Module 22:
Inductance and Magnetic Field Energy

Module 22: Outline

Self Inductance

Energy in Inductors

Circuits with Inductors: RL Circuit

Faraday's Law of Induction

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

Changing magnetic flux *induces* an EMF

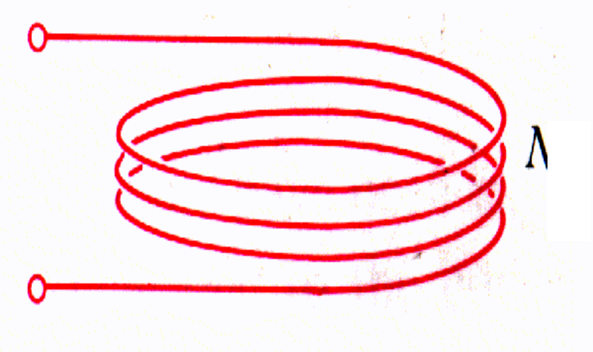
Lenz: Induction ***opposes*** change

Self Inductance

Self Inductance

What if is the effect of putting current into coil 1?

There is “self flux”:

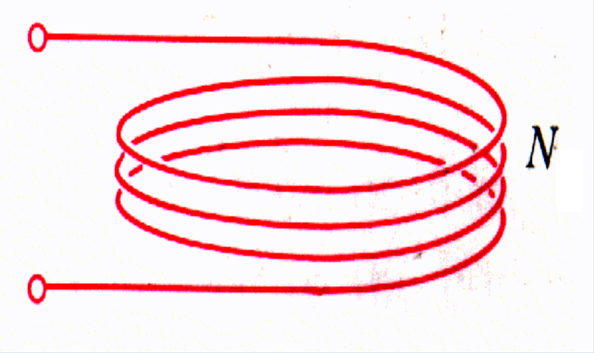


$$\Phi_B \equiv LI$$

Faraday’s Law →

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

Calculating Self Inductance



$$\boxed{L = \frac{\Phi_{B,self}^{total}}{I}}$$

Unit: Henry

$$1\,\mathrm{H} = 1\,\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}$$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

Problem: Solenoid

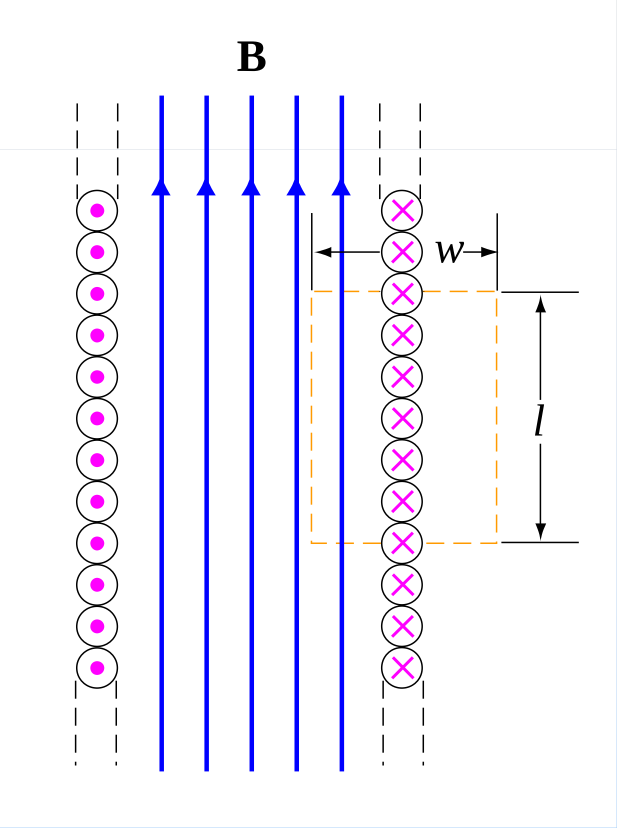
Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ, radius R)

$$L = \frac{\Phi_{B,self}^{total}}{I}$$

REMEMBER

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

Solenoid Inductance



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 I_{enc} = \mu_0 \left(n\ell\right) I$$

