Energy and Momentum in EM Waves
Challenge Problem Solutions

Problem 1:

As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ….) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations!)

Design a solar observatory. Specifically, we want an observatory that does not have to orbit but rather can just sit still, hovering over the sun. We will balance out gravity with radiation pressure. You need to estimate the mass of the observatory and choose suitable dimensions for it.

Some possibly useful numbers:

$G = 6.67\times10^{-11}\,\text{N m}^2\ \text{kg}^{-2} \qquad c = 3\times10^8\ \text{m/s}$

Sail materials have to be very lightweight. Current materials have areal mass densities of about 1 g/m^2, but proposed materials are projected to be as low as 0.05 g/m^2.
The sun has mass 2 x 10^{30} kg and radius 7 x 10^8 m. It radiates power at a rate of 3.9 x 10^{26} Watts and has a rotation period of about 30 days.

Problem 1 Solution:

We need to at least balance out gravity with solar radiation pressure (we can always reduce the force of radiation pressure by simply tilting the sail) so:

$$F_{\text{radiation}} = PA = \frac{2S}{c}A = \frac{2A}{c}\cdot\frac{P_{\text{sun}}}{4\pi r^2} \overset{?}{\geq} F_{\text{Grav}} = \frac{GM_{\text{sun}}m_{\text{observatory}}}{r^2}$$

Note that the distance from the sun drops out, so we have:

$$\frac{m_{\text{observatory}}}{A} \overset{?}{\leq} \frac{2}{c}\cdot\frac{P_{\text{sun}}}{4\pi GM_{\text{sun}}} = \frac{2}{\left(3\times10^8\ \text{m/s}\right)}\cdot\frac{3.9\times10^{26}\ \text{Watts}}{4\pi\left(6.67\times10^{-11}\,\text{N m}^2\ \text{kg}^{-2}\right)\left(2\times10^{30}\ \text{kg}\right)} \sim 1.5\ \frac{\text{g}}{\text{m}^2}$$

Since this is more dense than the sail material itself, this can work. Let's break the mass of the observatory into the part the equipment is in (which I'll estimate has a mass of about 100 tons) and the sail itself, for which I'll use a pretty realistic mass density of 0.5 g/m^2.

$$\frac{m_{\text{equipment}}}{A} + \frac{m_{\text{sail}}}{A} \leq 1.5\ \frac{\text{g}}{\text{m}^2} \Rightarrow \frac{m_{\text{equipment}}}{A} \leq 1\ \frac{\text{g}}{\text{m}^2} \Rightarrow A \geq \frac{m_{\text{equipment}}}{1\ \text{g/m}^2} = \frac{\left(10^5\ \text{kg}\right)}{10^{-3}\ \text{kg/m}^2} = 10^8\ \text{m}^2$$

This corresponds to a sail that is 10^4 m (10 km) on a side. This is large, but not totally unreasonable. And certainly if I made the payload lighter and used a lighter sail material this number would come down (1 metric ton and 0.05 g/m^2 leads to a sail less than a km on a side.)

Extra Note: We can easily get power from solar power. We might need 100 kW, but even as far away as the Earth we have 1.4 kW/m^2, meaning our sail will catch order 10^{11} Watts! Even if we only absorb a small fraction of this we will still have plenty!

Problem 2:

You have designed a solar space craft of mass *m* that is accelerated by the force due to the 'radiation pressure' from the sun's light that fall on a perfectly reflective circular sail that it is oriented face-on to the sun. The time averaged radiative power of the sun is P_{sun}. The gravitational constant is *G*. The mass of the sun is m_s. The speed of light is c. Model the sun's light as a plane electromagnetic wave, traveling in the +z direction with the electric field given by

$$\vec{\mathbf{E}}(z,t) = E_{x,0}\cos(kz - \omega t)\,\hat{\mathbf{i}}\,.$$

You may express your answer in terms of the symbols m, $\langle P \rangle$, c, m_s, G, k, and ω as necessary.

a) What is the magnetic field $\vec{\mathbf{B}}$ associated with this electric field?

b) What is the Poynting vector $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ associated with this wave? What is the time averaged Poynting vector $\langle \vec{\mathbf{S}} \rangle = \frac{1}{T}\int_0^T \vec{\mathbf{S}}\, dt$ associated with this superposition, where T is the period of oscillation. What is the amplitude of the electric field at your starting point?

c) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun?

