Module 24:
Undriven RLC Circuits

Module 24: Outline

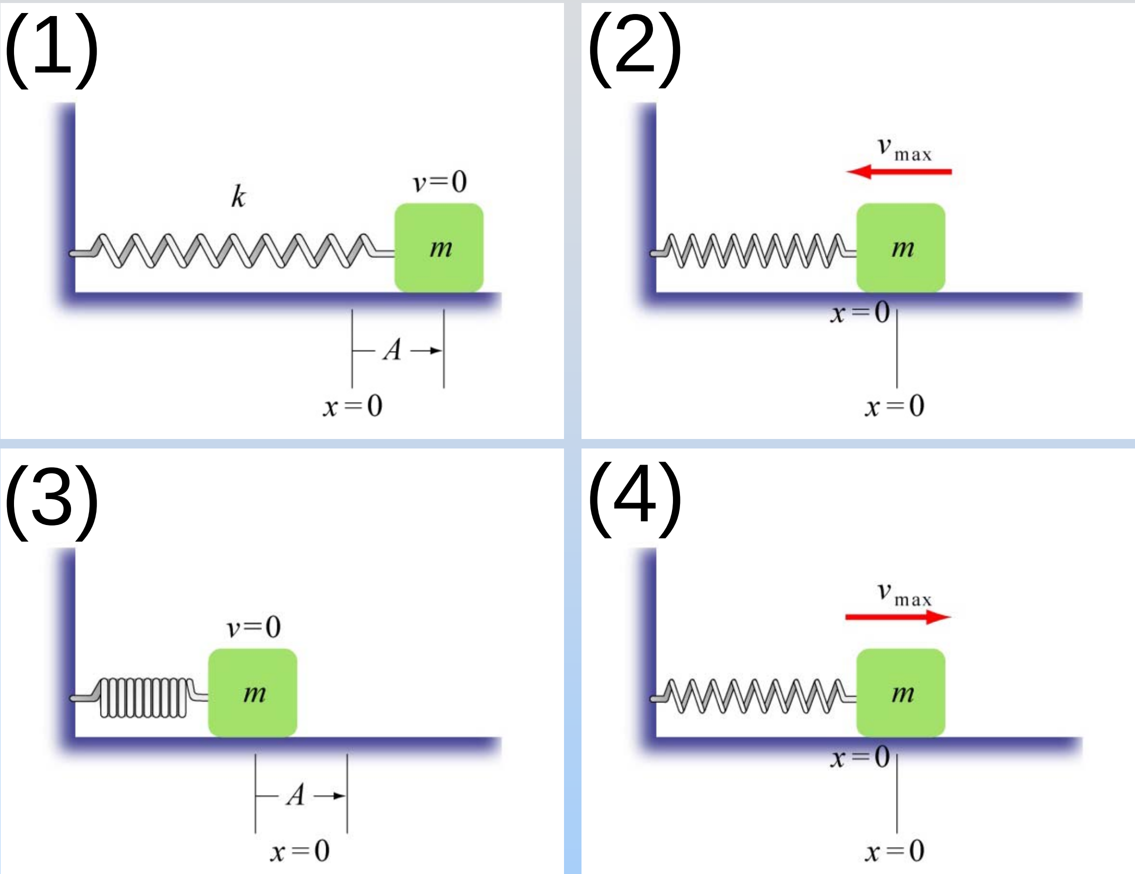
Undriven RLC Circuits

Expt. 8: Part 2:Undriven RLC Circuits

Circuits that Oscillate (LRC)

Mass on a Spring: Simple Harmonic Motion (Demonstration)

Mass on a Spring



What is Motion?

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

Simple Harmonic Motion

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

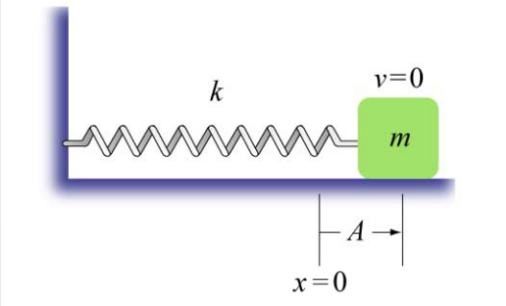
x_0: Amplitude of Motion

ϕ: Phase (time offset)

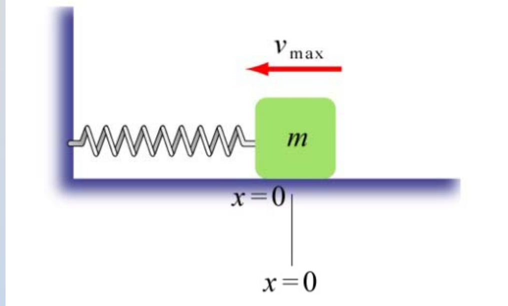
$$\omega_0 = \sqrt{\frac{k}{m}} = \text{Angular frequency}$$

Mass on a Spring: Energy

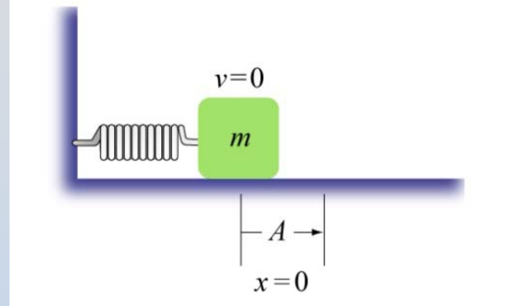
(1) Spring



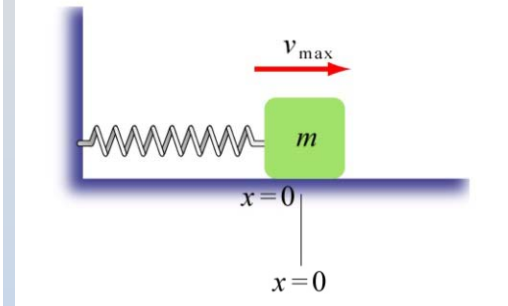
(2) Mass



(3) Spring



(4) Mass



$$x(t) = x_0 \cos(\omega_0 t + \phi) \qquad \frac{dx}{dt} = v_x(t) = -\omega_0 x_0 \sin(\omega_0 t + \phi)$$

