# Driven RLC Circuits
## Challenge Problem Solutions

**Problem 1:**

Using the same circuit as in problem 6, only this time leaving the function generator on and driving below resonance, which in the following pairs leads (if either):

(a) Voltage across the capacitor or voltage across the resistor

(b) Voltage across the function generator or voltage measured across the inductor

(c) Current or voltage across the resistor

(d) Current or voltage across the function generator

**Problem 1 Solutions:**

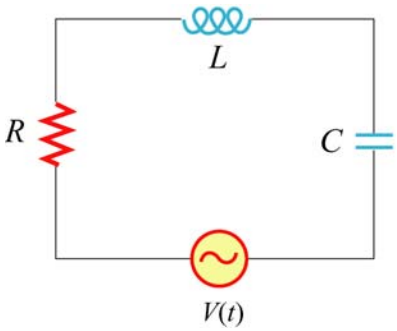
(a) **Resistor voltage** (which is always in phase with the current) always **leads capacitor** voltage by 90º, regardless of the drive frequency

(b) Below resonance the capacitor dominates, so the current leads the function generator voltage. We don't know by how much, but by less than 90º. The inductor voltage always leads the current by 90º. So the **inductor leads the function generator** by between 90º and 180º

(c) For a resistor, current and voltage are always **in phase**.

(d) As mentioned in (b) below resonance the capacitor dominates, so the **current leads the function generator voltage**.

**Problem 2:**




Consider the circuit at left, consisting of an AC function generator ($V(t) = V_0 \sin(\omega t)$, with $V_0$ = 5 V), an inductor $L$ = 8.5 mH, resistor $R$ = 5 Ω, capacitor $C$ = 100 μF and switch $S$.

The circuit has been running in equilibrium for a long time. We are now going to shut off the function generator (instantaneously replace it with a wire).

a) Assuming that our driving frequency ω is not necessarily on resonance, what is the frequency with which the system will ring down (in other words, that current will oscillate at after turning off the function generator)? Feel free to use an approximation if you wish, just make sure you know you are.

(b) What (numerical) frequency $f$ should we drive at to maximize the peak magnetic energy in the inductor?

(c) In this case, if we time the shut off to occur when the magnetic energy in the inductor peaks, after how long will the electric energy in the capacitor peak?

(d) Approximately how much energy will the resistor have dissipated during that time?

**Problem 2 Solutions:**

(a) The ringdown frequency is independent of the drive frequency. It will always ring down at the natural frequency. Actually, when there is resistance in the circuit the natural frequency is modified slightly (the approximation I mention in the question is ignoring the effect of the resistance). So the ring down frequency is:
$\omega_0 = 1/\sqrt{LC} \approx 1100 \text{ s}^{-1}$

(b) The peak magnetic energy depends on the peak current, which is maximized when we drive on resonance, in other words, we should drive at $f_0 = \omega_0/2\pi \approx 175 \text{ Hz}$.

(c) The electric energy is max when the current goes to zero, which is a quarter period after the current is a maximum. So after $t = T/4 = 1/(4f_0) = 1.4 \text{ ms}$

(d) The current will be $I = (V_0/R)\sin \omega_0 t$ so the average power dissipation (recalling that the average of $\sin^2(x) = ½$) will be $\overline{P} = V_0^2/2R = 2.5 \text{ Watts}$. So over 1.4 ms, about 3.6 mJ.

**Problem 3:**
A series *RLC* circuit with $R = 10.0\ \Omega$, $L = 400$ mH and $C = 2.0\ \mu$F is connected to an AC voltage source $V(t) = V_0 \sin\omega t$ which has a maximum amplitude $V_0 = 100$ V.

(a) What is the resonant frequency $\omega_0$?

(b) Find the rms current at resonance.

(c) Let the driving frequency be $\omega = 4000$ rad/s. Assume the current response is given by $I(t) = I_0 \sin(\omega t - \phi)$. Calculate the amplitude of the current and the phase shift between the current and the driving voltage.

