Inductance & Magnetic Energy
Challenge Problem Solutions

Problem 1:

A very long solenoid consisting of N turns has radius R and length d ($d >> R$). Suppose the number of turns is halved keeping all the other parameters fixed. The self inductance

a) remains the same.

b) doubles.

c) is halved.

d) is four times as large.

e) is four times as small.

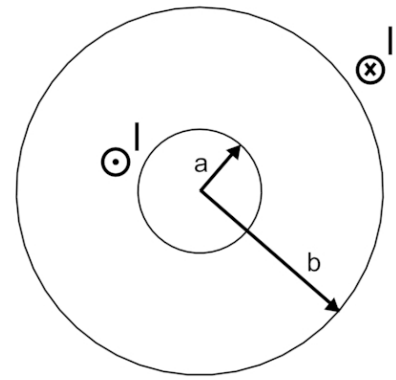
f) None of the above.

Problem 1 Solution:

e. The self-induction of the solenoid is equal to the total flux through the object which is the product of the number of turns time the flux through each turn. The flux through each turn is proportional to the magnitude of magnetic field. By Ampere's Law the magnitude of the magnetic field is proportional to the number of turns per unit length or hence proportional to the number of turns. Hence the self-induction of the solenoid is proportional to the square of the number of turns. If the number of turns is halved keeping all the other parameters fixed then he self inductance is four times as small.

Problem 2:

An inductor consists of two very thin conducting cylindrical shells, one of radius *a* and one of radius *b*, both of length *h*. Assume that the inner shell carries current *I* out of the page, and that the outer shell carries current *I* into the page, distributed uniformly around the circumference in both cases. The *z*-axis is out of the page along the common axis of the cylinders and the *r*-axis is the radial cylindrical axis perpendicular to the *z*-axis.



a) Use Ampere's Law to find the magnetic field between the cylindrical shells. Indicate the direction of the magnetic field on the sketch. What is the magnetic energy density as a function of *r* for $a < r < b$?

b) Calculate the inductance of this long inductor recalling that $U_B = \frac{1}{2}LI^2$ and using your results for the magnetic energy density in (a).

c) Calculate the inductance of this long inductor by using the formula $\Phi = LI = \int_{open\ surface} \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{A}}$ and your results for the magnetic field in (a). To do this you must choose an appropriate open surface over which to evaluate the magnetic flux. Does your result calculated in this way agree with your result in (b)?

Problem 2 Solutions:

(a)The enclosed current I_{enc} in the Ampere's loop with radius *r* is given by

$$I_{\text{enc}} = \begin{cases} 0, & r < a \\ I, & a < r < b \\ 0, & r > b \end{cases}$$

Applying Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{\text{enc}}$, we obtain

$$\vec{\mathbf{B}} = \begin{cases} 0, & r < a \\ \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}}, & a < r < b \quad \text{(counterclockwise in the figure)} \\ 0, & r > b \end{cases}$$

The magnetic energy density for $a < r < b$ is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

It is zero elsewhere.

(b)

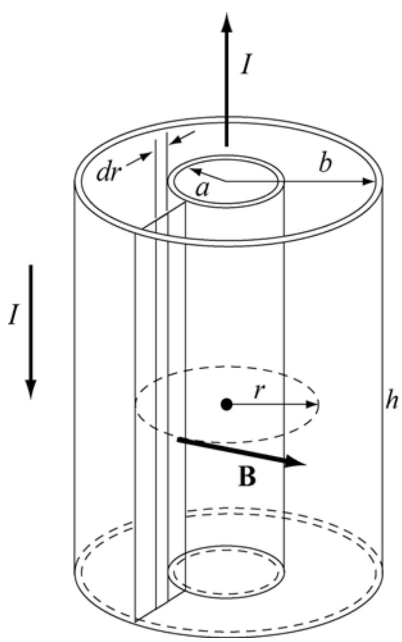
The volume element in this case is $2\pi rhdr$. The magnetic energy is :

$$U_B = \int_V u_B d\text{Vol} = \int_a^b \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right) 2\pi h\, rdr = \frac{\mu_0 I^2 h}{4\pi}\ln\left(\frac{b}{a}\right)$$

Since $U_B = \frac{\mu_0 I^2 l}{4\pi}\ln\left(\frac{b}{a}\right) = \frac{1}{2}LI^2$, the inductance is

$$\boxed{L = \frac{\mu_0 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(c)



The magnetic field is perpendicular to a rectangular surface shown in the figure. The magnetic flux through a thin strip of area $dA = ldr$ is

$$d\Phi_B = BdA = \left(\frac{\mu_0 I}{2\pi r}\right)(h\,dr) = \frac{\mu_0 Ih}{2\pi r}dr$$

Thus, the total magnetic flux is

$$\Phi_B = \int d\Phi_B = \int_a^b \frac{\mu_0 Ih}{2\pi r}dr = \frac{\mu_0 Ih}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 Ih}{2\pi}\ln\left(\frac{b}{a}\right)$$

Thus, the inductance is

$$L = \frac{\Phi_B}{I} = \boxed{\frac{\mu_0 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

which agrees with that obtained in (b).