Undriven RLC Circuits
Challenge Problems

Problem 1:

Show that

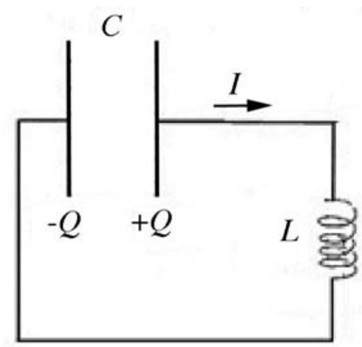
$$A\cos\omega t + B\sin\omega t = Q_{\mathrm{m}}\cos(\omega t + \phi),$$

where

$$Q_{\mathrm{m}} = \left(A^2 + B^2\right)^{1/2}, \text{ and } \phi = \tan^{-1}\left(-B/A\right).$$

Problem 2:

At the moment depicted in the LC circuit the current is non-zero and the capacitor plates are charged (as shown in the figure below). The energy in the circuit is stored



a) only in the electric field and is decreasing.

b) only in the electric field and is constant.

c) only in the magnetic field and is decreasing.

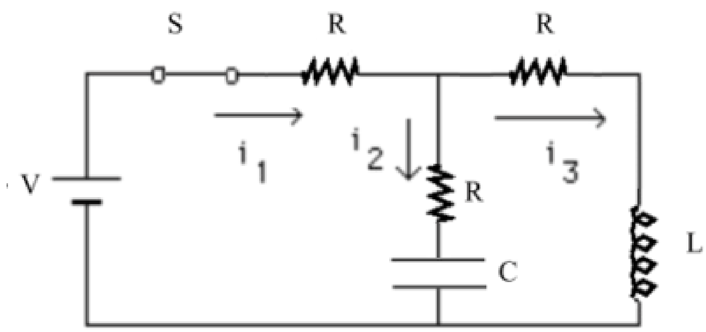
d) only in the magnetic field and is constant.

e) in both the electric and magnetic field and is constant.

f) in both the electric and magnetic field and is decreasing.

Problem 3:

A circuit consists of a battery with emf V, an inductor with inductance L, a capacitor with capacitance C, and three resistors, each with resistance R, as shown in the sketch. The capacitor is initially uncharged and there is no current flowing anywhere in the circuit. The switch S has been open for a long time, and is then closed, as shown in the diagram. If we wait a long time after the switch is closed, the currents in the circuit are given by:



a) $i_1 = \dfrac{2V}{3R}$ $i_2 = \dfrac{V}{3R}$ $i_3 = \dfrac{V}{3R}$.

b) $i_1 = \dfrac{V}{2R}$ $i_2 = 0$ $i_3 = \dfrac{V}{2R}$.

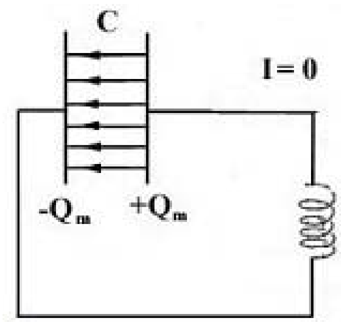
c) $i_1 = \dfrac{V}{3R}$ $i_2 = 0$ $i_3 = \dfrac{V}{3R}$.

d) $i_1 = \dfrac{V}{2R}$ $i_2 = \dfrac{V}{2R}$ $i_3 = 0$.

e) None of the above.

Problem 4:

In an LC circuit, the electric and magnetic fields are shown in the figure. Which of the following is true? **Explain your answer** At the moment depicted in the figure, the energy in the circuit is stored in



1. the electric field and is decreasing
2. the electric field and is constant.
3. the magnetic field and is decreasing.
4. the magnetic field and is constant.
5. in both the electric and magnetic field and is constant.
6. in both the electric and magnetic field and is decreasing.

Problem 5:

In a freely oscillating *LC* circuit, (no driving voltage), suppose the maximum charge on the capacitor is $Q_{\max}$. Assume the circuit has zero resistance.

a) In terms of the maximum charge on the capacitor, what value of charge is present on the capacitor when the energy in the magnetic field is three times the energy in the electric field.

b) How much time has elapsed from when the capacitor is fully charged for this condition to arise?

c) If the resistance is non-zero, will the natural frequency of oscillation compared to the natural frequency of the ideal *LC* circuit (with zero resistance)