**Problem 3 Solutions:**
(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\,\text{mH})(2.0\,\mu\text{F})}} = \frac{1}{\sqrt{8.0\times10^{-7}\,\text{s}}} = 1.1\times10^3 \text{ rad/s}$$

(b)
At resonance, $Z = R$. Therefore,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{(V_0/\sqrt{2})}{R} = \frac{(100V/\sqrt{2})}{10.0\Omega} = 7.07\text{A}$$

(c)

$$X_C = \frac{1}{\omega C} = \frac{1}{(4000\,\text{rad/s})(2.0\,\mu\text{F})} = 125\ \Omega$$

$$X_L = \omega L = (4000\,\text{rad/s})(400\,\text{mH}) = 1600\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(10.0\Omega)^2 + (1600\Omega - 125\Omega)^2} = 1475\ \Omega$$

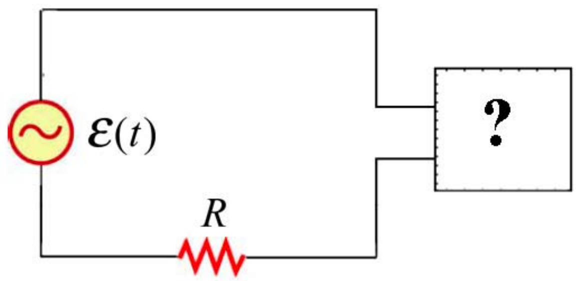
**So**

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{100\text{ V}}{\sqrt{(10.0\Omega)^2 + (1600\Omega - 125\Omega)^2}} = \frac{100\text{ V}}{1475\ \Omega} = 6.8\times10^{-2}\text{ A}$$

$$\phi = \tan\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{1600-125}{10.0}\right) = \tan^{-1}(147.5) = 89.6^\circ$$

**Problem 4:**

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin\omega t$, a resistor with resistance $R = 6\,\Omega$, and a "black box", which contains ***either*** an inductor ***or*** a capacitor, ***or both***. The amplitude of the driving emf is $\mathcal{E}_0 = 6\text{ volt}$. We measure the current in the circuit at an angular frequency $\omega = 2\text{ rad}\cdot\text{s}^{-1}$ and find that it is exactly in phase with the driving emf. We measure the current in the circuit at an angular frequency $\omega = 1\text{ rad}\cdot\text{s}^{-1}$ and find that it is out of phase from the driving emf by exactly $\pi/4$ radians.




a) What does the black box contain – an inductor or a capacitor, or both? Explain your reasoning. Does current lead or lag at $\omega = 1\text{ rad}\cdot\text{s}^{-1}$?

b) What is the numerical value of the capacitance *or* of the inductance, *or of both*, as the case may be? Indicate units. Your answer(s) will involve simple fractions only, you will not need a calculator to find the value(s).

c) What is ratio of the amplitudes of the current $\dfrac{I_0(\omega = 2\text{ rad}\cdot\text{s}^{-1})}{I_0(\omega = 1\text{ rad}\cdot\text{s}^{-1})}$ ?

**Problem 4 Solutions:**

(a) Since the current is exactly in phase with the inductor at $\omega = 2\text{ rad}\cdot\text{s}^{-1}$, the circuit is resonance and therefore the box must contain both an inductor and capacitor in series. For $\omega = 1\text{ rad}\cdot\text{s}^{-1}$, the driving angular frequency is below resonance, therefore the circuit is acting capacitively, which means the current leads the driving emf.

(b) When $\omega = 1\text{ rad}\cdot\text{s}^{-1}$, the phase angle $\phi = -\pi/4$. So $\tan\phi = \tan(-\pi/4) = -1$. Since

$$\tan\phi = \frac{X_L - X_C}{X_R} = \frac{\omega L - \dfrac{1}{\omega C}}{R} = -1\,.$$

(c) At resonance $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$:

$$I_0(\omega = 2\ \text{rad}\cdot\text{s}^{-1}) = \frac{\varepsilon_0}{R} = \frac{6\ \text{V}}{6\ \Omega} = 1\ \text{A}\,.$$

Below resonance

$$I_0(\omega = 1\ \text{rad}\cdot\text{s}^{-1}) = \frac{\varepsilon_0}{\left(R^2 + (X_L - X_C)^2\right)^{1/2}} = \frac{\varepsilon_0}{\left(R^2 + (\omega L - \frac{1}{\omega C})^2\right)^{1/2}}\,.$$