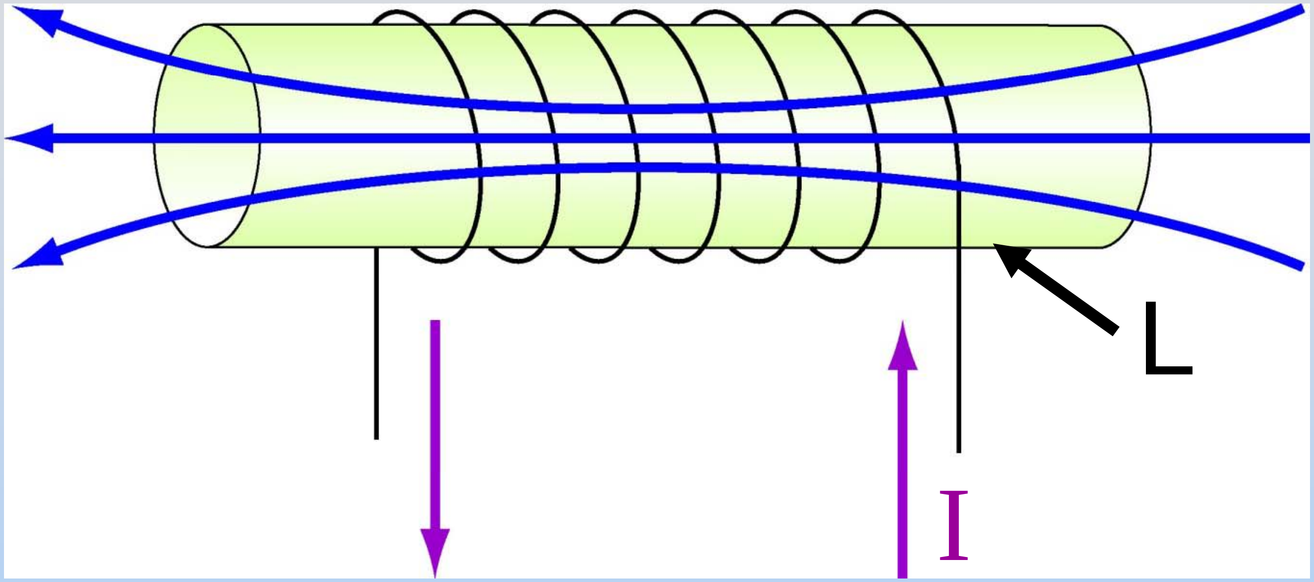
$$B = \mu_0 n I$$

$$\Phi_{B,turn} = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA = \mu_0 n I \pi R^2$$