Problem 2 Solutions:

(a) $\vec{E} = E_{x,0} \cos(kz - \omega t)\, \hat{i}$

$\vec{B} = \frac{E_{x,0}}{c} \cos(kz - \omega t)\, \hat{j}$

note dir $(\vec{E} \times \vec{B})$ = dir (propagation)

$(\pm \hat{i}) \times (\pm \hat{j}) = \hat{k}$

(b) $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \frac{E_{x0}^2}{c} \cos^2(kz - \omega t)\, \hat{k}$

time averaged Poynting vector

$|\langle \vec{S} \rangle| = \frac{1}{2} \frac{E_{x0}^2}{\mu_0 c}$

time average → $\frac{1}{2}$

time averaged power $\langle P \rangle$ of sun

$\frac{\langle P \rangle}{4\pi r^2} = |\langle \vec{S} \rangle| = \frac{1}{2} \frac{E_{x0}^2}{\mu_0 c}$

$\Rightarrow \quad E_{x,0} = \left(\frac{2 \langle P \rangle \mu_0 c}{4\pi r^2} \right)^{1/2}$

(c) radiation pressure for a perfectly reflecting sail

$\langle P_{rad} \rangle = \frac{2 |\langle \vec{S} \rangle|}{c} = \frac{|\langle \vec{F}_{rad} \rangle|}{\text{Area}} \quad \Rightarrow$

$|\langle \vec{F}_{rad} \rangle| = \frac{2 \langle |\vec{S}| \rangle}{c} \text{Area} = \frac{2 \langle P \rangle}{4\pi r^2 c} (\text{Area})$

$$\vec{F}_{rad} + \vec{F}_{grav} = m\frac{d^2\hat{r}}{dt^2}$$

when $|\vec{F}_{rad}| = |\vec{F}_{grav}|$

$$\frac{2\langle P\rangle (\text{Area})_{min}}{4\pi r^2 c} = \frac{G m m_s}{r^2}$$

$$\Rightarrow \quad (\text{Area})_{min} = \frac{G m m_s \, 4\pi c}{2\langle P\rangle}$$

$$c = \frac{\omega}{k} \quad \Rightarrow$$

$$(\text{Area})_{min} = \frac{G m m_s \, 2\pi}{\langle P\rangle} \frac{\omega}{k}$$

Problem 3:

Consider a plane electromagnetic wave with the electric and magnetic fields given by

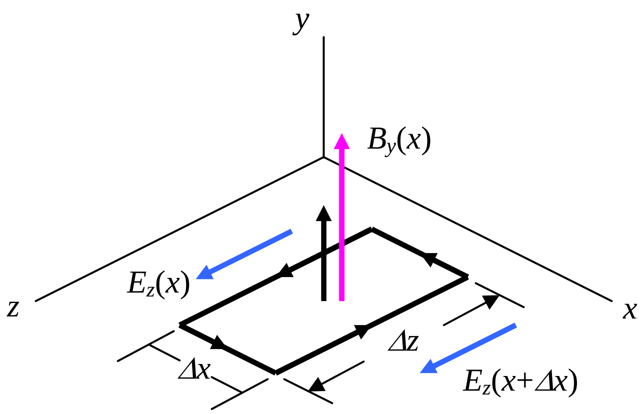
$$\vec{\mathbf{E}}(x,t) = E_z(x,t)\hat{\mathbf{k}}, \quad \vec{\mathbf{B}}(x,t) = B_y(x,t)\hat{\mathbf{j}}$$

Applying arguments similar to that presented in Section 13.4 of the *Course Notes*, show that the fields satisfy the following relationships:

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}, \quad \frac{\partial B_y}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t}$$

Problem 3 Solutions:

Consider a rectangular loop in the *xz* plane depicted in the figure below, with a unit normal $\hat{n} = \hat{j}$.