Energy has 2 parts: (Mass) Kinetic and (Spring) Potential

$$K = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 = \frac{1}{2} k x_0^2 \sin^2(\omega_0 t + \phi)$$

$$U_s = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^2 \cos^2(\omega_0 t + \quad)$$

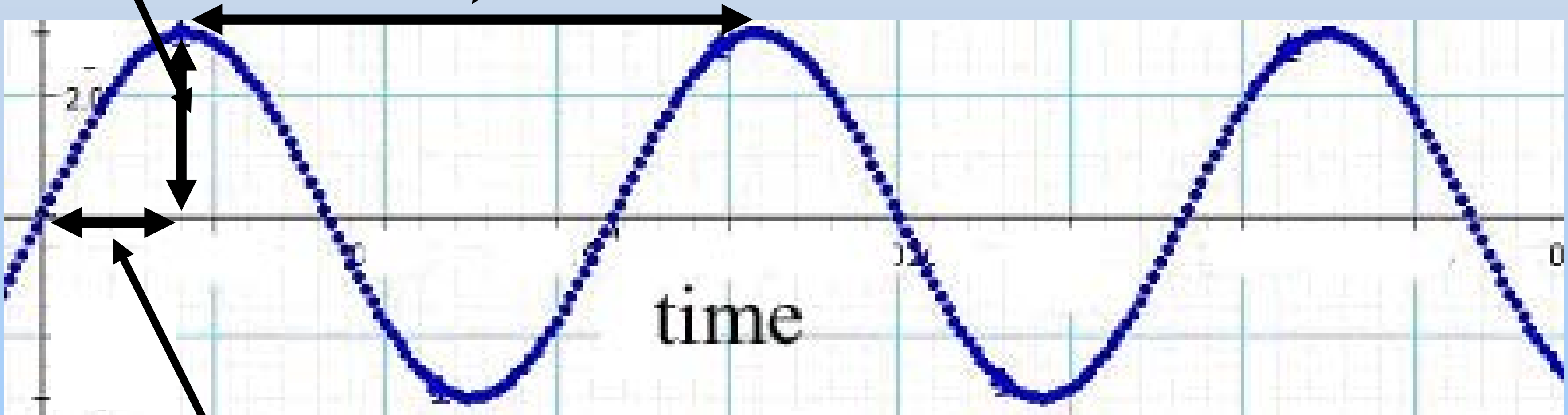
Energy sloshes back and forth

Simple Harmonic Motion

$$\text{Period} = \frac{1}{\text{frequency}} \rightarrow T = \frac{1}{f}$$

$$\text{Period} = \frac{2\pi}{\text{angular frequency}} \rightarrow T = \frac{2\pi}{\omega}$$

Amplitude (x_0)



$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

$$\text{Phase Shift } (\phi) = \frac{-\pi}{2}$$

Electronic Analog:
LC Circuits

Analog: LC Circuit

Mass doesn't like to accelerate

Kinetic energy associated with motion

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}; \quad E = \frac{1}{2}mv^2$$

Inductor doesn't like to have current change

Energy associated with current

$$\varepsilon = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}; \quad E = \frac{1}{2}LI^2$$

Analog: LC Circuit

Spring doesn't like to be compressed/extended

Potential energy associated with compression

$$F = -kx; \quad E = \frac{1}{2}kx^2$$

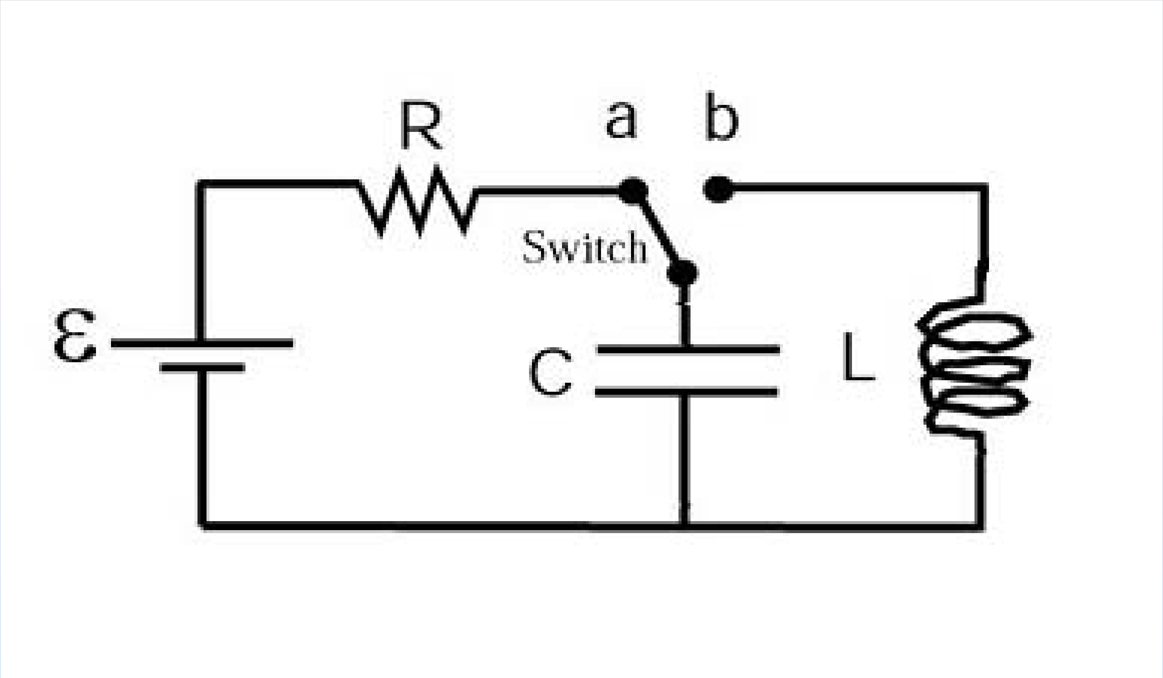
Capacitor doesn't like to be charged (+ or -)

