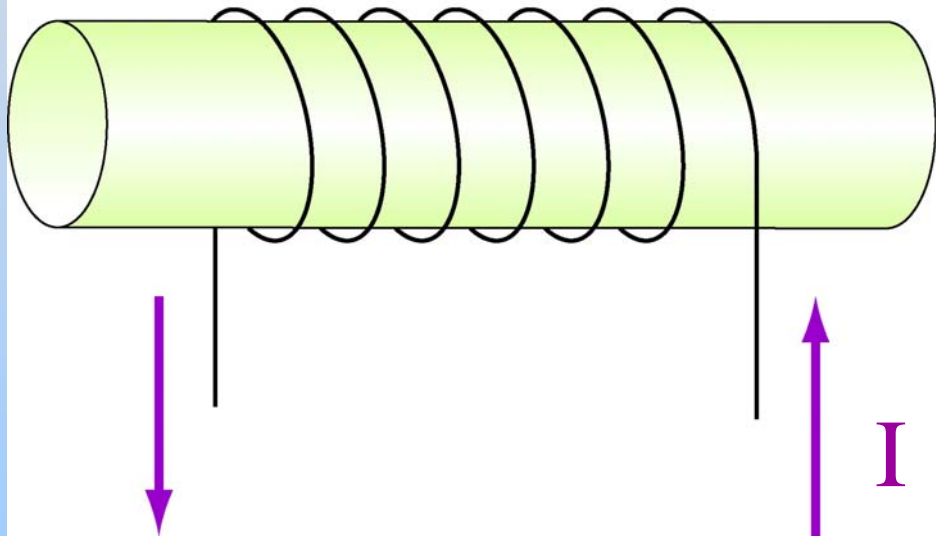
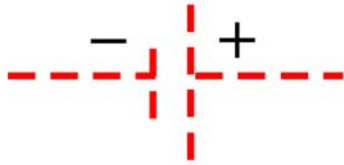


$$\mathcal{E} = -L \frac{dI}{dt}$$

Inductor with constant current does nothing

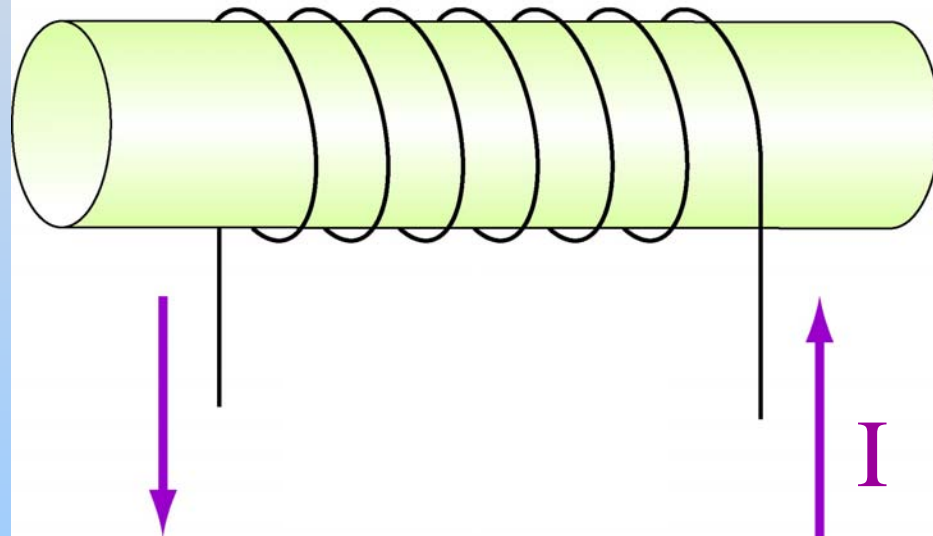
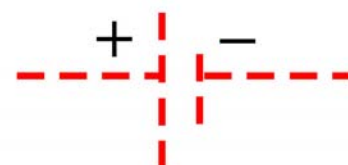
Back EMF $\mathcal{E} = -L \frac{dI}{dt}$