From the phase condition $\omega L - \frac{1}{\omega C} = -R$, the amplitude at $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$ becomes

$$I_0(\omega = 1\ \text{rad}\cdot\text{s}^{-1}) = \frac{\varepsilon_0}{\left(R^2 + R^2\right)^{1/2}} = \frac{\varepsilon_0}{\sqrt{2}R} = \frac{1}{\sqrt{2}}\ \text{A}\,.$$

Therefore the ratio of the amplitudes of the current is

$$\frac{I_0(\omega = 2\ \text{rad}\cdot\text{s}^{-1})}{I_0(\omega = 1\ \text{rad}\cdot\text{s}^{-1})} = \frac{\varepsilon_0 / R}{\varepsilon_0 / \sqrt{2}R} = \sqrt{2}\,.$$

We have that for $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$

$$L - \frac{1}{C} = -R\,.$$

We can divide through by $L$ in the above equation yielding

$$1\ \text{rad}\cdot\text{s}^{-1} - \frac{1}{LC(1\ \text{rad}\cdot\text{s}^{-1})} = -\frac{R}{L}$$

We also have the resonance condition at $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$,

$$\omega = 2\ \text{rad}\cdot\text{s}^{-1} = \frac{1}{\sqrt{LC}}\,.$$

Thus

$$\omega^2 = 4\ \text{rad}^2\cdot\text{s}^{-2} = \frac{1}{LC}$$

Substituting that in the above equation yields

$$1\ \text{rad}\cdot\text{s}^{-1} - 4\ \text{rad}\cdot\text{s}^{-1} = -\frac{R}{L}.$$

Solving for $L$ then yields

$$L = \frac{R}{3\ \text{rad}\cdot\text{s}^{-1}} = \frac{6\ \Omega}{3\ \text{rad}\cdot\text{s}^{-1}} = 2\ \text{H}.$$

From the resonance condition

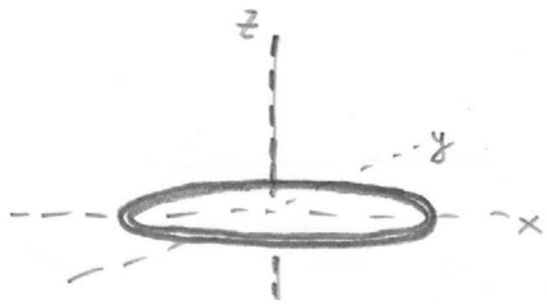
$$4\ \text{rad}^2\cdot\text{s}^{-2}C = \frac{1}{L(4\ \text{rad}^2\cdot\text{s}^{-2})} = \frac{1}{(2\ \Omega)(4\ \text{rad}^2\cdot\text{s}^{-2})} = \frac{1}{8}\ \text{F}.$$

**Problem 5:**

***Loop Antenna***. An electromagnetic wave propagating in air has a magnetic field given by

$$B_x = 0 \qquad B_y = 0 \qquad B_z = B_0 \cos(\omega t - kx).$$

It encounters a circular loop antenna of radius $a$ centered at the origin $(x, y, z) = (0, 0, 0)$ and lying in the x-y plane. The radius of the antenna $a << \lambda$ where $\lambda$ is the wavelength of the wave. So you can assume that at any time $t$ the magnetic field inside the loop is approximately equal to its value at the center of the loop.




a) What is the magnetic flux, $\Phi_{mag}(t) \equiv \iint_{disk} \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$, through the plane of the loop of the antenna?

The loop has a self-inductance $L$ and a resistance $R$. Faraday's law for the circuit is

$$IR = -\frac{d\Phi_{mag}}{dt} - L\frac{dI}{dt} .$$

b) Assume a solution for the current of the form $I(t) = I_0 \sin(\omega t - \phi)$ where $\omega$ is the angular frequency of the electromagnetic wave, $I_0$ is the amplitude of the current, and $\phi$ is a phase shift between the changing magnetic flux and the current.. Find expressions for the constants $\phi$ and $I_0$.

c) What is the magnetic field created at the center of the loop by this current $I(t)$?