Using Faraday's law

$$\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

the left-hand-side can be written as

$$\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = E_z(x)\Delta z - E_z(x+\Delta x)\Delta z = -\left[E_z(x+\Delta x) - E_z(x)\right]\Delta z = -\frac{\partial E_z}{\partial x}\Delta x \Delta z$$

where we have made the expansion

$$E_z(x+\Delta x) = E_z(x) + \frac{\partial E_z}{\partial x}\Delta x + \ldots$$

On the other hand, the rate of change of magnetic flux on the right-hand-side is given by

$$-\frac{d}{dt}\iint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = -\left(\frac{\partial B_y}{\partial t}\right)\Delta x \Delta z$$

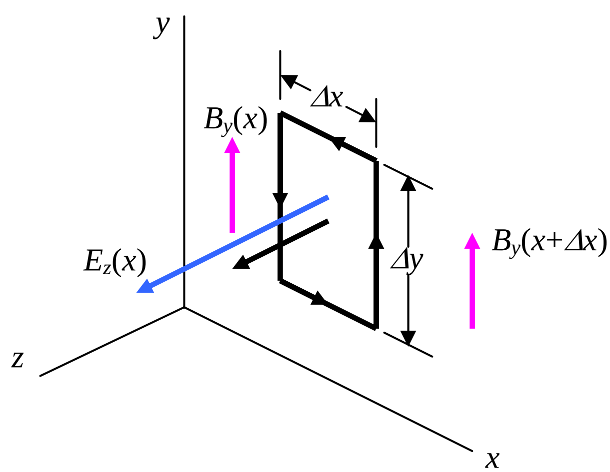
Equating the two expressions and dividing through by the area $\Delta x \Delta z$ yields

$$\boxed{\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}}$$

The second condition on the relationship between the electric and magnetic fields may be deduced by using the Ampere-Maxwell equation:

$$\oint \vec{\mathbf{B}}\cdot d\,\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt}\iint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$$

Consider a rectangular loop in the *xy* plane depicted in the figure below, with a unit normal $\hat{n} = \hat{k}$.



The line integral of the magnetic field is

$$\oint \vec{\mathbf{E}}\cdot d\,\vec{\mathbf{s}} = B_y(x+\Delta x)\Delta y - B_y(x)\Delta y = \left[B_y(x+\Delta x) - B_y(x)\right]\Delta y = \left(\frac{\partial B_y}{\partial x}\right)\Delta x \Delta y$$

On the other hand, the time derivative of the electric flux is

$$\mu_0 \varepsilon_0 \frac{d}{dt}\iint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \mu_0 \varepsilon_0 \left(\frac{\partial E_z}{\partial t}\right)\Delta x \Delta y$$

Equating the two equations and dividing by $\Delta x \Delta y$, we have

$$\boxed{\frac{\partial B_y}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t}}$$

Problem 4:

The magnetic field of a plane electromagnetic wave is described as follows:

$$\vec{\mathbf{B}} = B_0 \sin(kx - \omega t)\,\hat{\mathbf{j}} \tag{0.1}$$

a) What is the wavelength λ of the wave?

b) Write an expression for the electric field $\vec{\mathbf{E}}$ associated to this magnetic field. Be sure to indicate the direction with a unit vector and an appropriate sign (+ or –).

c) What direction is this wave moving?

d) What is the direction and magnitude Poynting vector associated with this wave? Give appropriate units, as well as magnitude.

e) This wave is totally reflected by the thin perfectly conducting sheet lying in the *y*-*z* plane at $x = 0$. What is the resulting radiation pressure on the sheet? Give appropriate units, as well as magnitude.

f) The component of an electric field parallel to the surface of an ideal conductor must be zero. Using this fact, find expressions for the electric and magnetic fields for the reflected wave. Check that the sum of your transmitted and reflected wave must satisfies the condition that the electric field is zero at the conducting sheet (located at $x = 0$).

Problem 4 Solutions:

(a) The wavelength is given by

$$\lambda = \frac{2\pi}{k} \tag{0.2}$$

(b) The amplitude of the electric field is related to the amplitude of the magnetic field by

$$E_0 = cB_0 \tag{0.3}$$

The direction of the electric field, magnetic field, and propagation direction are related by

$$dir(\vec{\mathbf{E}} \times \vec{\mathbf{B}}) = dir(propagation) \tag{0.4}$$

Since $-\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$, the electric field is given by

$$\vec{\mathbf{E}} = cB_0 \sin(kx - \omega t)(-\hat{\mathbf{k}}) \tag{0.5}$$

Alternatively, the differential version of the Generalized Ampere's Law for a magnetic field that has a y-component that is only a function of x and t (as given in Eq. (0.1)) is

$$\frac{\partial B_y}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t}, \tag{0.6}$$