$$L = \frac{N\Phi_{B,turn}}{I} = N\mu_0 n \pi R^2 = \mu_0 n^2 \pi R^2 l$$

Energy in Inductors

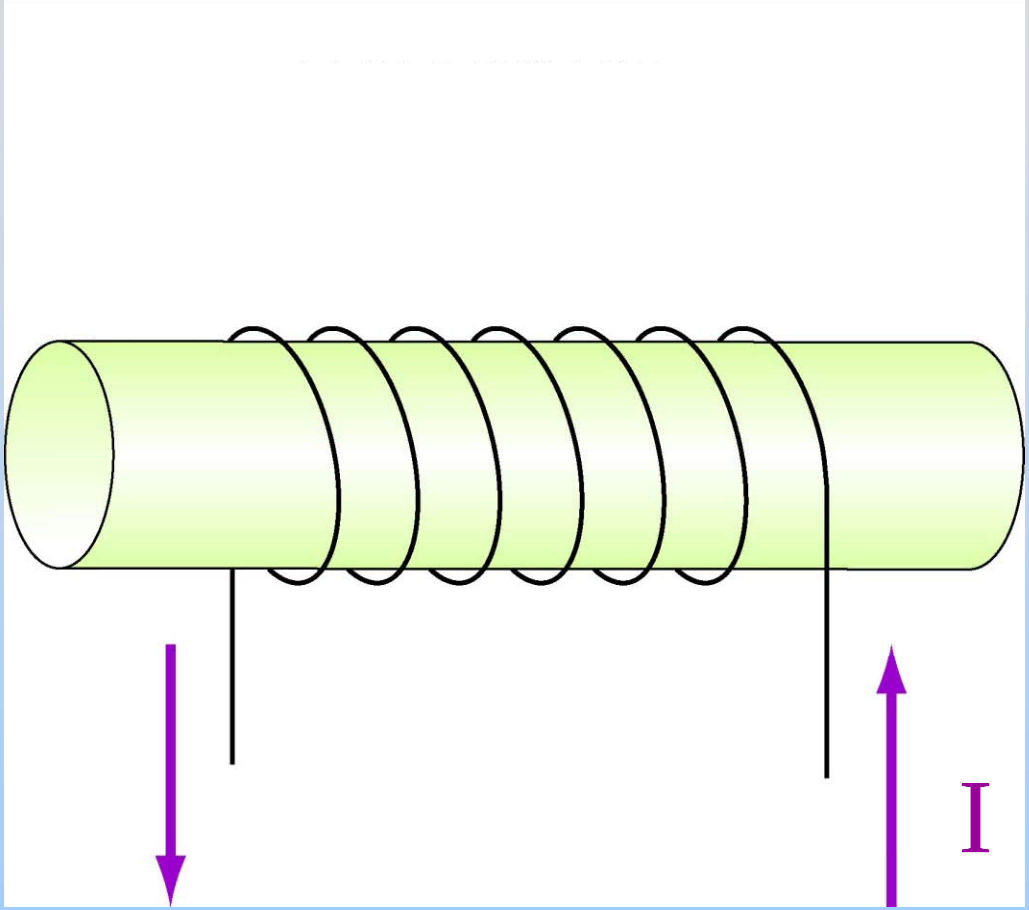
Inductor Behavior



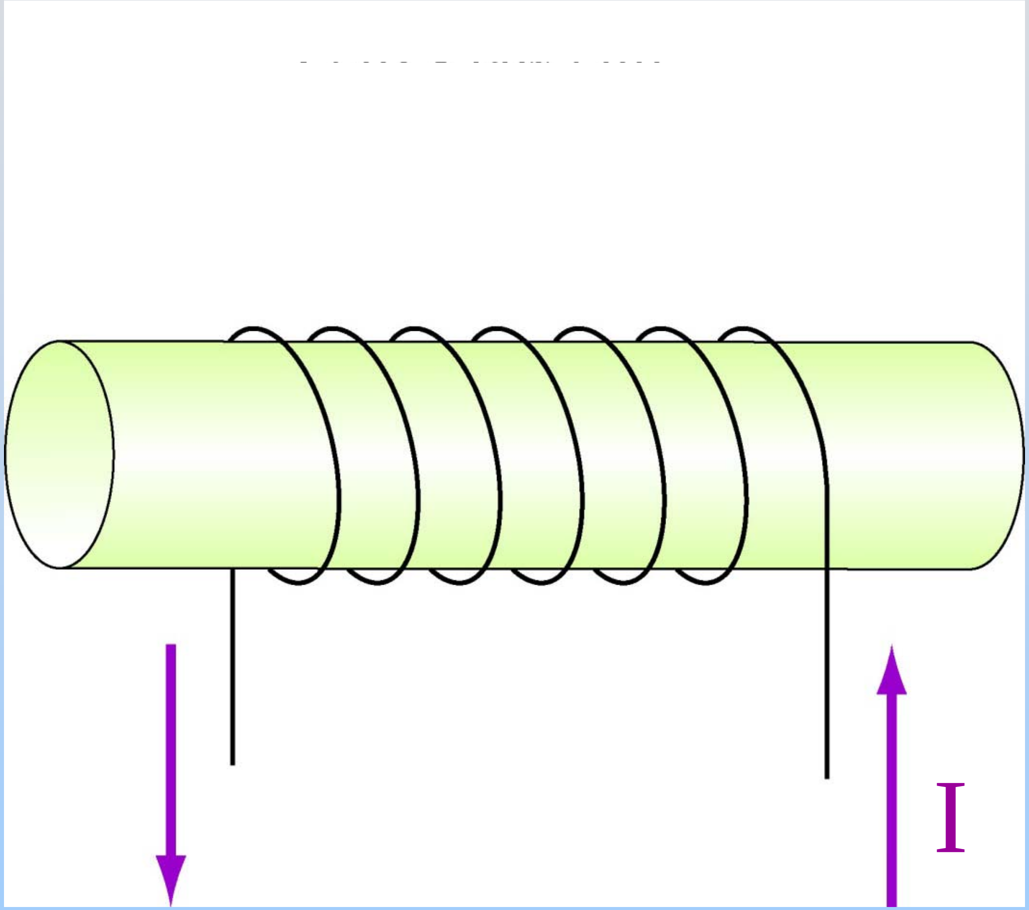
$$\mathcal{E} = -\, L\frac{dI}{dt}$$

Inductor with constant current does nothing

Back EMF $\varepsilon = -L\frac{dI}{dt}$



$$\frac{dI}{dt} > 0,\ \varepsilon_L < 0$$



$$\frac{dI}{dt} < 0,\ \varepsilon_L > 0$$

Energy To “Charge” Inductor

1. Start with “uncharged” inductor
2. Gradually increase current. Must work:

$$dW = P dt = \varepsilon I\, dt = L\frac{dI}{dt} I\, dt = LI\, dI$$

3. Integrate up to find total work done:

$$W = \int dW = \int_{I=0}^{I} LI\, dI = \tfrac{1}{2} L I^2$$

Energy Stored in Inductor

$$\boxed{U_L = \frac{1}{2} L I^2}$$

But where is energy stored?