Energy associated with stored charge

$$\varepsilon = \frac{1}{C}q; \quad E = \frac{1}{2}\frac{1}{C}q^2$$

$$\boxed{F \to \varepsilon;\ x \to q;\ v \to I;\ m \to L;\ k \to C^{-1}}$$

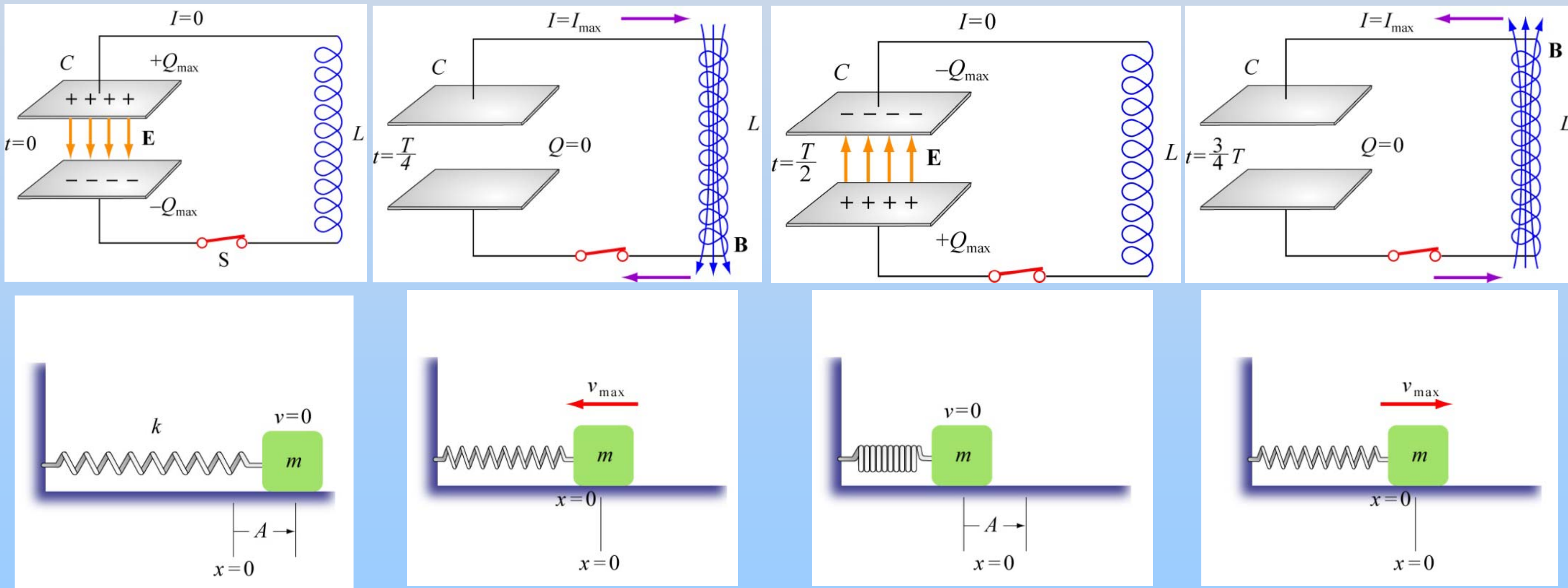
LC Circuit



1. Set up the circuit above with capacitor, inductor, resistor, and battery.
2. Let the capacitor become fully charged.
3. Throw the switch from a to b
4. What happens?

LC Circuit

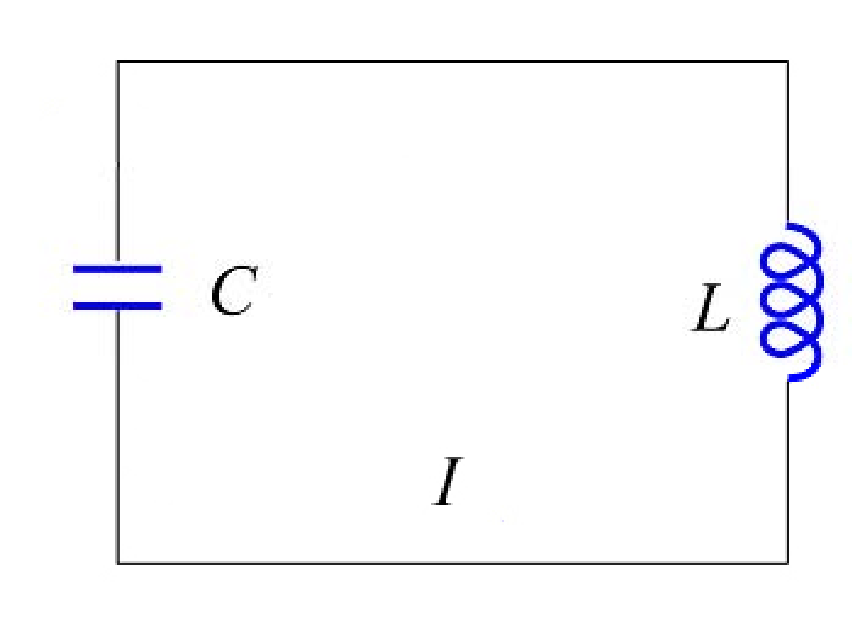
It undergoes simple harmonic motion, just like a mass on a spring, with trade-off between charge on capacitor (Spring) and current in inductor (Mass)



