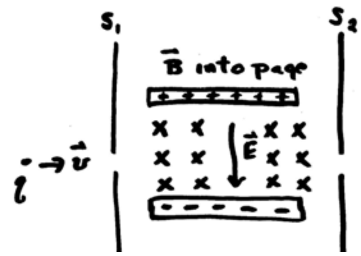
Magnetic Forces
Challenge Problem Solutions

Problem 1:
A particle with charge q and velocity $\vec{\mathbf{v}}$ enters through the hole in screen 1 and passes through a region with non-zero electric and magnetic fields (see sketch). If $q < 0$ and the magnitude of the electric field E is greater than the product of the magnitude of the initial velocity v and the magnitude of the magnetic field B, that is $E > vB$, then the force on the particle



a) is zero and the particle will move in a straight line and pass through the hole on screen 2.

b) is constant and the particle will follow a parabolic trajectory hitting the screen 2 above the hole.

c) is constant and the particle will follow a parabolic trajectory hitting screen 2 below the hole.

d) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 above the hole.

e) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 below the hole.

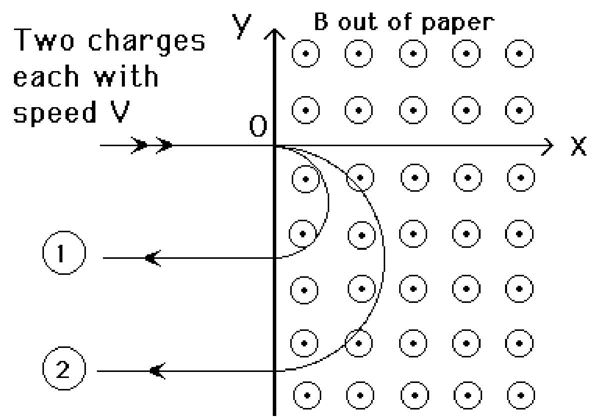
f) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 above the hole.

g) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 below the hole.

Problem 1 Solution:

f. When the particle enters the region where the fields are non-zero, the electric force is points upwards for a negatively charged particle and is greater in magnitude then the downward magnetic force. Both electric and magnetic forces are perpendicular to the particle's velocity and the particle starts to curve upwards. The electric force is always upwards but the magnetic force changes direction as the particle moves along a curved trajectory, so the direction of the force changes. It turns out that the magnitude of the force while the particle is between the plates is constant but does not point to a central point so the trajectory of the particle is not circular. Assuming that the time it takes the particle to cross the plates is smaller than $-\pi m / qB$, when the particle leaves the region between the plates the slope of the trajectory of the particle points upward, and so the particle will strike screen 2 above the hole. Because the fields in this region outside the plates are now zero, the force is zero so the magnitude of the force has changed.

Problem 2:
The entire x-y plane to the right of the origin O is filled with a uniform magnetic field of magnitude B pointing out of the page, as shown. Two charged particles travel along the negative x axis in the positive x direction, each with velocity $\vec{v}$, and enter the magnetic field at the origin O. The two particles have the same mass m, but have different charges, q_1 and q_2. When in the magnetic field, their trajectories both curve in the same direction (see sketch), but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly twice as big as the radius of the semi-circle traced out by particle 1.



(a) Are the charges of these particles positive or negative? Explain your reasoning.

(b) What is the ratio q_2 / q_1?

Problem 2 Solution:
(a)Because $\vec{F}_B = q\vec{v} \times \vec{B}$, the charges of these particles are POSITIVE.

(b) We first find an expression for the radius R of the semi-circle traced out by a particle with charge q in terms of q, $v \equiv |\vec{v}|$, B, and m. The magnitude of the force on the charged particle is qvB and the magnitude of the acceleration for the circular orbit is v^2 / R. Therefore applying Newton's Second Law yields

$$qvB = \frac{mv^2}{R}.$$

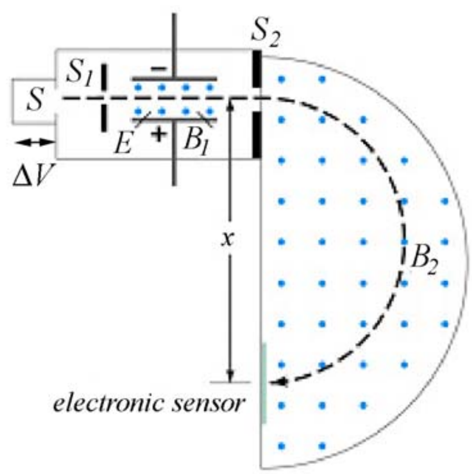
We can solve this for the radius of the circular orbit

$$R = \frac{mv}{qB}$$

Therefore the charged ratio

$$\frac{q_2}{q_1} = \left(\frac{mv}{R_2 B}\right) \Bigg/ \left(\frac{mv}{R_1 B}\right) = \frac{R_1}{R_2}.$$