Noting that

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}, \tag{0.7}$$

Eq. (0.6) can be integrated with respect to time to find the electric field

$$E_z = c^2 \int \frac{\partial B_y}{\partial x} dt \,. \tag{0.8}$$

The partial derivative in the integrand can be calculated using Eq. (0.1),

$$\frac{\partial B_y}{\partial x} = kB_0 \cos(kx - \omega t) \,. \tag{0.9}$$

Substituting Eq. (0.9) into the integrand in Eq. (0.8) and integrated yields

$$E_z = c^2 \int kB_0 \cos(kx - \omega t) dt = -c^2 \frac{k}{\omega} B_0 \sin(kx - \omega t) \,. \tag{0.10}$$

Finally,

$$\frac{1}{c} = \frac{k}{\omega} \,. \tag{0.11}$$

Substituting Eq. (0.11) into Eq. (0.10) yields

$$E_z = -cB_0 \sin(kx - \omega t) \tag{0.12}$$

In agreement with Eq. (0.5) for the z-component for the electric field.

(c) The wave is moving in the positive x-direction.

(d) The Poynting vector is given by

$$\vec{\mathbf{S}} = \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}} \,. \tag{0.13}$$

Substituting Eq. (0.5) and Eq. (0.1) into Eq. (0.13) yields

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\left(cB_0 \sin(kx-\omega t)(-\hat{\mathbf{k}})\right)\times\left(B_0 \sin(kx-\omega t)\,\hat{\mathbf{j}}\right) = \frac{cB_0^2}{\mu_0}\sin^2(kx-\omega t)\,\hat{\mathbf{i}} \qquad (0.14).$$

The magnitude of the Poynting vector is

$$\left|\vec{\mathbf{S}}\right| = \frac{cB_0^2}{\mu_0}\sin^2(kx-\omega t) \qquad (0.15)$$

and the units are $[\vec{\mathbf{S}}] = [\mathrm{W}\cdot\mathrm{m}^{-2}]$. The time averaged Poynting vector is then

$$\left\langle\vec{\mathbf{S}}\right\rangle = \frac{1}{2}\frac{cB_0^2}{\mu_0}\hat{\mathbf{i}}\,. \qquad (0.16)$$

Suppose you place a transmitter some distance in front of a perfectly conducting sheet, oriented so that the propagation direction of the waves hitting the reflector is perpendicular to the plane of the reflector (so that they'll reflect straight back out towards the transmitter). Assume that the wave is a plane wave with magnetic field given by Eq. (0.1) and has an electric field which you found in part a).

(e) Since the electric field is totally reflected, the magnitude of the radiation pressure is given by (use Eq. (0.16) for the time averaged Poynting vector)

$$P_{pressure} = \frac{2\left|\left\langle\vec{\mathbf{S}}\right\rangle\right|}{c} = \frac{B_0^2}{\mu_0} \qquad (0.17)$$

and the units are

$$[P_{pressure}] = \left[\frac{\mathrm{W}\cdot\mathrm{m}^{-2}}{\mathrm{m}\cdot\mathrm{s}^{-1}}\right] = \left[\frac{\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}\cdot\mathrm{m}^{-2}}{\mathrm{m}\cdot\mathrm{s}^{-1}}\right] = [\mathrm{N}\cdot\mathrm{m}^{-2}] \qquad (0.18)$$

(f) The incident electric field will reflect from the sheet. The electric field is the linear superposition of the incident field and the reflected field,

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{incident} + \vec{\mathbf{E}}_{reflected}\,. \qquad (0.19)$$

Substituting Eq. (0.5) for the incident electric field into Eq. (0.19) yields

$$\vec{\mathbf{E}} = cB_0 \sin(kx-\omega t)(-\hat{\mathbf{k}}) + \vec{\mathbf{E}}_{reflected}\,. \qquad (0.20)$$

In particular, the electric field must vanish on the conducting sheet located at $x = 0$, (recall that for a perfect conductor the electric field tangent to the surface is zero).