Concept Question Questions: LC Circuit

Concept Question: LC Circuit

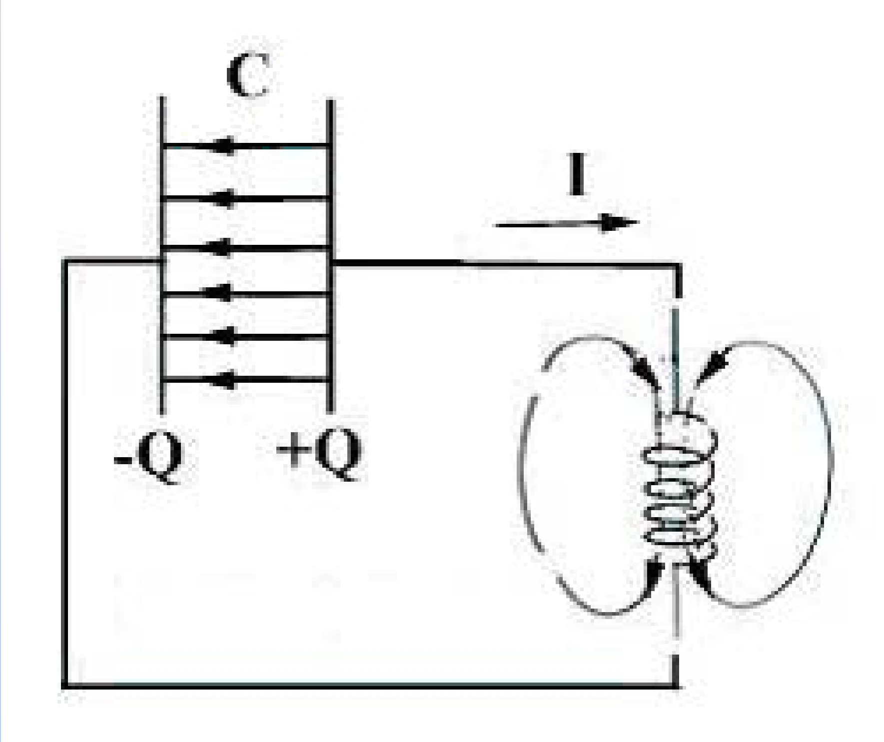
Consider the LC circuit at right. At the time shown the current has its maximum value. At this time



1. The charge on the capacitor has its maximum value
2. The magnetic field is zero
3. The electric field has its maximum value
4. The charge on the capacitor is zero
5. Don’t have a clue

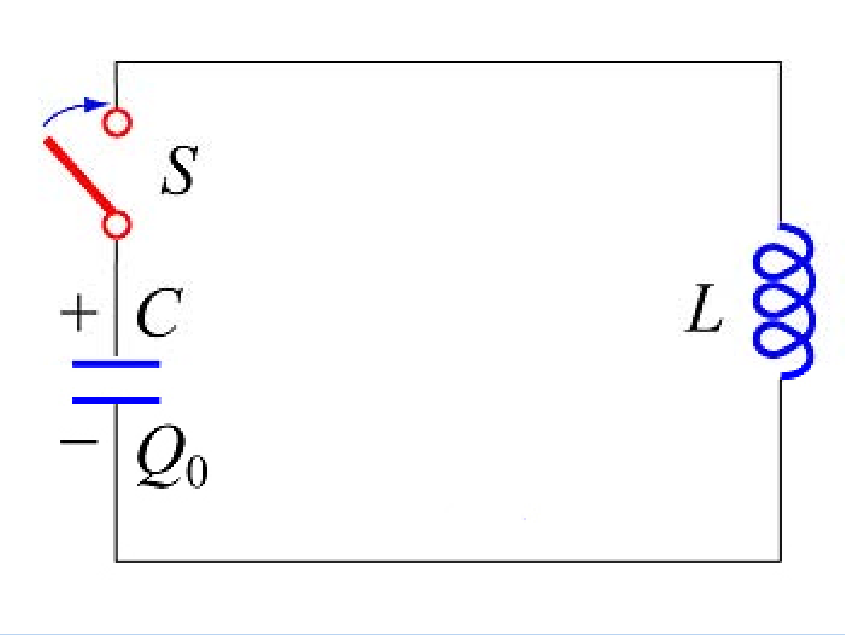
Concept Question: LC Circuit

In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,



1. I is increasing and Q is increasing
2. I is increasing and Q is decreasing
3. I is decreasing and Q is increasing
4. I is decreasing and Q is decreasing
5. Don’t have a clue

LC Circuit



$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \quad ; \quad I = -\frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