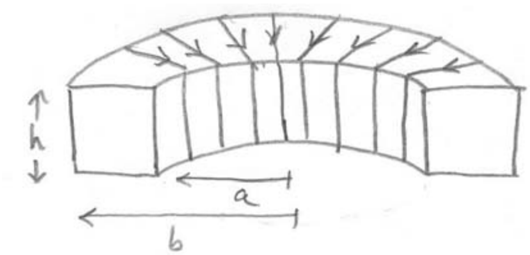
i) increase

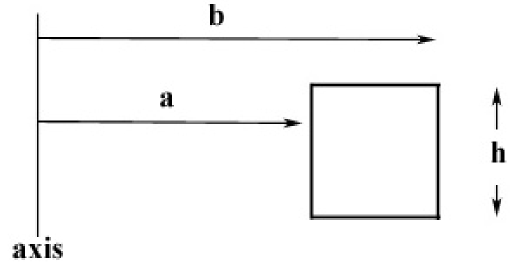
ii) stay the same

iii) decrease

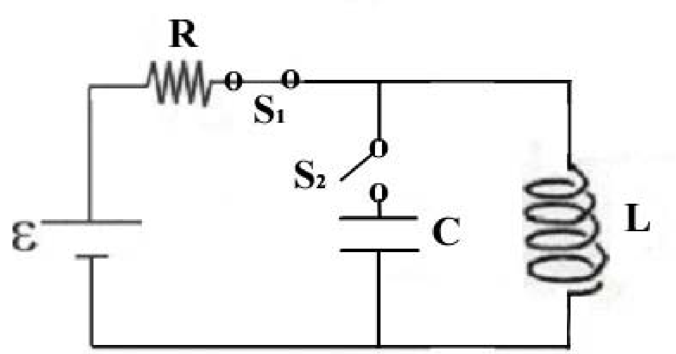
Problem 6:

A toroidoil coil has N turns, and an inner radius a, outer radius b, and height h. The coil has a rectangular cross section shown in the figures below.





The coil is connected via a switch, S_1, to an ideal voltage source with electromotive force $\mathcal{E}$. The circuit has total resistance R. Assume all the self-inductance L in the circuit is due to the coil. At time $t = 0$ S_1 is closed and S_2 remains open.



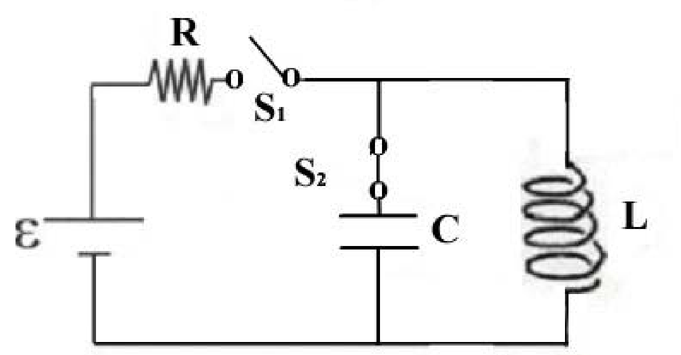
a) When a current I is flowing in the circuit, find an expression for the magnitude of the magnetic field inside the coil as a function of distance r from the axis of the coil.

b) What is the self-inductance L of the coil?

c) What is the current in the circuit a very long time $(t >> L/R)$ after S_1 is closed?

d) How much energy is stored in the magnetic field of the coil a very long time $(t >> L/R)$ after S_1 is closed?

For the next two parts, assume that a very long time $(t >> L/R)$ after the switch S_1 was closed, the voltage source is disconnected from the circuit by opening S_1, and by simultaneously closing S_2 the toroid is connected to a capacitor of capacitance C. Assume there is negligible resistance in this new circuit.



e) What is the maximum amount of charge that will appear on the capacitor?

f) How long will it take for the capacitor to first reach a maximal charge after S_2 has been closed?

Problem 7:

For the underdamped RLC circuit, $R^2 < 4L/C$, let $\gamma = \left(1/LC - R^2/4L^2\right)^{1/2}$ and $\alpha = R/2L$.

(a) Show by direct substitution that the equation

$$0 = L\frac{d^2Q}{dt^2} + \frac{dQ}{dt}R + \frac{Q}{C}$$

has a solution of the form

$$Q(t) = Ae^{-\alpha t}\cos(\gamma t + \phi)$$

(b) Denote the current by

$$I(t) = \frac{dQ(t)}{dt} = Fe^{-\alpha t}\cos(\gamma t + \phi + \beta)$$

Find the constants F and β in terms of R, L and C as needed.

Problem 8:
***LC* Circuit**

An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increase linearly in time as described by $I = Kt$. The capacitor initially has no charge. Determine

(a) the voltage across the inductor as a function of time,

(b) the voltage across the capacitor as a function of time, and

(c) the time when the energy stored in the capacitor first exceeds that in the inductor.

Problem 9:
***LC* Circuit**

(a) Initially, the capacitor in a series *LC* circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time T the energy stored in the capacitor is one-fourth its initial value. Determine L if C and T are known.

(b) A capacitor in a series *LC* circuit has an initial charge Q_0 and is being discharged. The inductor is a solenoid with N turns. Find, in terms of L and C, the flux through each of the N turns in the coil at time t, when the charge on the capacitor is $Q(t)$.

(c) An *LC* circuit consists of a 20.0-mH inductor and a $0.500\text{-}\mu\text{F}$ capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?