Example: Solenoid

Ideal solenoid, length *l*, radius *R*, *n* turns/length, current *I:*

$$B = \mu_0 n I \qquad\qquad L = \mu_o n^2 \pi R^2 l$$

$$U_B = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\left(\mu_o n^2 \pi R^2 l\right) I^2$$

$$U_B = \left(\frac{B^2}{2\mu_o}\right) \pi R^2 l$$

Energy Density

Volume

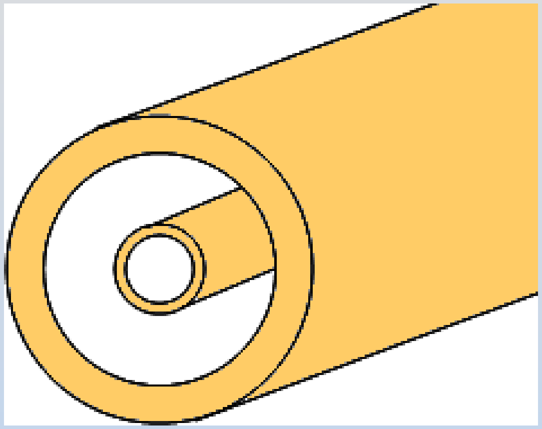
Energy Density

Energy is stored in the magnetic field!

$$\boxed{u_B = \frac{B^2}{2\mu_o}}$$: Magnetic Energy Density

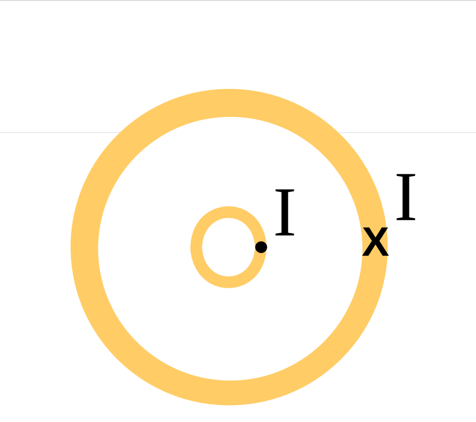
$$u_E = \frac{\varepsilon_o E^2}{2}$$: Electric Energy Density

Problem: Coaxial Cable



Inner wire: $r = a$

Outer wire: $r = b$



1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral
The EASIEST way to do (2) is to use (1)

Technology

Many Applications of
Faraday’s Law

Demos: Breaking Circuits

Big Inductor
Marconi Coil

The Question:
What happens if big ΔI, small Δt

The Point: Big EMF

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

Big L

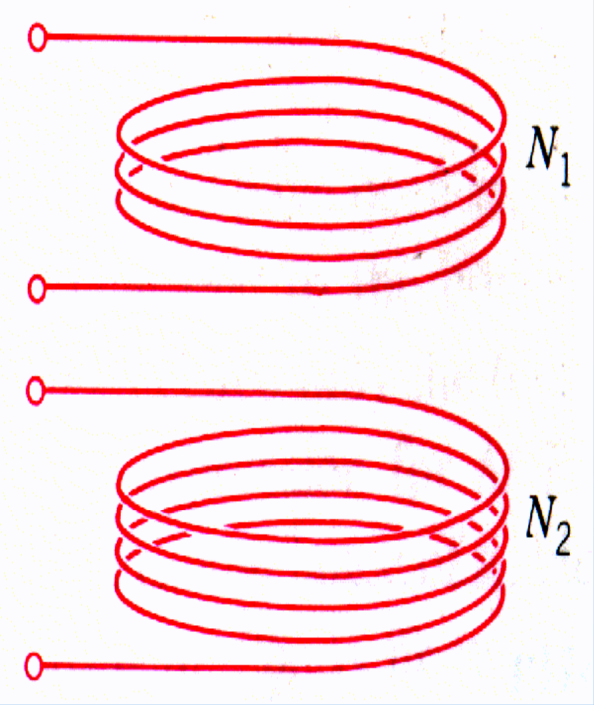
Big dI → Huge ε

Small dt

First:
Mutual Inductance

Demonstration:
Remote Speaker

Mutual Inductance



Current I_2 in coil 2, induces magnetic flux Φ_{12} in coil 1. "Mutual inductance" M_{12}:

$$\Phi_{12} \equiv M_{12} I_2$$

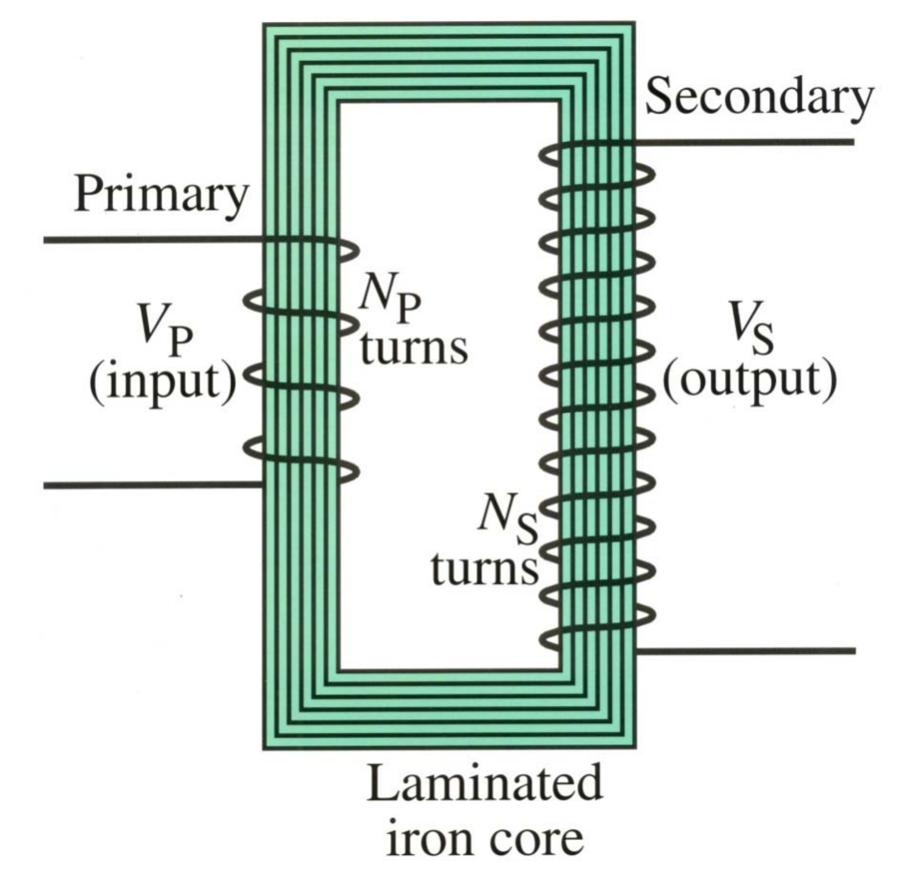
$$M_{12} = M_{21} = M$$

Change current in coil 2? Induce EMF in coil 1:

$$\boxed{\mathcal{E}_{12} \equiv -M_{12} \frac{dI_2}{dt}}$$

Transformer

Step-up transformer



Flux Φ through each turn same:

$$\varepsilon_p - N_p \frac{d\Phi}{dt}; \quad \varepsilon_s - N_s \frac{d\Phi}{dt}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

$N_s > N_p$: step-up transformer

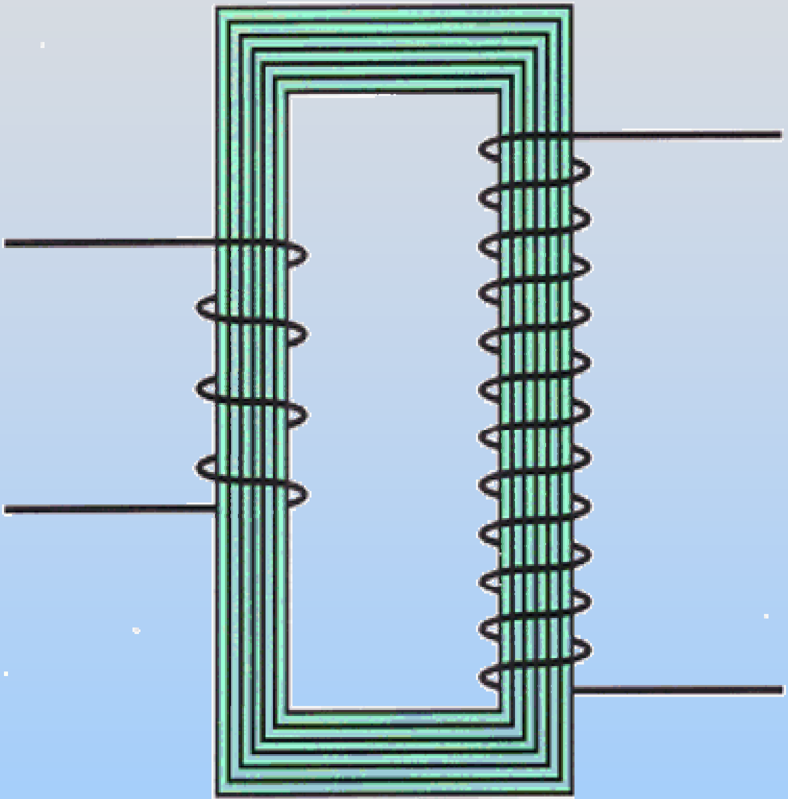
$N_s < N_p$: step-down transformer

Demonstrations:

One Turn Secondary:
Nail

Many Turn Secondary:
Jacob’s Ladder

Concept Question: Residential Transformer

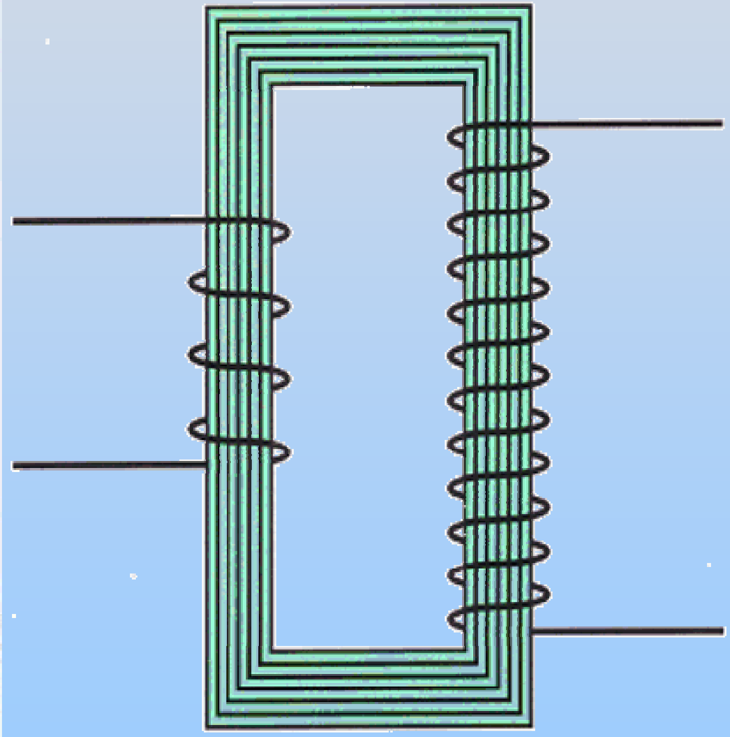


If the transformer in the can looks like the picture, how is it connected?

1. House=Left, Line=Right
2. Line=Left, House=Right
3. I don't know

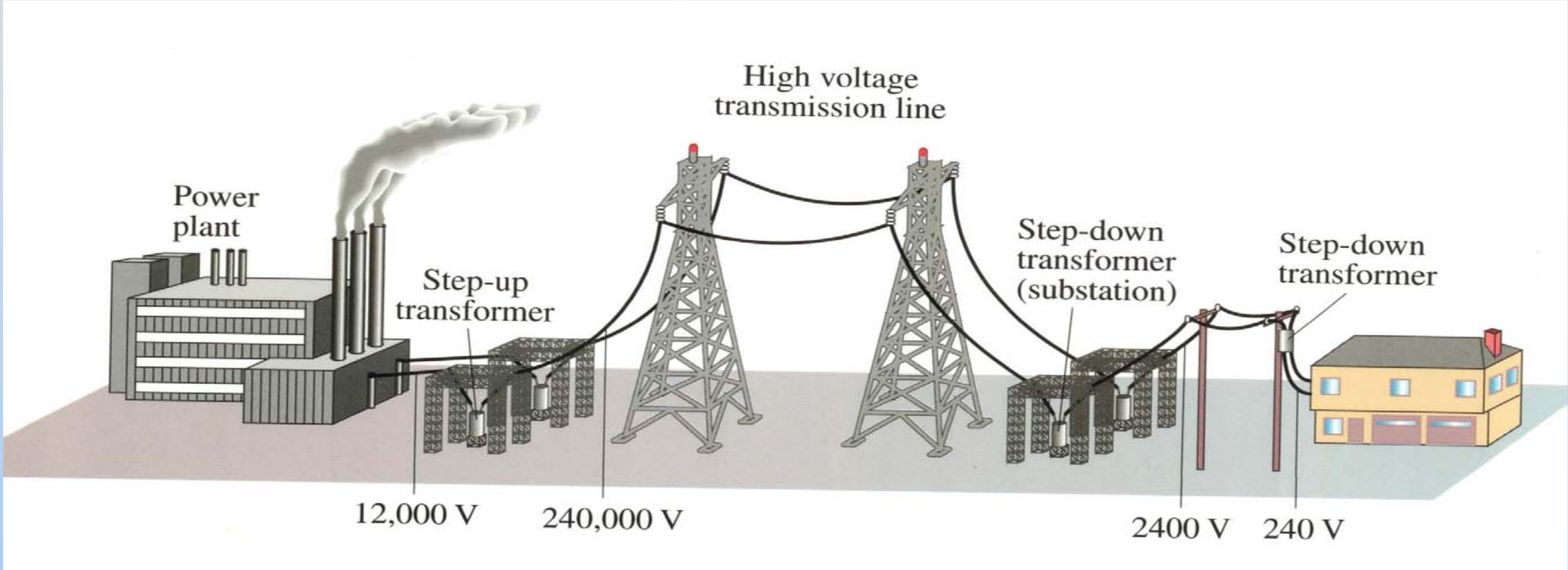
Answer: Residential Transformer

Answer: 1. House on left, line on right



The house needs a lower voltage, so we step down to the house (fewer turns on house side)