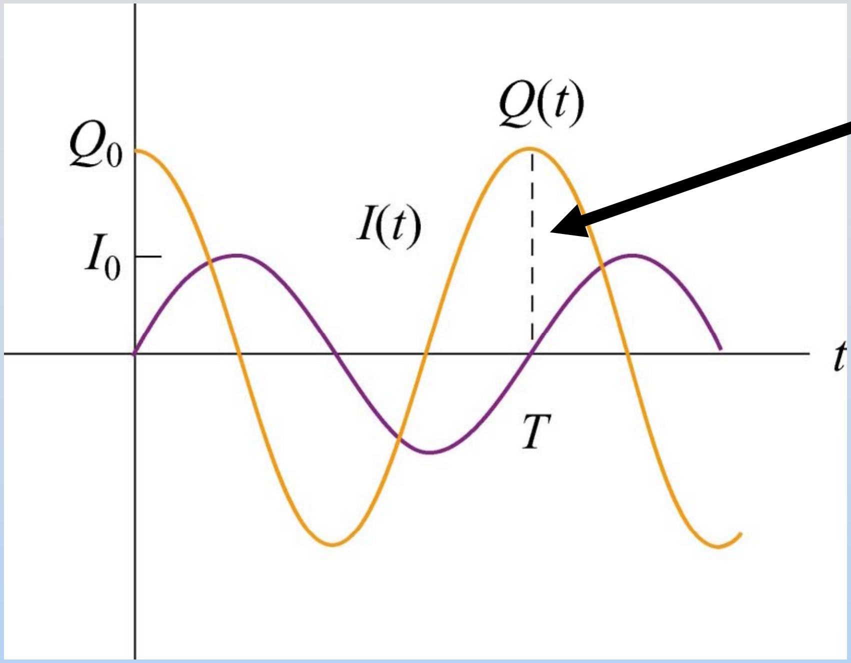
Simple Harmonic Motion

$$Q(t) = Q_0\cos(\omega_0 t + \phi) \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

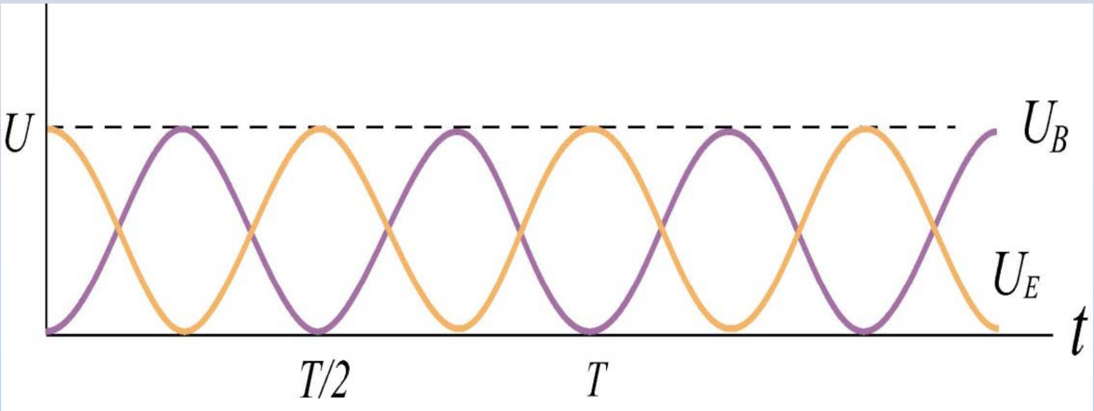
Q_0: Amplitude of Charge Oscillation

ϕ: Phase (time offset)