$$\vec{\mathbf{E}}(x=0) = \vec{\mathbf{0}} \tag{0.21}$$

Using the condition set by Eq. (0.21) in Eq. (0.20) yields

$$\vec{\mathbf{E}}(x=0) = \vec{\mathbf{0}} = cB_0 \sin(-\omega t)\,(-\hat{\mathbf{k}}) + \vec{\mathbf{E}}_{reflected}\,. \tag{0.22}$$

Thus we can solve Eq. (0.22) for the reflected wave at $x = 0$, noting that $\sin(-\omega t) = -\sin(\omega t)$

$$\vec{\mathbf{E}}_{reflected}\,(x=0) = cB_0 \sin(\omega t)\,(-\hat{\mathbf{k}})\,. \tag{0.23}$$

Since the reflected wave is traveling in the negative-x direction, the reflected electric field everywhere is given by

$$\vec{\mathbf{E}}_{reflected} = cB_0 \sin(kx + \omega t)\,(-\hat{\mathbf{k}})\,. \tag{0.24}$$

The associated reflected magnetic field is given by

$$\vec{\mathbf{B}}_{reflected} = B_0 \sin(kx + \omega t)\,(-\hat{\mathbf{j}})\,. \tag{0.25}$$

We can substitute Eq. (0.24) into Eq. (0.20) to fins that the sum of the incident and reflected electric field is

$$\vec{\mathbf{E}} = cB_0 (\sin(kx - \omega t) + \sin(kx + \omega t))(-\hat{\mathbf{k}})\,. \tag{0.26}$$

Similarly, the sum of the incident and reflected magnetic fields is then

$$\vec{\mathbf{B}} = B_0 (-\sin(kx - \omega t) + \sin(kx + \omega t))\,(-\hat{\mathbf{j}})\,. \tag{0.27}$$

On the plane $x = 0$, Eq. (0.26) and Eq. (0.27) become respectively,

$$\vec{\mathbf{E}}(x=0) = \vec{\mathbf{0}} \tag{0.28}$$

and

$$\vec{\mathbf{B}}(x=0) = 2B_0 \sin(\omega t)\,(-\hat{\mathbf{j}})\,. \tag{0.29}$$

Problem 5:

The electric field of an electromagnetic wave is given by the superposition of two waves

$$\vec{\mathbf{E}} = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}} \,. \tag{0.30}$$

You may find the following identities useful

$$\cos(kz + \omega t) = \cos(kz)\cos(\omega t) - \sin(kz)\sin(\omega t) \tag{0.31}$$

$$\sin(kz + \omega t) = \sin(kz)\cos(\omega t) + \cos(kz)\sin(\omega t) \,. \tag{0.32}$$

a) What is the associated magnetic field $\vec{\mathbf{B}}(x, y, z, t)$?

b) What is the energy per unit area per unit time (the Poynting vector $\vec{\mathbf{S}}$) transported by this wave?

c) What is the time average of the Poynting $\langle \vec{\mathbf{S}} \rangle$ vector? Explain your answer, (note: you may be surprised by your answer, but try to explain it). Recall that the time average of the Poynting vector is given by

$$\langle \vec{\mathbf{S}} \rangle \equiv \frac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt \,. \tag{0.33}$$

Problem 5 Solution:

(a) The electric field in Eq. (0.30) is the superposition of two traveling waves, one in the positive z-direction and the other in the negative z-direction. The amplitude of the associated magnetic field is diminished by a factor of $1/c$. By the right hand rule, the magnetic field associated with the wave traveling in the positive z-direction points in the $+\hat{\mathbf{j}}$ (when the cos factor is positive), and the magnetic field associated with the wave traveling in the negative z-direction points in the $-\hat{\mathbf{j}}$ (when the cos factor is positive). Hence the magnetic field is given by

$$\vec{\mathbf{B}} = \frac{E_0}{c}(\cos(kz - \omega t) - \cos(kz + \omega t))\,\hat{\mathbf{j}} \tag{0.34}$$

Using the identity in Eq. (0.31), Eq. (0.34) can be rewritten as

$$\begin{aligned}\vec{\mathbf{B}} &= \frac{E_0}{c}(\cos(kz-\omega t)-\cos(kz+\omega t))\,\hat{\mathbf{j}} \\ &= \frac{E_0}{c}\{[\cos(kz)\cos(\omega t)+\sin(kz)\sin(\omega t)]-[\cos(kz)\cos(\omega t)-\sin(kz)\sin(\omega t)]\}\,(\hat{\mathbf{j}}) \\ &= \frac{2E_0}{c}\sin(kz)\sin(\omega t)\hat{\mathbf{j}}\end{aligned} \quad (0.35)$$