**Problem 5 Solutions:**

a) $\Phi_{mag} = \iint \vec{B} \cdot d\vec{a} \simeq B_o \cos(\omega t - kz)\, \pi a^2$

b) $$\frac{d\Phi_{mag}}{dt} = -IR - L\frac{dI}{dt}$$

$$-B_o \omega \sin(\omega t - kz)\, \pi a^2 = -IR - L\frac{dI}{dt}$$

$$B_o \omega \pi a^2 \sin(\omega t - kz) = IR + L\frac{dI}{dt}$$

Set $x = 0$ we solved this equation for the case

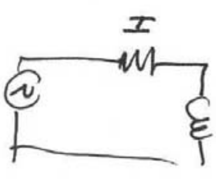
$$V(t) = V_0 \sin(\omega t - kx) \quad , \quad kx \doteq \text{constant}$$

$$V(t) = V_0 \sin \omega t \quad , I(t) = I_0 \sin(\omega t - \phi)$$

$$V_0 = B_0 \omega \pi a^2$$

$$V(t) = IR + L\frac{dI}{dt} \quad , \quad Z^T = R + i\omega L = |Z^T| e^{i\delta}$$

$$\delta = \tan^{-1}\left(\frac{\omega L}{R}\right), \; |Z^T| = (R^2 + (\omega L)^2)^{1/2}$$




$$I = I_m I_0 \, e^{i\omega t} e^{-i\phi}$$

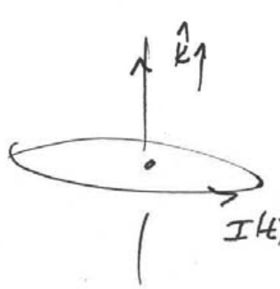
$$V = I_m V_0 \, e^{i\omega t}$$

$$V_0 e^{i\omega t} = Z^T I = I_0 |Z^T| e^{i\delta} e^{i\omega t} e^{-i\phi}$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \delta = \phi \quad , \quad I_0 = \frac{V_0}{|Z^T|} = \frac{V_0}{(R^2 + (\omega L)^2)^{1/2}}$$

$$I(t) = \frac{B_0 \omega \pi a^2}{(R^2 + (\omega L)^2)^{1/2}} \sin(\omega t - \phi)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad\qquad \hat{\theta} \times -\hat{r} = \hat{k}$$




From Biot-Savart

$$\vec{B}_{center} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

$$= \frac{\mu_0}{4\pi} I \int \frac{a \, d\theta \, \hat{\theta} \times (-a\hat{r})}{a^3}$$

$$\vec{B} = \frac{\mu_0 I}{2a} \hat{k}$$

$$\vec{B}_{center} = \frac{\mu_0}{2a} \frac{B \omega \pi a^2}{(R^2 + (\omega L)^2)^{1/2}} \sin(\omega t - \phi) \; \hat{k}$$

**Problem 6:**

***Driven RLC circuit***

Suppose you want a series $RLC$ circuit to tune to your favorite FM radio station that broadcasts at a frequency of $89.7\,MHz$. You would like to avoid the obnoxious station that broadcasts at $89.5\,MHz$. In order to achieve this, for a given input voltage signal from your antenna, you want the width of your resonance to be narrow enough at $89.7\,MHz$ such that the current flowing in your circuit will be $10^{-2}$ times less at $89.5\,MHz$ than at $89.7\,MHz$. You cannot avoid having a resistance of $R = 0.1\Omega$, and practical considerations also dictate that you use the minimum $L$ possible.

a) In terms of your circuit parameters, $L$, $R$, and $C$, what is the amplitude of your current in your circuit as a function of the angular frequency of the input signal?

b) What is the angular frequency of the input signal at the desired resonance?

c) What values of $L$ and $C$ must you use?

d) What is the quality factor for this resonance?

e) Show that at resonance, the ratio of the amplitude of the voltage across the inductor with the driving signal amplitude is the quality of the resonance.

f) Show that at resonance the ratio of the amplitude of the voltage across the capacitor with the driving signal amplitude is the quality of the resonance

g) What is the time averaged power at resonance that the signal delivers to the circuit?

h) What is the phase shift for the input signal at $89.5\,MHz$?

i) What is the time averaged power for the input signal at $89.5\,MHz$?

j) Is the circuit acting capacitatively or inductively at $89.5\,MHz$?