LC Oscillations: Energy



Notice relative phases

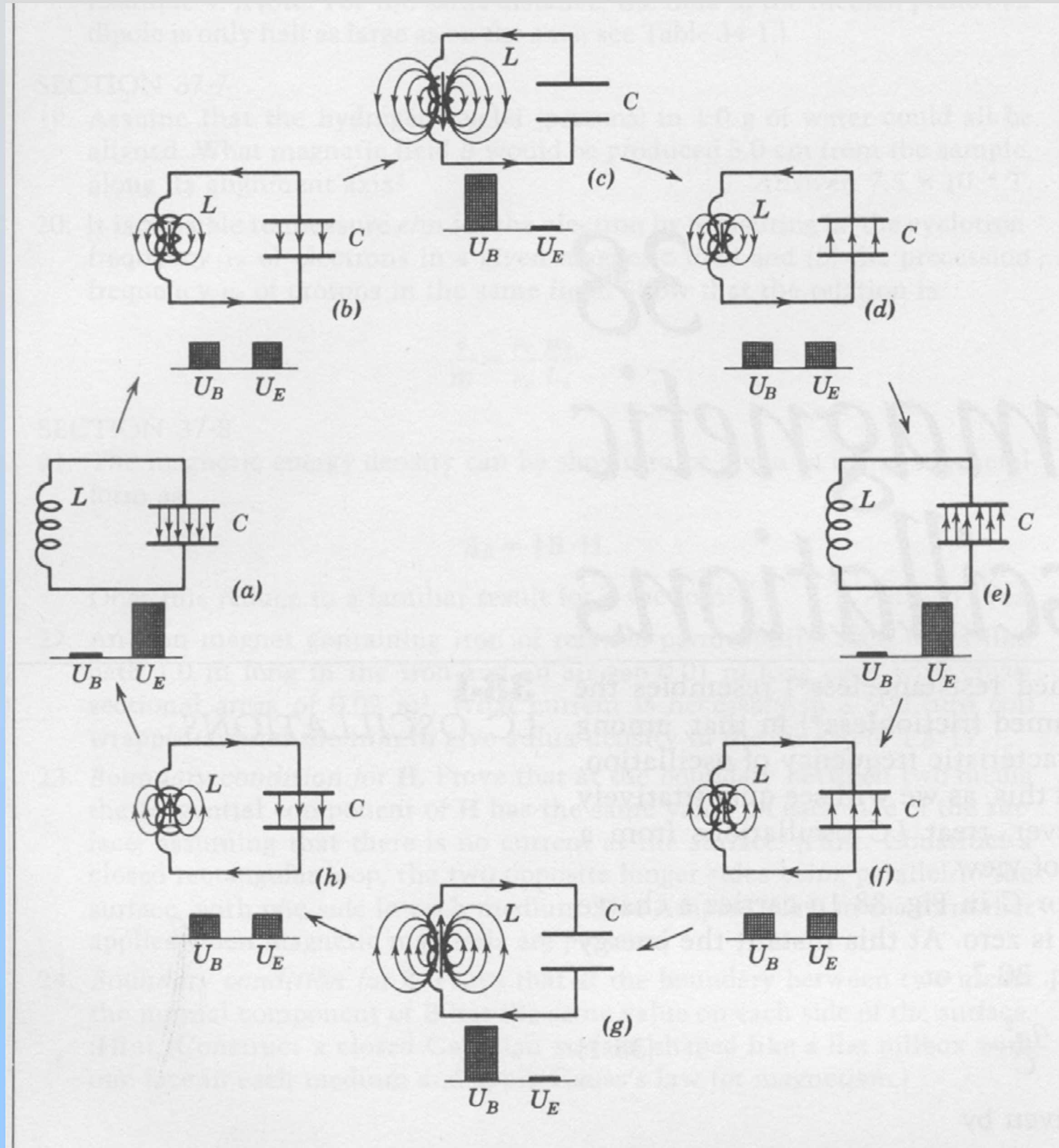


$$U_E = \frac{Q^2}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2\omega_0 t \qquad U_B = \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2\sin^2\omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2\omega_0 t$$

$$U = U_E + U_B = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q_0^2}{2C}$$

Total energy is conserved !!

Summary: The Ideal LC Circuit



Adding Damping: RLC Circuits

The Real RLC Circuit: Energy Considerations

Include finite resistance:

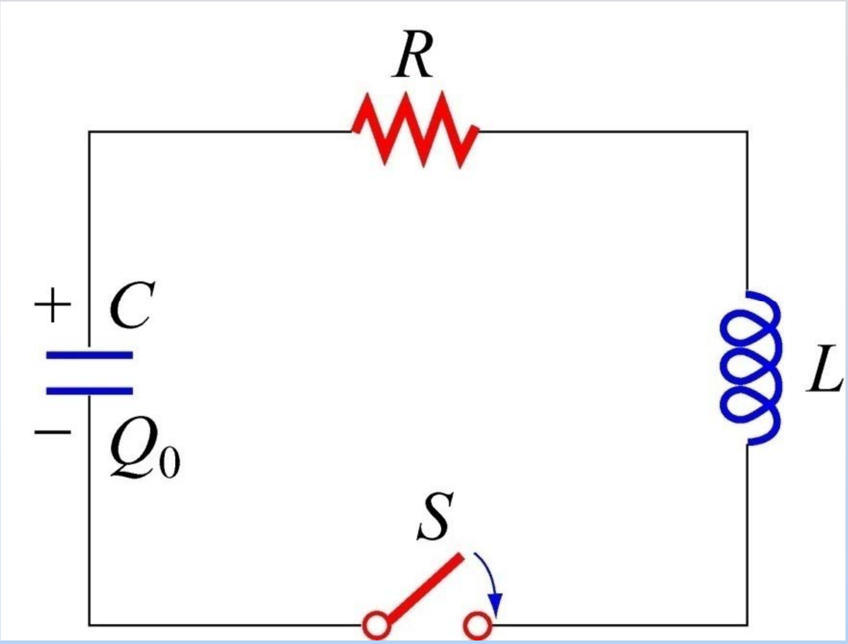
$$\frac{Q}{C} + I\,R + L\frac{dI}{dt} = 0$$