Lenz's law emf



$$\frac{dI}{dt} > 0, \quad \mathcal{E}_L < 0$$

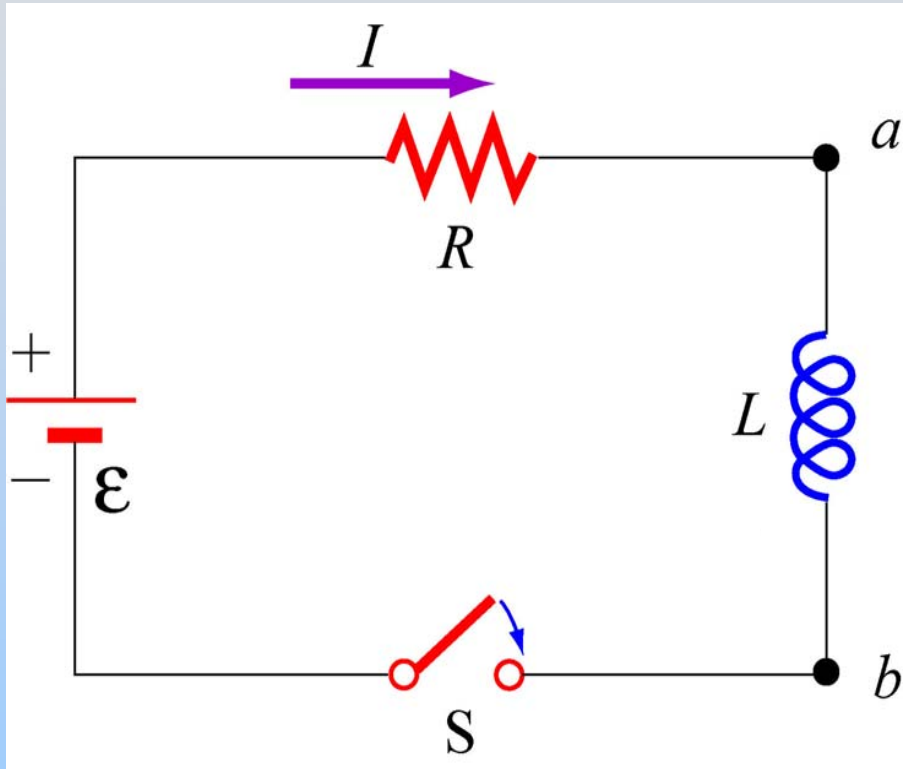
Lenz's law emf



$$\frac{dI}{dt} < 0, \quad \mathcal{E}_L > 0$$

Inductors in Circuits

Inductor: Circuit element which exhibits self-inductance



Symbol: 

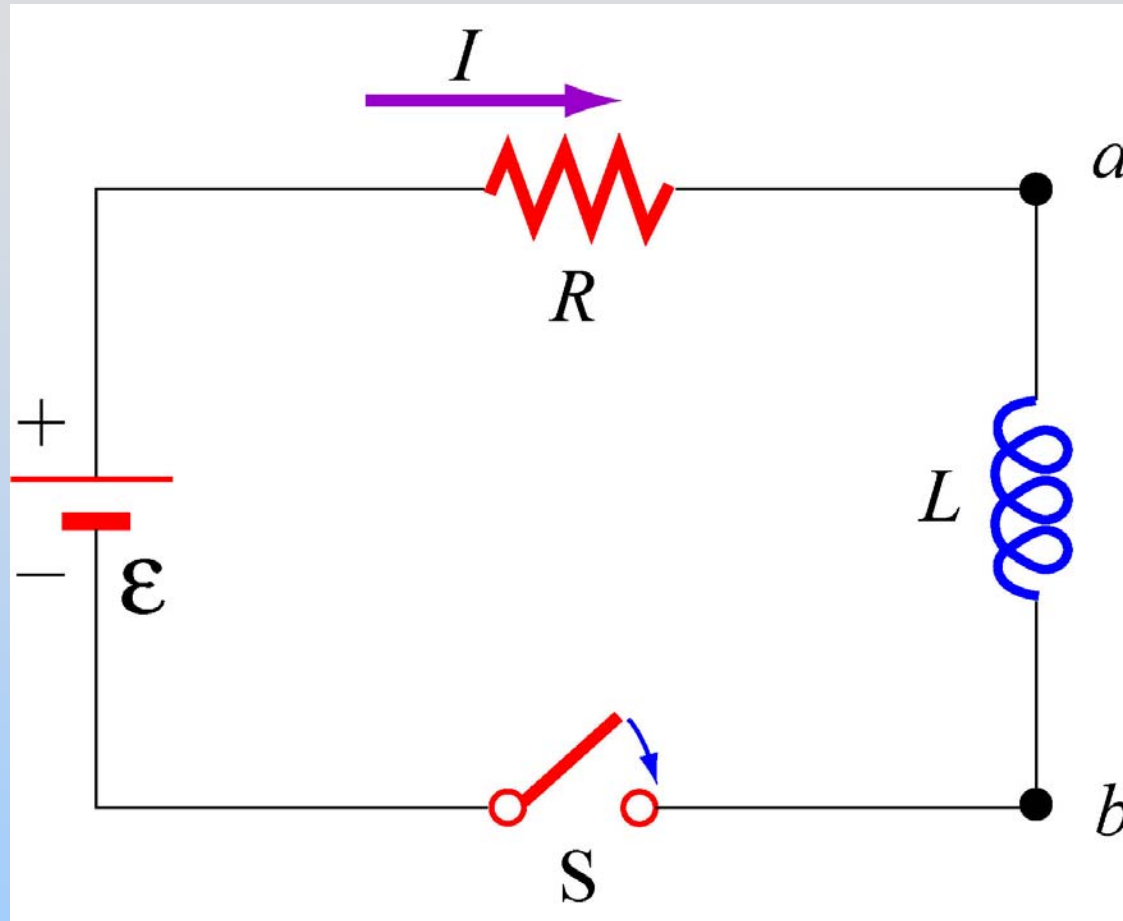
When traveling in direction of current:

$$\mathcal{E} = -L \frac{dI}{dt}$$

Inductors hate change, like steady state
They are the opposite of capacitors!