**Problem 6 Solutions:**

a)The amplitude of the current as a function of angular frequency of the input signal is

$$I_0 = \frac{V_0}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}$$

b)

$$\omega_0 = 1/\sqrt{LC}\ .$$

c) The condition in part b) implies that

$$L = \frac{1}{C\omega_0^{\ 2}}$$

We also require the condition that the current at $\omega_1 = 89.5\,MHz$ is equal to 0.01 the value at resonance. The current at resonance is $I_{\max,0} = V_0/R$. The amplitude of the current at $\omega_1 = 89.5\,MHz$ is

$$I_0(\omega_1) = \frac{V_0}{\left(R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2\right)^{1/2}} = 0.01 I_{\max,0} = 0.01\frac{V_0}{R}.$$

This requires that

$$\left(R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2\right)^{1/2} = 100R.$$

This implies that

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 >> R^2.$$

So we can set

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = 100R\ .$$

Then

$$\omega_1 L - \frac{1}{\omega_1 C} = \pm 100R\ .$$

This equation imposes a condition on $L$,

$$L = \frac{1}{\omega_1{}^2 C} \pm \frac{100R}{\omega_1}.$$

We use our other condition $L = 1/\left(C\omega_0{}^2\right)$

Then

$$\frac{1}{\omega_0{}^2 C} = \frac{1}{\omega_1{}^2 C} \pm \frac{100R}{\omega_1}$$

We can solve this equation for the capacitance choosing the negative root,

$$C = \frac{\omega_1}{100R}\left(\frac{1}{\omega_1{}^2} - \frac{1}{\omega_0{}^2}\right) = \frac{(2\pi)\left(89.5\times10^6\,Hz\right)}{\left(4\pi^2\right)(100)(0.1\Omega)}\left(\frac{1}{\left(89.5\times10^6\,Hz\right)^2} - \frac{1}{\left(89.7\times10^6\,Hz\right)^2}\right) = 7.92\times10^{-13}\,F$$

The self-inductance is then

$$L = \frac{1}{C\omega_0{}^2} = \frac{1}{\left(7.92\times10^{-13}\,F\right)\left(4\pi^2\right)\left(89.7\times10^6\,Hz\right)^2} = 4.0\times10^{-6}\,H$$

d)

$$Q_{quality} = \frac{L\omega_0}{R} = \frac{\left(4.0\times10^{-6}\,H\right)(2\pi)\left(89.7\times10^6\,Hz\right)}{0.1\Omega} = 2.2\times10^4$$

e) The voltage across the inductor is

$$V_L(t) = L\frac{dI}{dt}.$$

Since the current at resonance is

$$I = I_0 \sin(\omega_0 t)$$

We have that

$$V_L(t) = L\frac{dI}{dt} = LI_0\omega_0\cos\left(\omega_0 t\right).$$

Therefore

$$V_{L,0} = LI_0\omega_0$$

Thus

$$\frac{V_{L,0}}{V_0} = \frac{L\omega_0 I_0}{V_0} = \frac{L\omega_0}{R} = Q$$

f) The voltage across the capacitor is given by

$$V_C(t) = \frac{q}{C}.$$

The charge is the integral of the current,

$$q = \int I(t)dt = \int I_0 \sin(\omega_0 t)dt = -\frac{I_0\cos(\omega_0 t)}{\omega_0}$$

So

$$\frac{V_{C,0}}{V_0} = \frac{I_0}{\omega_0 C}\frac{1}{V_0} = \frac{V_0}{\omega_0 CR}\frac{1}{V_0} = \frac{\sqrt{LC}}{CR} = \frac{\sqrt{L}}{R\sqrt{C}} = \frac{L}{R\sqrt{LC}} = \frac{L\omega_0}{R} = Q_{quality}$$

g) The time-averaged power is

$$\langle P(t)\rangle = \frac{1}{T}\int_0^T P(t)dt = \frac{1}{T}\int_0^T \frac{V_0^{\,2}}{R}\sin^2\left(\frac{1}{\sqrt{LC}}t\right)dt = \frac{V_0^{\,2}}{2R}$$

h)

$$\phi = \tan^{-1}\left(\left(\omega_1 L - \frac{1}{\omega_1 C}\right)\Big/R\right) = \tan^{-1}\left(-100R/R\right) = -\tan^{-1}\left(100\right) = -89.4^0$$

i) We calculated that the time-averaged power is

$$\langle P\rangle = \frac{1}{T}\int_0^T P(t)dt = \frac{1}{2}\frac{V_0^{\,2}}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}\cos(\phi).$$