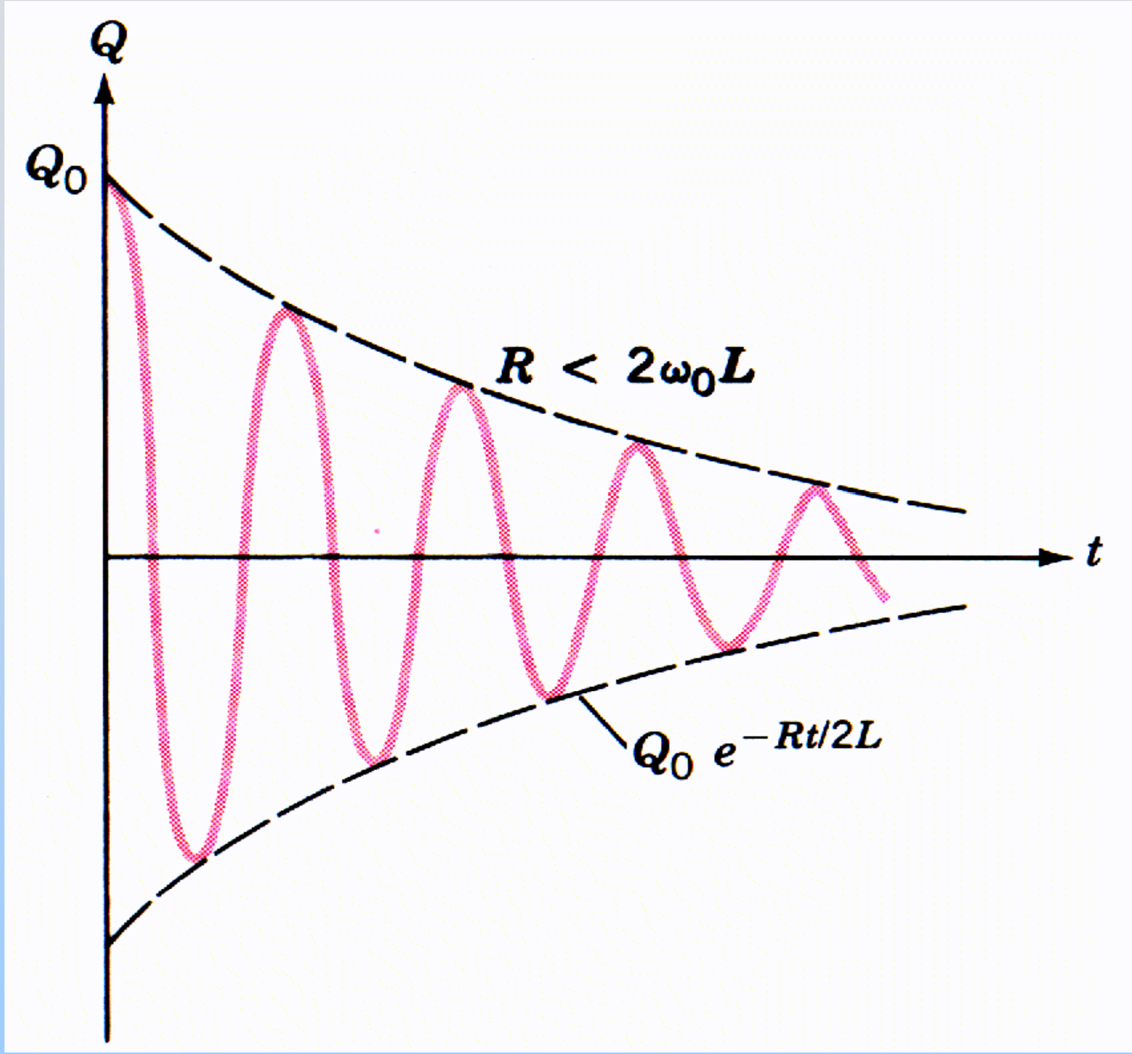
Multiply by I and after a little work:

$$\frac{d}{dt}\left[\frac{Q^2}{2C} + \frac{1}{2}L\,I^2\right] = -\,I^2 R$$

$$\frac{d}{dt}\left[\text{Total Energy}\right] = -\,I^2 R$$

Damped LC Oscillations



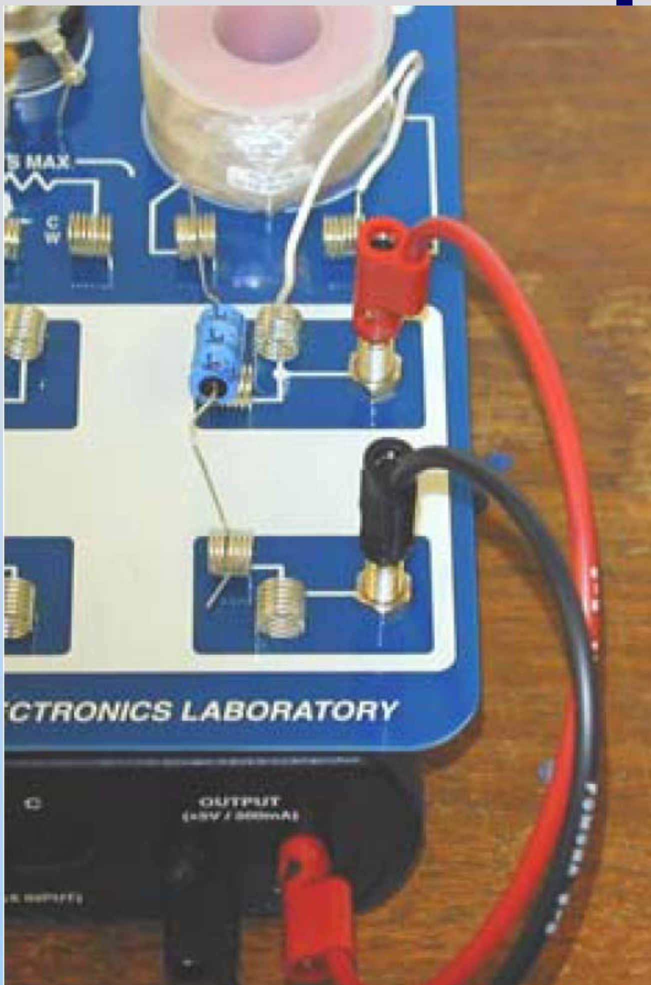


Resistor dissipates energy and system rings down over time

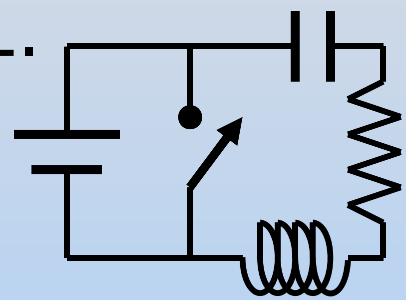
Also, frequency decreases: $\omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$

Experiment 8:
Part 2 Undriven RLC Circuits

Concept Question: Expt. 8



In today's lab the battery turns on and off. Which circuit diagram is most representative of our circuit?

1.