Problem 3:
Shown below are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass m and charge $q = +e$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, S_1, in which there is an adjustable magnetic field $\vec{\mathbf{B}}_1$, pointing out of the page and a deflecting electric field $\vec{\mathbf{E}}$, pointing from positive to negative plate. Only particles of a uniform velocity $\vec{\mathbf{v}}$ leave the selector. The emerging particles at S_2, enter a second magnetic field $\vec{\mathbf{B}}_2$, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance x from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{\mathbf{E}}|$, e, x, m, $B_2 \equiv |\vec{\mathbf{B}}_2|$, and ΔV.



a) What magnetic field $\vec{\mathbf{B}}_1$ in the selector chamber is needed to insure that the particle travels straight through?

b) Find an expression for the mass of the particle after it has hit the electronic sensor at a distance x from the entry slit

Problem 3 Solution:

(a) We first find an expression for the speed of the particle after it is accelerated by the potential difference ΔV, in terms of m, e, and ΔV. The change in kinetic energy is $\Delta K = (1/2)mv^2$. The change in potential energy is $\Delta U = -e\Delta V$ From conservation of energy, $\Delta K = -\Delta U$, we have that

$$(1/2)mv^2 = e\Delta V .$$

So the speed is

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Inside the selector the force on the charge is given by

$$\vec{\mathbf{F}}_e = e(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_1).$$

If the particle travels straight through the selector then force on the charge is zero, therefore

$$\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}_1.$$

Since the velocity is to the right in the figure above (define this as the $+\hat{\mathbf{i}}$ direction), the electric field points up (define this as the $+\hat{\mathbf{j}}$ direction) from the positive plate to the negative plate, and the magnetic field is pointing out of the page (define this as the $+\hat{\mathbf{k}}$ direction) . Then

$$E\hat{\mathbf{j}} = -v\hat{\mathbf{i}} \times B_1\hat{\mathbf{k}} = vB_1\hat{\mathbf{j}}.$$

So

$$\vec{\mathbf{B}}_1 = \frac{E}{v}\hat{\mathbf{k}} = \sqrt{\frac{m}{2e\Delta V}} E\,\hat{\mathbf{k}}$$

(b) The force on the charge when it enters the magnetic field $\vec{\mathbf{B}}_2$ is given by

$$\vec{\mathbf{F}}_e = ev\hat{\mathbf{i}} \times B_2\hat{\mathbf{k}} = -evB_2\hat{\mathbf{j}}.$$

This force points downward and forces the charge to start circular motion. You can verify this because the magnetic field only changes the direction of the velocity of the particle and not its magnitude which is the condition for circular motion. When in circular motion the acceleration is towards the center. In particular when the particle just enters the field $\vec{\mathbf{B}}_2$, the acceleration is downward

$$\vec{\mathbf{a}} = -\frac{v^2}{x/2}\hat{\mathbf{j}}.$$

Newton's Second Law becomes

$$-evB_2\hat{\mathbf{j}} = -m\frac{v^2}{x/2}\hat{\mathbf{j}}.$$

Thus the particle hits the electronic sensor at a distance

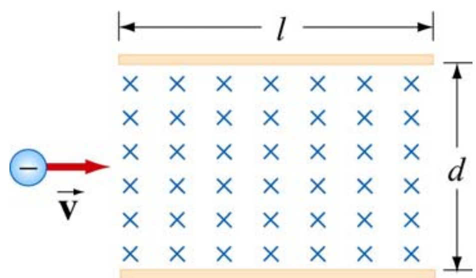
$$x = \frac{2mv}{eB_2} = \frac{2}{eB_2}\sqrt{2e\Delta V m}$$

from the entry slit. The mass of the particle is then

$$m = \frac{eB_2^{\,2}x^2}{8\Delta V} .$$

Problem 4:

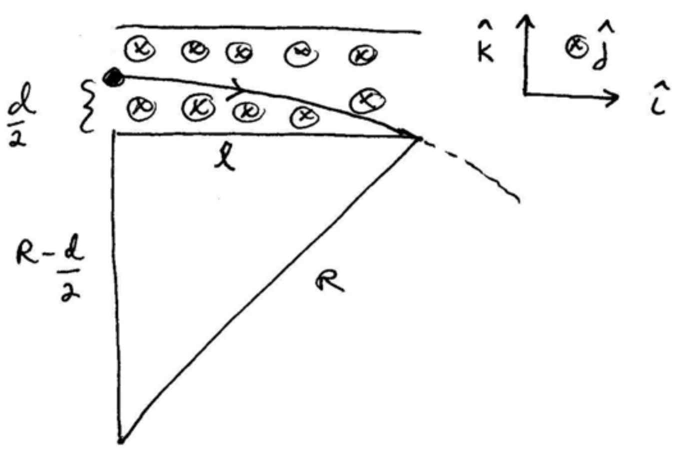
particle of charge $-e$ is moving with an initial velocity $\vec{\mathbf{v}}$ when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in the figure below. You may ignore effects of the gravitational force.



(a) Is the trajectory of the particle deflected upward or downward?

(b) What is the magnitude of the velocity of the particle if it just strikes the end of the plate?

Problem 4 Solution: Choose unit vectors as shown in the figure.



The force on the particle is given by

$$\vec{\mathbf{F}} = -e\left(v\,\hat{\mathbf{i}} \times B\hat{\mathbf{j}}\right) = -evB\hat{\mathbf{k}} .$$

so the direction of the force is downward. Remember that when a charged particle moves through a uniform magnetic field, the magnetic force on the charged particle only changes the direction of the velocity hence leaves the speed unchanged so the particle undergoes circular motion. Therefore we can use Newton's second law in the form

$$evB = m\frac{v^2}{R}.$$

The speed of the particle is then

$$v = \frac{eBR}{m}.$$

.

In order to determine the radius of the orbit we note that the particle just hits the end of the plate. From the figure above, by the Pythagorean theorem, we have that

$$R^2 = (R - d/2)^2 + l^2.$$

.

Expanding the above equation yields

$$R^2 = R^2 - Rd + d^2/4 + l^2$$

.

which we can solve for the radius of the circular orbit:

$$R = \frac{d}{4} + \frac{l^2}{d}.$$

We can now substitute the our result for the radius into our expression for the speed and find the speed necessary for the particle to just hit the end of the plate:

$$v = \frac{eB}{m}\left(\frac{d}{4} + \frac{l^2}{d}\right).$$

Problem 5:

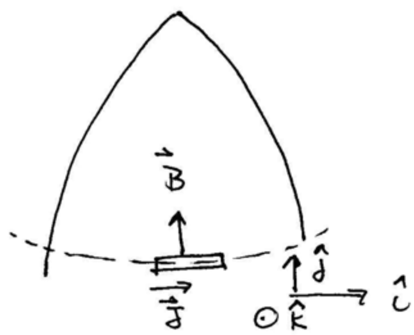
A copper wire of diameter d carries a current density $\vec{\mathbf{J}}$ at the earth's equator where the earth's magnetic field is horizontal, points north, and has magnitude $\left|\vec{\mathbf{B}}_{earth}\right| = 0.5 \times 10^{-4}\ \mathrm{T}$. The wire lies in a plane that is parallel to the surface of the earth and is oriented in the east-west direction. The density of copper is $\rho_{Cu} = 8.9 \times 10^{3}\ \mathrm{kg \cdot m^{-3}}$. The resistivity of copper is $\rho_r = 1.7 \times 10^{-8}\,\Omega \cdot \mathrm{m}$.

a) How large must $\vec{\mathbf{J}}$ be, and which direction must it flow in order to levitate the wire? Use $g = 9.8\,\mathrm{m \cdot s^{-2}}$

b) When the wire is floating how much power will be dissipated per cubic centimeter?

Problem 5 Solution:

At the equator, the magnetic field is pointing north. Choose unit vectors such that $\hat{\mathbf{i}}$ points east, $\hat{\mathbf{j}}$ points north, and $\hat{\mathbf{k}}$ points up. Let $\vec{\mathbf{J}} = J_x\,\hat{\mathbf{i}}$ (with the sign of J_x to be determined), $\vec{\mathbf{B}}_{earth} = B_{earth}\,\hat{\mathbf{j}}$. .