PRS Question: Closing a Switch

LR Circuit

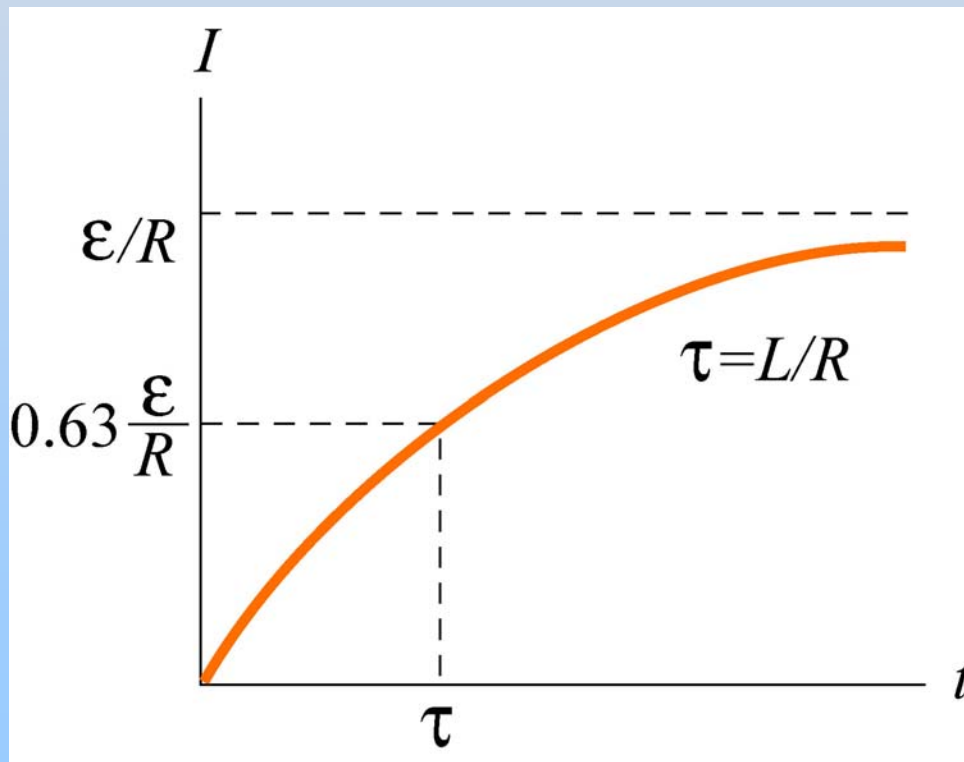


$$\sum_i V_i = \mathcal{E} - IR - L \frac{dI}{dt} = 0$$

LR Circuit

$$\varepsilon - IR - L \frac{dI}{dt} = 0 \Rightarrow \frac{L}{R} \frac{dI}{dt} = - \left(I - \frac{\varepsilon}{R} \right)$$