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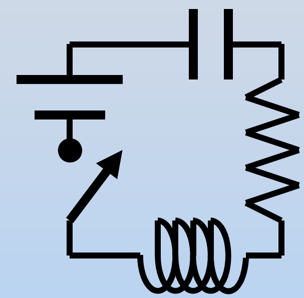

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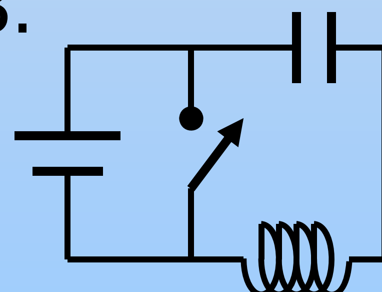

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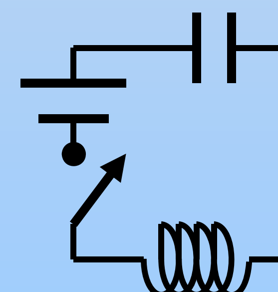
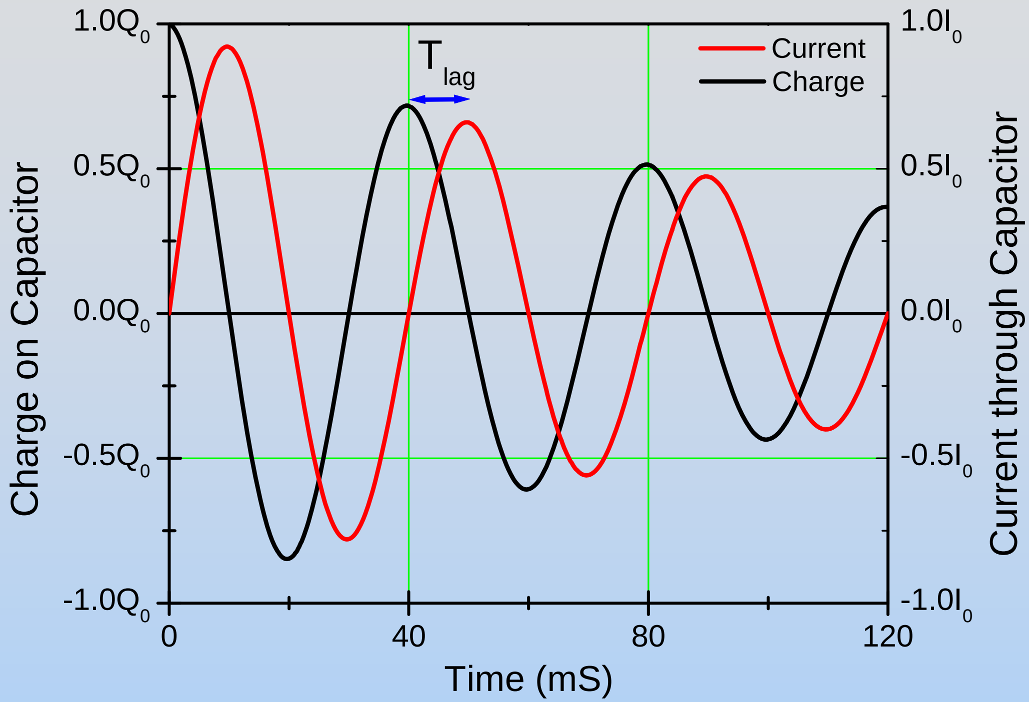


Load lab while waiting…

Concept Question Questions: Undriven Circuits

Concept Question: LC Circuit

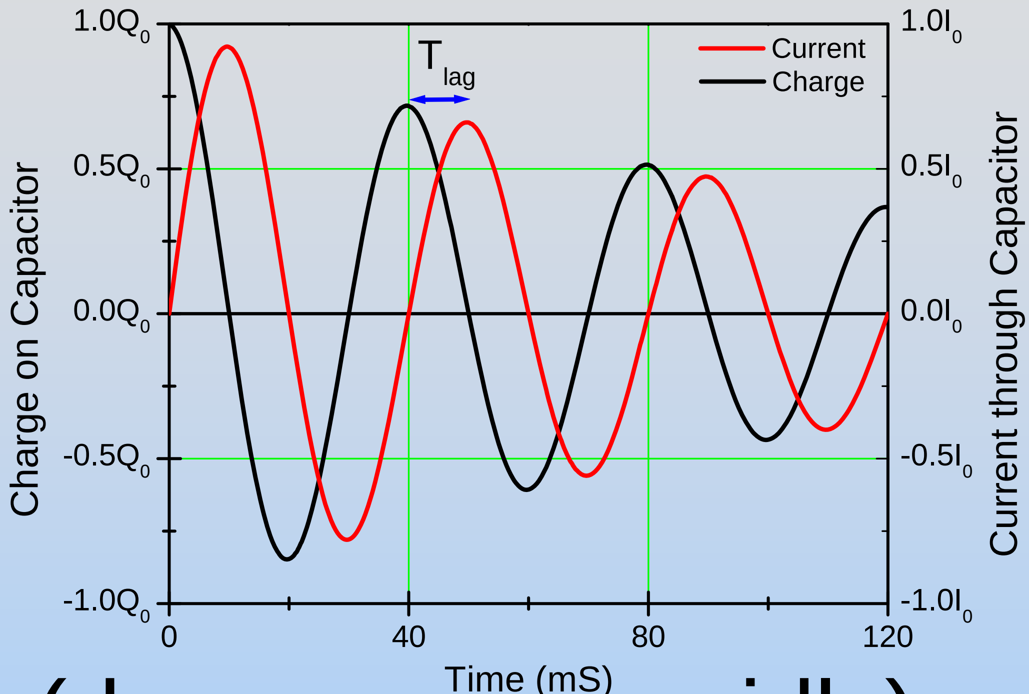
The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{Lag}?



1. It will increase
2. It will decrease
3. It will stay the same
4. I don't know

Concept Question: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?



1. It will increase (decay more rapidly)
2. It will decrease (decay less rapidly)
3. It will stay the same
4. I don't know