Then the magnetic force $d\vec{\mathbf{F}}_{mag}$ on the a small volume of wire dV_{vol} is

$$d\vec{\mathbf{F}}_{mag} = \vec{\mathbf{J}}dV_{vol} \times \vec{\mathbf{B}}_{earth} = J_x dV_{vol}\,\hat{\mathbf{i}} \times B_{earth}\hat{\mathbf{j}} = J_x B_{earth} dV_{vol}\,\hat{\mathbf{k}}\,.$$

In order to balance the gravitational force this must point upwards hence $J_x > 0$; the current flows from west to east in the wire. The total force on the small element of the wire is zero so

$$\vec{\mathbf{0}} = d\vec{\mathbf{F}}_{grav} + d\vec{\mathbf{F}}_{mag} = \rho_{Cu} dV_{vol}\, g(-\hat{\mathbf{k}}) + J_x B_{earth} dV_{vol}\,\hat{\mathbf{k}}\,.$$

We can solve the above equation for J_x:

$$J_x = \frac{\rho_{Cu} g}{B_{earth}}$$

$$J_x = \frac{(8.9\times10^3\ \mathrm{kg\cdot m^{-3}})(9.8\ \mathrm{m\cdot s^{-2}})}{(0.5\times10^{-4}\ \mathrm{T})} = 1.74\times10^9\ \mathrm{A\cdot m^{-2}}.$$

(b) Let $A = \pi(d/2)^2$ denote the cross-sectional area of the wire. The power dissipated per volume $dV_{vol} = Adl$ where dl is a unit length of wire is given by

$$\frac{P}{dV_{vol}} = \frac{I^2 R}{dV_{vol}}.$$

.

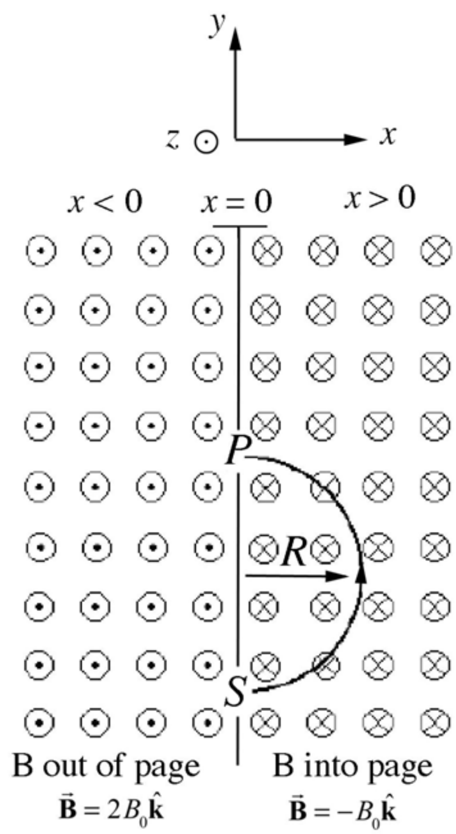
Let The current that flows in the wire is given by is given by $I = J_x A$. The resistance per unit length dl is given by $R = \rho_r dl / A$. So the above equation becomes

$$\begin{aligned}
\frac{P}{dV_{vol}} &= \frac{(J_x A)^2(\rho_r dl/A)}{Adl} = \rho_r J_x{}^2 \\
&= (1.7\times10^{-8}\ \Omega\cdot\mathrm{m})(1.74\times10^9\ \mathrm{A\cdot m^{-2}})^2. \\
&= (5.2\times10^{10}\ \mathrm{W\cdot m^{-3}}) \\
&\cong 50\ \mathrm{kW\cdot cm^{-3}}
\end{aligned}$$

The wire will get very hot!

Problem 6:

The $x-y$ plane for $x<0$ is filled with a uniform magnetic field pointing out of the page, $\vec{\mathbf{B}} = 2B_0\,\hat{\mathbf{k}}$ with $B_0 > 0$, as shown. The $x-y$ plane for $x>0$ is filled with a uniform magnetic field $\vec{\mathbf{B}} = -B_0\,\hat{\mathbf{k}}$, pointing into the page, as shown. A charged particle with mass m and charge q is initially at the point S at $x=0$, moving in the positive x-direction with speed v. It subsequently moves counterclockwise in a circle of radius R, returning to $x=0$ at point P, a distance $2R$ from its initial position, as shown in the sketch.