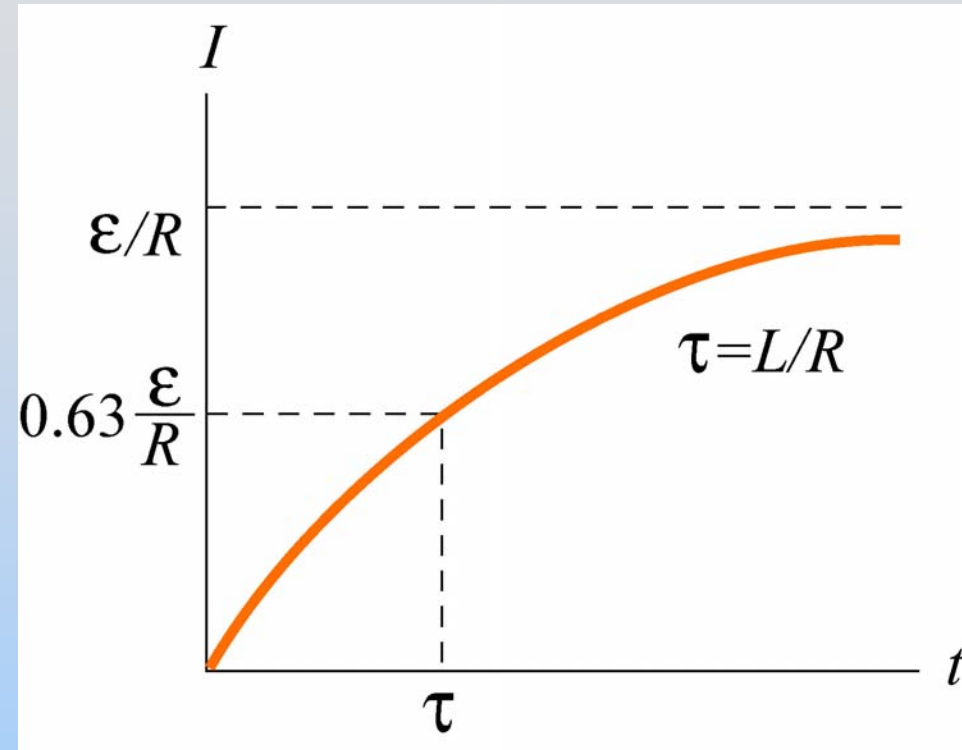
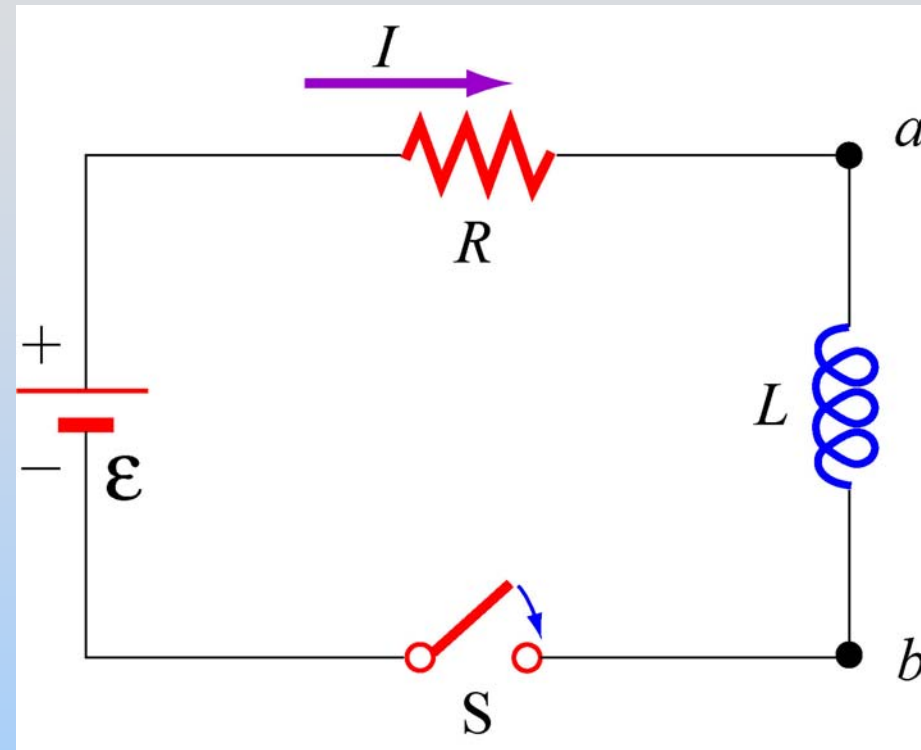
Solution to this equation when switch is closed at $t = 0$:



$$I(t) = \frac{\varepsilon}{R} \left(1 - e^{-t/\tau} \right)$$

$\tau = \frac{L}{R}$: LR time constant

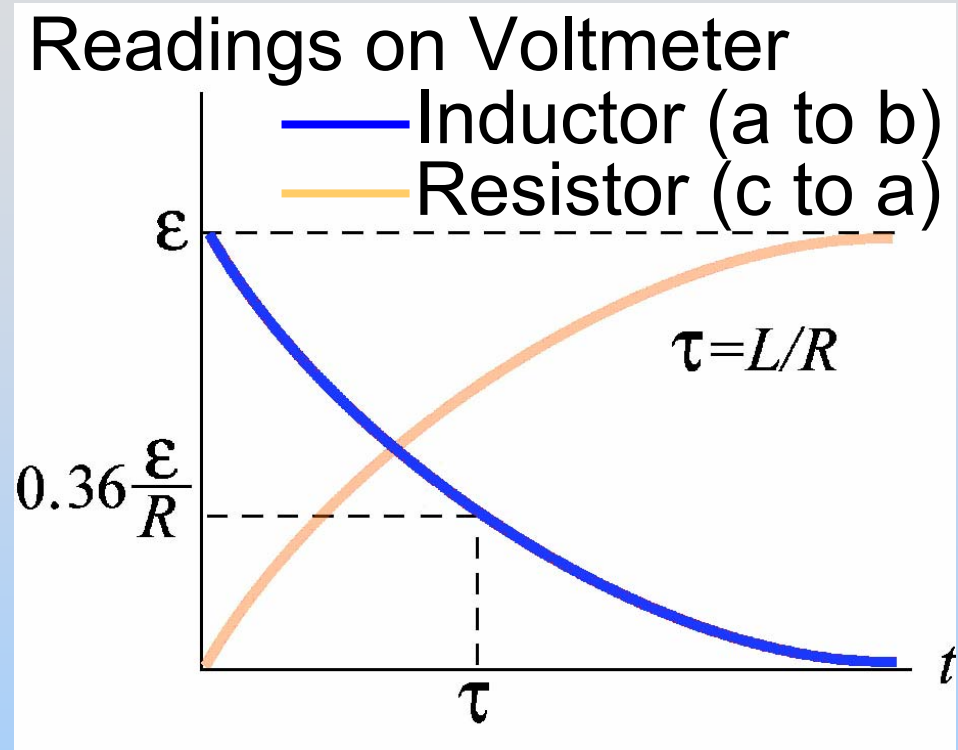
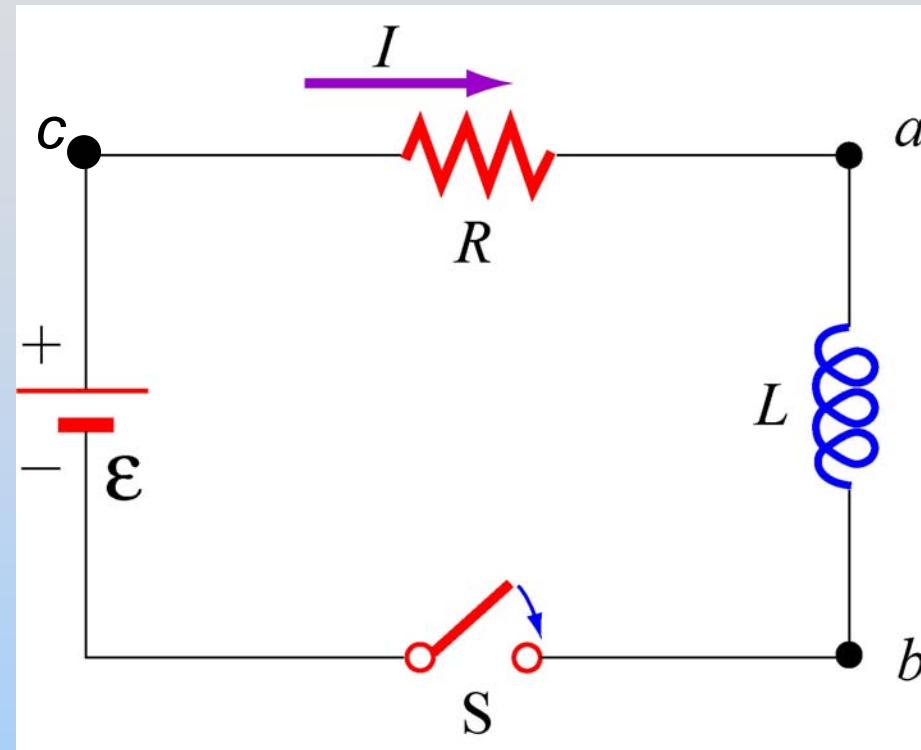
LR Circuit



$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

$t=\infty$: Current is steady. Inductor does nothing.