The phase shift introduces a factor $\cos(\phi)$, which we can calculate by noting that

$$\tan\phi = \frac{y}{x} = \left(\left(\omega L - \frac{1}{\omega C}\right)\Big/R\right).$$

Therefore

$$\cos(\phi) = \frac{x}{\left(x^2 + y^2\right)^{1/2}} = (\frac{R}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}})$$

The time-averaged power is

$$\langle P \rangle = \frac{1}{2} \frac{R V_0^{\,2}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{1}{2} \frac{R V_0^{\,2}}{\left(10 \times 10^4\right) R^2} = 5.0 \times 10^{-5} \frac{V_0^{\,2}}{R}.$$

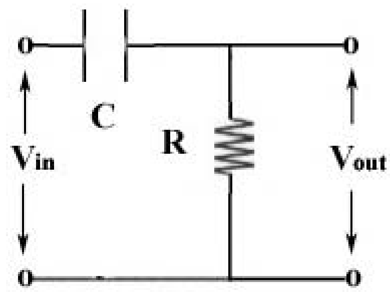
j) The phase shift is negative, we are below resonance, so the circuit acts capacitatively.

**Problem 7:**

Consider a circuit consisting of a resistor and a capacitor with an ac sinusoidal input,

$$V_{in}(t) = V_0 \sin(\omega t)$$

and two output terminals.




a) What is the total impedance for this circuit?

b) What is the amplitude and phase of the current $I(t) = I_0 \sin(\omega t - \phi)$ in the circuit?

c) What is the amplitude and phase of the output voltage $V_{out}(t)$ across the resistor?

d) What is the ratio of the magnitudes of the output signal amplitude to the input signal amplitude $\left|V_{out,0}\right| / \left|V_{in,0}\right|$?

e) Explain why this type of circuit element is referred to as a *'high pass filter'*.

**Problem 7 Solutions:**

(a) The total impedance for this circuit is

$$\left|z^T\right| = \left(\left|z_R\right|^2 + \left|z_C\right|^2\right)^{1/2} = \left(R^2 + \left(1/\omega C\right)^2\right)^{1/2} \quad (0.1)$$

(b) The amplitude of the current is given by

$$I_0 = \frac{V_{in,0}}{\left|z^T\right|} = \frac{V_{in,0}}{\left(R^2 + \left(1/\omega C\right)^2\right)^{1/2}} \quad (0.2)$$

The phase angle $\phi$ is given by

$$\phi = \tan^{-1}\left(-1/\omega CR\right) = -\tan^{-1}\left(1/\omega CR\right).$$

Note that the phase angle is negative so the current leads the voltage as it should for a "capacitive" circuit.

(c) The amplitude of the voltage across the resistor is given by

$$V_{out,0} = |z_R| I_0 = \frac{|z_R| V_{in,0}}{\left(|z_R|^2 + |z_C|^2\right)^{1/2}} = \frac{R V_{in,0}}{\left(R^2 + (1/\omega C)^2\right)^{1/2}} \tag{0.3}$$

(d) Divide through by the input voltage in Eq. (0.3) to find the ratio of amplitude of the output voltage to the input voltage:

$$\frac{V_{out,0}}{V_{in,0}} = \frac{R}{\left(R^2 + (1/\omega C)^2\right)^{1/2}}. \tag{0.4}$$

(e) Note that for small values of $\omega$, the denominator is Eq. (0.4) becomes very large hence the ratio of amplitudes becomes very small. For very large values of $\omega$, we can ignore the second term in the denominator and the ratio approaches unity. Only large angular frequencies will pass through this filter. The small angular frequency signals are attenuated. We call this type of circuit element a *'high pass filter'*.

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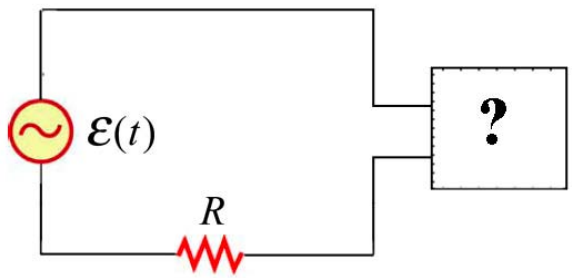
**Problem 8:**

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, a resistor with resistance $R$, and a "black box", which contains either an inductor or a capacitor, but not both. The amplitude of the driving emf, $\mathcal{E}_0$, is 200 volts, and the angular frequency $\omega = 5 \text{ rad} \cdot \text{s}^{-1}$. We measure the current in the circuit and find that it is given as a function of time by the expression:

$$I(t) = (10 \text{ A}) \sin(\omega t - \pi / 4) .$$




a) Does the current lead or lag the emf?

b) What is the unknown circuit element in the black box – an inductor or a capacitor?