Transmission of Electric Power



Power loss can be greatly reduced if transmitted at high voltage

Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^2 V} = 500A$$

83% loss!!

$$P_L = I^2 R = (500A)^2(0.40\Omega) = 100kW$$

(b) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^4 V} = 5.0A$$

0.0083% loss

$$P_L = I^2 R = (5.0A)^2(0.40\Omega) = 10W$$

Transmission lines

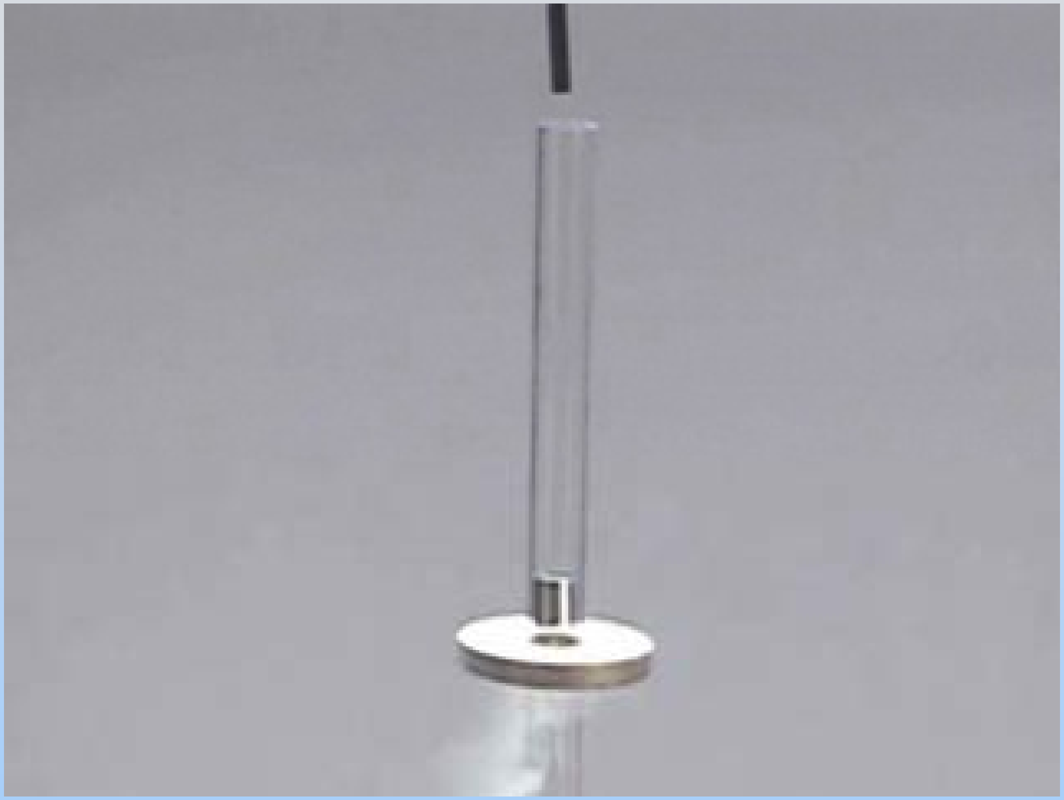
We just calculated that I^2R is smaller for bigger voltages.

What about V^2/R? Isn’t that bigger?

Why doesn’t that matter?

Brakes

Magnet Falling Through a Ring



Link to movie

What happened to kinetic energy of magnet?