LR Circuit

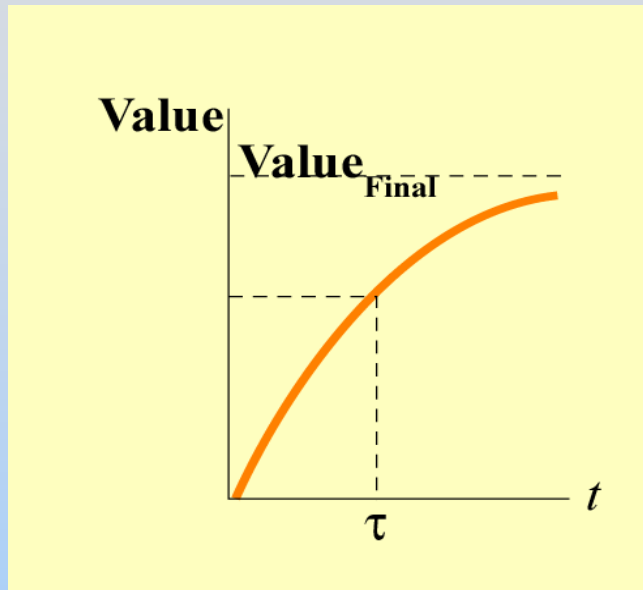


$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

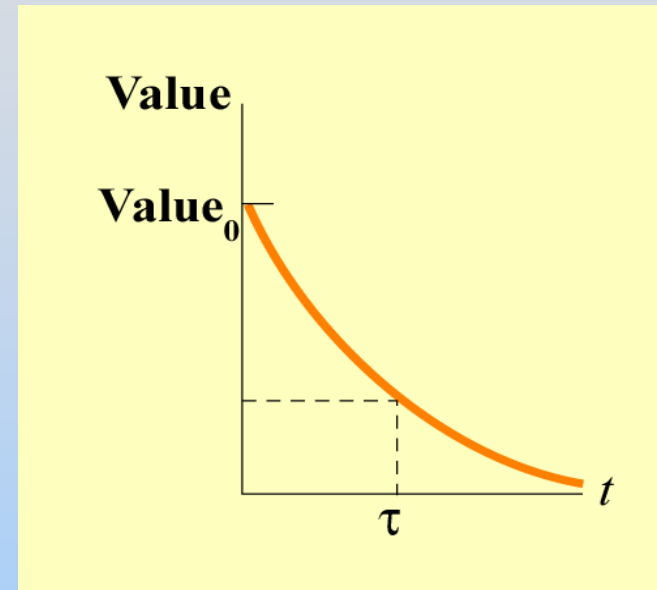
$t=\infty$: Current is steady. Inductor does nothing.

General Comment: LR/RC

All Quantities Either:



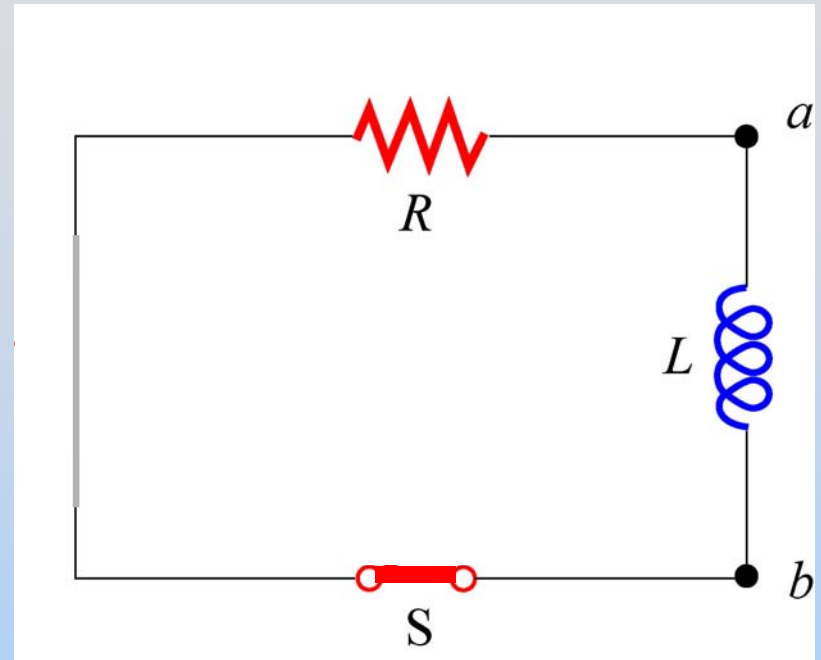
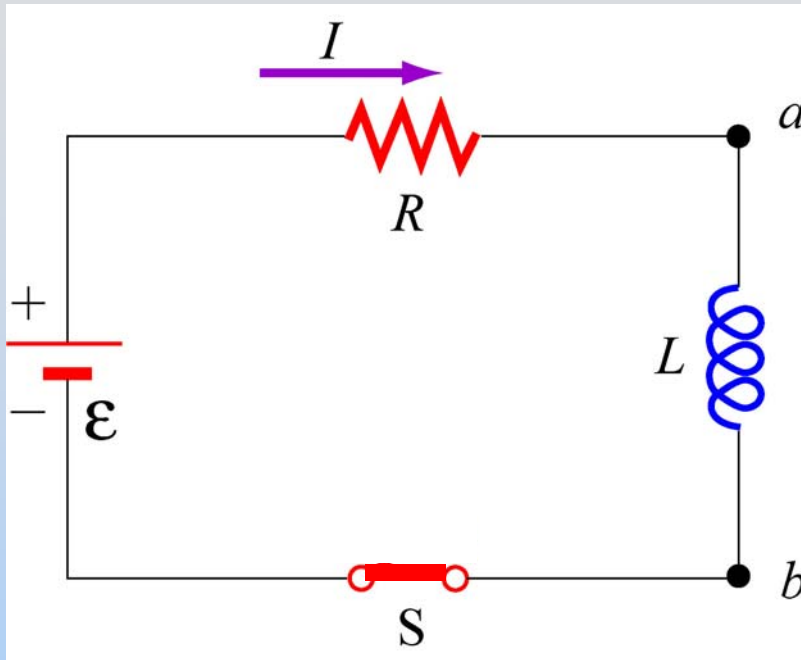
$$\text{Value}(t) = \text{Value}_{\text{Final}} \left(1 - e^{-t/\tau}\right)$$