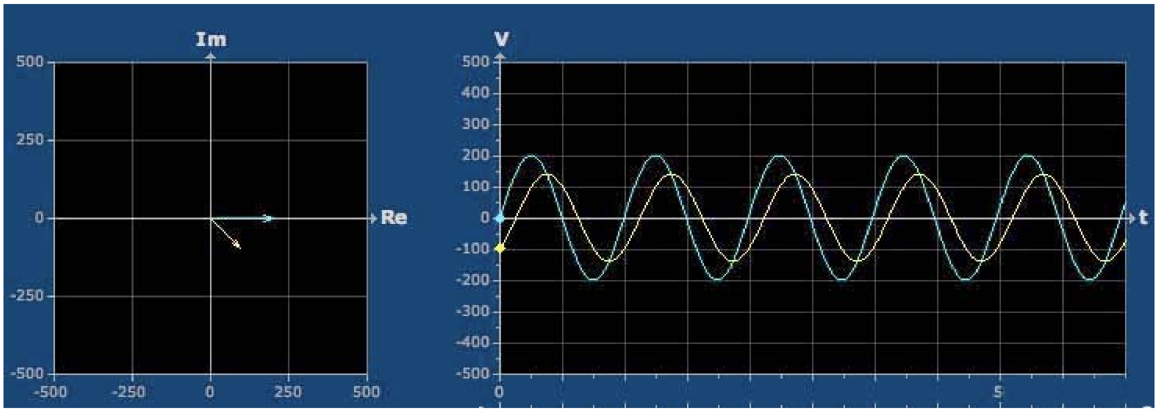
c) What is the numerical value of the resistance $R$? Include units.

d) What is the numerical value of the capacitance or inductance, as the case may be? Include units.

e) What value of $C$ or $L$ should be connected in series for the circuit to be in resonance with the given driving angular frequency? What would be the amplitude $I_0$ of the new current?

**Problem 8 Solution:**

(a) In the figure below we show on the right a plot of the generator emf (blue) and the current (white), and on the left the phase relation $\phi = \pi / 4$ between the emf (blue arrow) and the current (white).




The current lags the voltage since the voltage peaks at a slightly earlier time than the current.

(b) Since the current lags the voltage, the unknown circuit element must be an inductor.

(c) The phase $\phi = \pi / 4$ for the LR circuit so

$$1 = \tan(\pi / 4) = \frac{\omega L}{R} . \qquad (0.5)$$

The amplitude for the current in a sinusoidally driven $LR$ circuit is

$$I_0 = \frac{V_0}{\left(R^2 + (\omega L)^2\right)^{1/2}} . \qquad (0.6)$$

From Eq. (0.5), $\omega L = R$ which we can substitute into Eq. (0.6) yielding

$$I_0 = \frac{V_0}{\sqrt{2} R} . \qquad (0.7)$$

Solve Eq. (0.7) for the resistance:

$$R = \frac{V_0}{\sqrt{2} I_0} = \frac{200 \text{ V}}{\sqrt{2}(10 \text{ A})} = 14\,\Omega . \qquad (0.8)$$

(d) We can solve Eq. (0.5) for the inductance:

$$L = \frac{R}{\omega} = \frac{14\,\Omega}{5 \text{ rad} \cdot \text{s}^{-1}} = 2.8 \text{ H} . \qquad (0.9)$$

(e) A sinusoidally driven series $RLC$ circuit is in resonance when it is driven with an angular frequency

$$\omega_{res} = \frac{1}{\sqrt{LC}}. \tag{0.10}$$

We can solve Eq. (0.10) for the capacitance,

$$C = \frac{1}{\omega_{res}^2 L} = \frac{1}{(5\ \mathrm{rad \cdot s^{-1}})^2 (2.8\ \mathrm{H})} = 1.4 \times 10^{-2}\ \mathrm{F}. \tag{0.11}$$