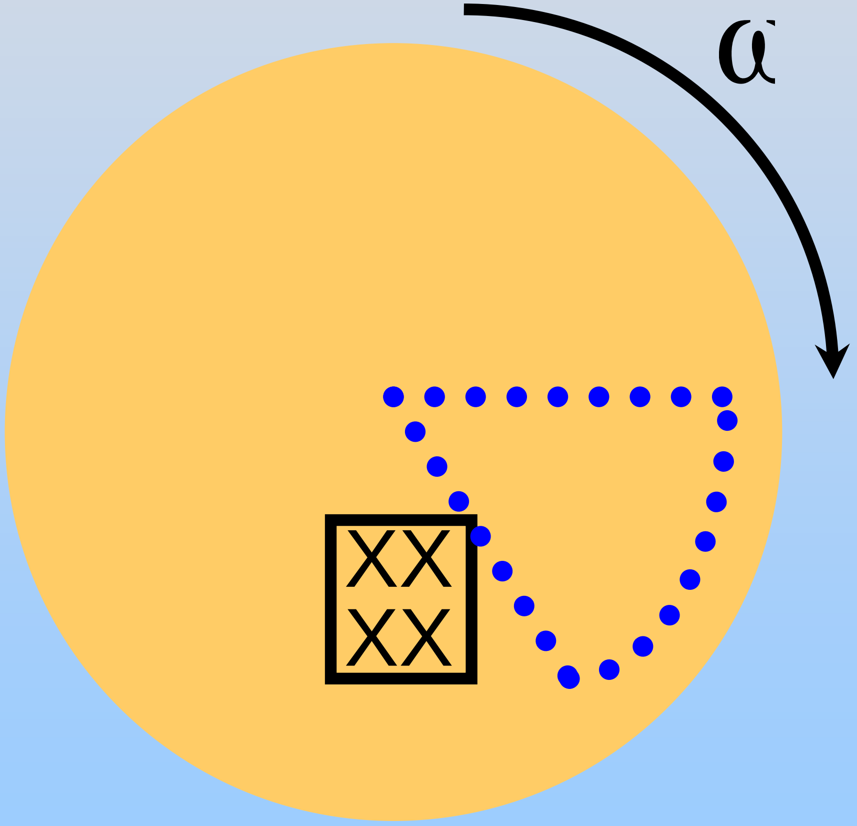
Demonstration:
Eddy Current Braking

Eddy Current Braking

What happened to kinetic energy of disk?

Eddy Current Braking

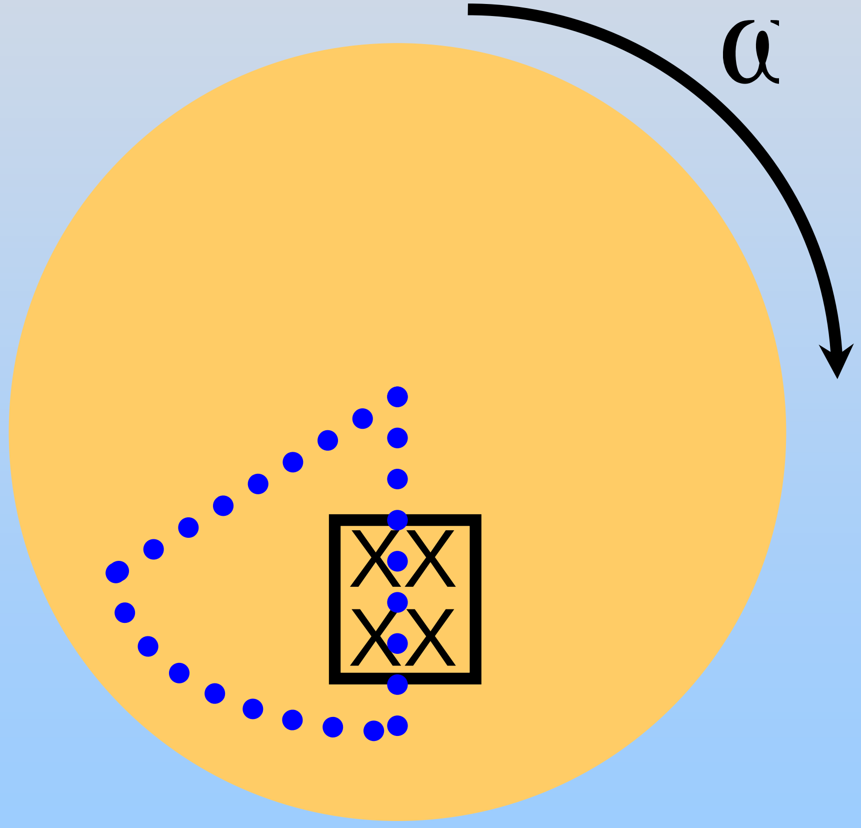
The magnet induces currents in the metal that dissipate the energy through Joule heating:



1. Current is induced counter-clockwise (out from center)
2. Force is opposing motion (creates slowing torque)

Eddy Current Braking

The magnet induces currents in the metal that dissipate the energy through Joule heating:



1. Current is induced clockwise (out from center)
2. Force is opposing motion (creates slowing torque)
3. EMF proportional to ω
4. .

$$F \propto \frac{\mathcal{E}^2}{R}$$

Demonstration: Levitating Magnet

[Link to Movie]

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