$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

τ can be obtained from differential equation
(prefactor on d/dt) e.g. $\tau = L/R$ or $\tau = RC$

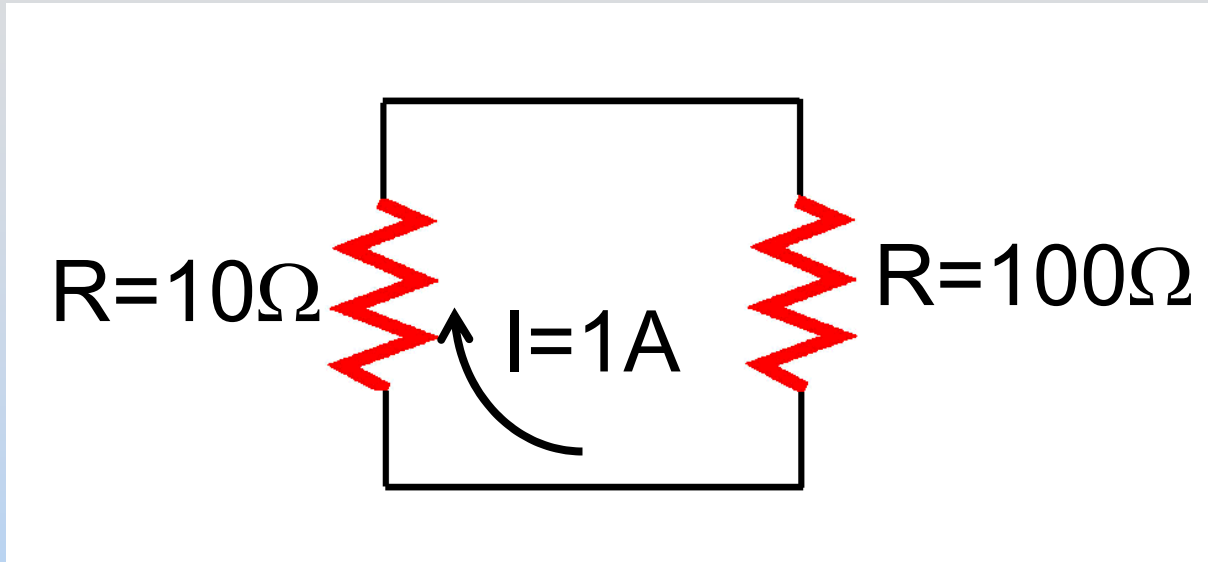
Group Problem: LR Circuit



1. What direction does the current flow just after turning off the battery (at $t=0+$)? At $t=\infty$?
2. Write a differential equation for the circuit
3. Solve and plot I vs. t and voltmeters vs. t

PRS Questions: LR Circuit & Problem...

Non-Conservative Fields



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is no longer a conservative field –
Potential now meaningless

**This concept
(& next 3 slides)
are complicated.
Bare with me and try not to
get confused**

Kirchhoff's Modified 2nd Rule

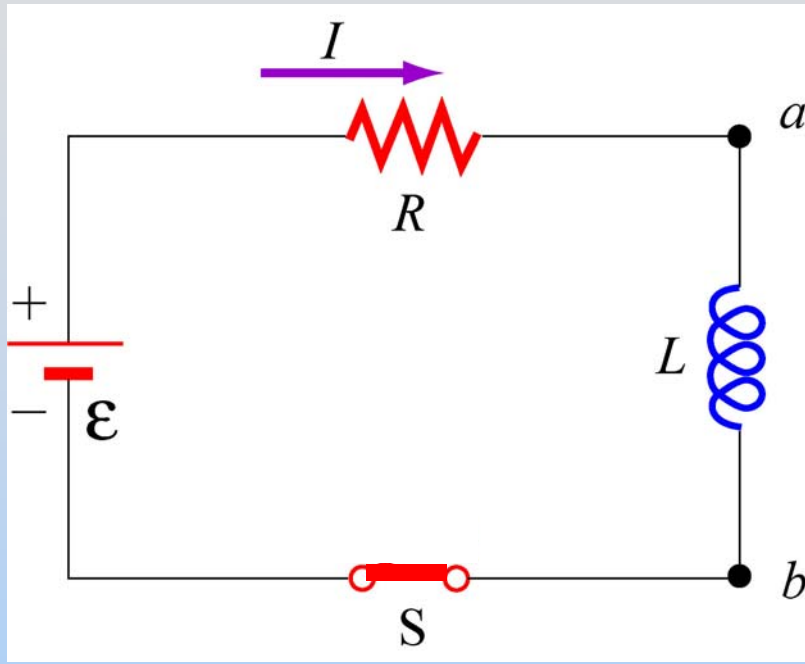
$$\sum_i \Delta V_i = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = +N \frac{d\Phi_B}{dt}$$

$$\Rightarrow \sum_i \Delta V_i - N \frac{d\Phi_B}{dt} = 0$$

If all inductance is 'localized' in inductors then our problems go away – we just have:

$$\sum_i \Delta V_i - L \frac{dI}{dt} = 0$$

Ideal Inductor



BUT, EMF generated in an inductor is not a voltage drop across the inductor!

$$\epsilon = -L \frac{dI}{dt}$$

$$\Delta V_{\text{inductor}} \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Because resistance is 0, E must be 0!