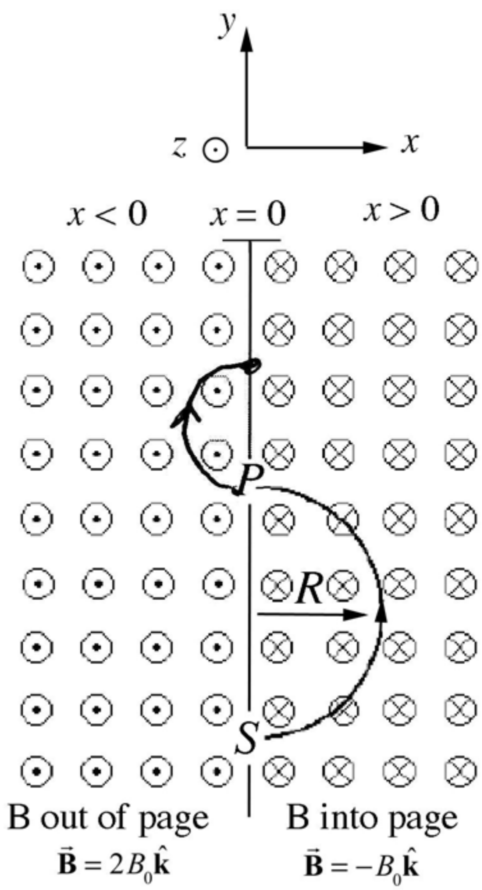


a) Is the charge positive or negative? Briefly explain your reasoning.

b) Find an expression for the radius R of the trajectory shown, in terms of v, m, q, and B_0.

c) How long does the particle take to return to the plane $x=0$ at point P?

d) Describe and sketch the subsequent trajectory of the particle on the figure below after it passes point P. Be sure to define any relevant distances in terms of v, m, q, and B_0.



Problem 6 Solution:

(a) Because the orbit is counterclockwise the force $\vec{F} = q\vec{v}\times\vec{B}$ must point up when the particle is at point S. The $\vec{v}\times\vec{B} = v\hat{i}\times(-B_0\,\hat{k}) = vB_0\,\hat{j}$ points up therefore the charge of the particle must be positive in order for $\vec{F} = q\vec{v}\times\vec{B}$ also to point up.

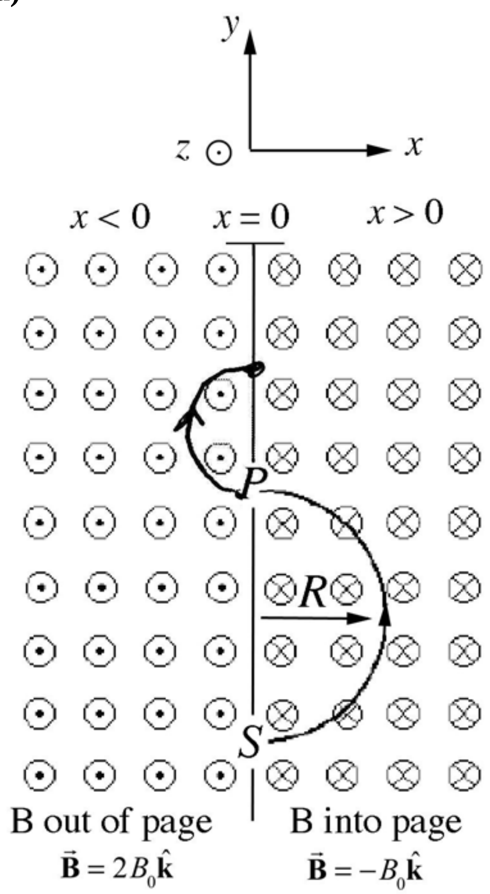
(b) The orbit is circular, so Newton's second Law becomes $qvB_0 = mv^2/R$. Thus the radius of the orbit is

$$R = \frac{mv}{qB_0}.$$

(c) The time t_P it takes the particle to complete a semicircular path from S to P is

$$t_P = \frac{\pi R}{v} = \frac{\pi m}{qB_0}.$$

(d)



When the particle is at point P, the force is still up because both the velocity and the magnetic field now point in opposite directions. Hence

$$\vec{F} = q\vec{v} \times \vec{B} = v(-\hat{i}) \times (2B_0\ \hat{k}) = 2vB_0\ \hat{j}.$$

Newton's Second Law is now $2vB_0 = mv^2 / R_2$. Hence the radius is now

$$R_2 = mv / 2B_0 = R/2.$$

8.02SC Physics II: Electricity and Magnetism
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