Conclusion:
Be mindful of physics
Don't think too hard doing it

Demos: Breaking circuits with inductors

Internal Combustion Engine

See figure 1:

<http://auto.howstuffworks.com/engine3.htm>

Ignition System

The Distributor:

<http://auto.howstuffworks.com/ignition-system4.htm>

- (A) High Voltage Lead
- (B) Cap/Rotor Contact
- (C) Distributor Cap
- (D) To Spark Plug

- (A) Coil connection
- (B) Breaker Points
- (D) Cam Follower
- (E) Distributor Cam

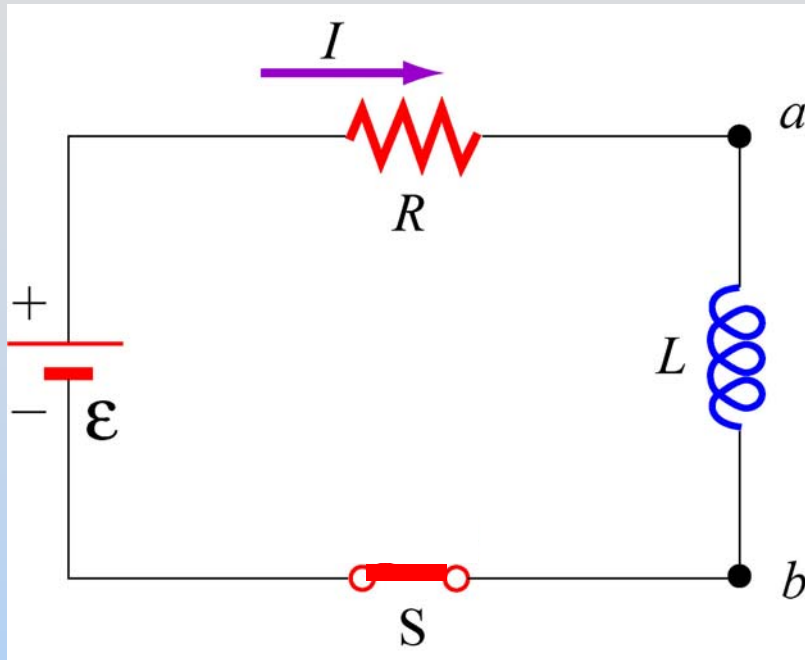
Modern Ignition

See figure:

<http://auto.howstuffworks.com/ignition-system.htm>

Energy in Inductor

Energy Stored in Inductor



$$\mathcal{E} = +IR + L \frac{dI}{dt}$$

$$I\mathcal{E} = I^2 R + LI \frac{dI}{dt}$$

$$I\mathcal{E} = I^2 R + \frac{d}{dt} \left(\frac{1}{2} LI^2 \right)$$

↗
↑
↑

Battery Resistor Inductor
 Supplies Dissipates Stores

Energy Stored in Inductor

$$U_L = \frac{1}{2} L I^2$$

But where is energy stored?

Example: Solenoid

Ideal solenoid, length l , radius R , n turns/length, current I :

$$B = \mu_0 n I \qquad L = \mu_0 n^2 \pi R^2 l$$

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2} \left(\mu_0 n^2 \pi R^2 l \right) I^2$$

$$U_B = \left(\frac{B^2}{2\mu_0} \right) \pi R^2 l$$

Energy Density Volume

Energy Density

Energy is stored in the magnetic field!

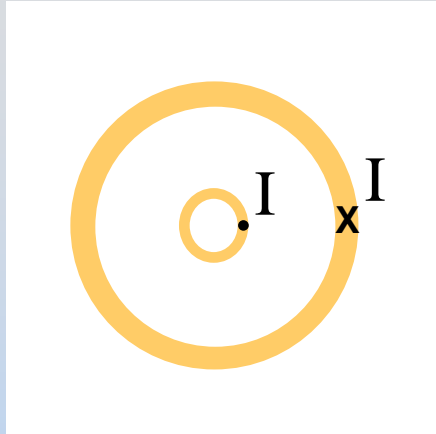
$$u_B = \frac{B^2}{2\mu_0}$$

: Magnetic Energy Density

$$u_E = \frac{\epsilon_0 E^2}{2}$$

: Electric Energy Density

Group Problem: Coaxial Cable



Inner wire: $r=a$

Outer wire: $r=b$

1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral

The EASIEST way to do (2) is to use (1)

Back to Back EMF

PRS Question: Stopping a Motor