In a similar fashion, using Eq. (0.31), the electric field in Eq. (0.30) can be rewritten as

$$\begin{aligned}\vec{\mathbf{E}} &= E_0(\cos(kz-\omega t)+\cos(kz+\omega t))\,\hat{\mathbf{i}} \\ &= E_0\{[\cos(kz)\cos(\omega t)+\sin(kz)\sin(\omega t)]+[\cos(kz)\cos(\omega t)-\sin(kz)\sin(\omega t)]\}\,(\hat{\mathbf{i}}) \\ &= 2E_0\cos(kz)\cos(\omega t)\hat{\mathbf{i}}\end{aligned} \quad (0.36)$$

Equivalently, Faraday's Law in differential form for the plane waves traveling in the $\pm z$ -direction is given by

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \quad (0.37)$$

Eq. (0.37) can be integrated with respect to time to find the electric field

$$B_y = -\int \frac{\partial E_x}{\partial z}\, dt\ . \quad (0.38)$$

The partial derivative in the integrand can be calculated using Eq. (0.36),

$$\frac{\partial E_x}{\partial z} = -2kE_0\sin(kz)\cos(\omega t)\,. \quad (0.39)$$

Substituting Eq. (0.39)into the integrand in Eq. (0.38)and integrated yields

$$B_y = 2kE_0\sin(kz)\int\cos(\omega t)dt = \frac{2kE_0}{\omega}\sin(kz)\sin(\omega t)\,. \quad (0.40)$$

Finally,

$$\frac{1}{c} = \frac{k}{\omega}\,. \quad (0.41)$$

Substituting Eq. (0.41)into Eq. (0.40)yields

$$B_y = \frac{2E_0}{c}\sin(kz)\sin(\omega t)\,, \quad (0.42)$$

in agreement with Eq. (0.35) for the y-component of the magnetic field.

(b) The Poynting vector is given by

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}. \qquad (0.43)$$

Substituting Eq. (0.35) and Eq. (0.36) into Eq. (0.43) yields

$$\begin{aligned}\vec{\mathbf{S}} &= \frac{1}{\mu_0}2E_0\cos(kz)\cos(\omega t)\hat{\mathbf{i}}\times\frac{2E_0}{c}\sin(kz)\sin(\omega t)\hat{\mathbf{j}} \\ &= \frac{E_o^2}{c\mu_0}(2\cos(kz)\sin(kz))(2\cos(\omega t)\sin(\omega t))\,\hat{\mathbf{k}} \quad .\end{aligned} \qquad (0.44)$$

Recall the identity

$$2\cos(a)\sin(a)) = \sin(2a). \qquad (0.45)$$

Use Eq. (0.45) twice in Eq. (0.44) yields

$$\vec{\mathbf{S}} = \frac{E_o^2}{c\mu_0}\sin(2kz)\sin(2\omega t)\,\hat{\mathbf{k}}. \qquad (0.46)$$

(c) Substitute Eq. (0.46) into Eq. (0.33) yielding

$$\left\langle\vec{\mathbf{S}}\right\rangle = \frac{E_o^2}{c\mu_0 T}\sin(2kz)\int_0^{T=2\pi/\omega}\sin(2\omega t)dt\,\hat{\mathbf{k}}. \qquad (0.47)$$

Integrate Eq. (0.47) finding

$$\begin{aligned}\left\langle\vec{\mathbf{S}}\right\rangle &= -\frac{E_o^2}{c\mu_0 2\omega T}\sin(2kz)\cos(2\omega t)\Big|_0^{T=2\pi/\omega}\,\hat{\mathbf{k}} \\ &= -\frac{E_o^2}{c\mu_0 2\omega T}\sin(2kz)(\cos(4\pi)-\cos(0))\,\hat{\mathbf{k}}. \\ &= \vec{\mathbf{0}}\end{aligned} \qquad (0.48)$$

Since the wave described by Eq. (0.35) and Eq. (0.36) is a plane standing wave, the wave is not propagating, therefore there is no time averaged energy transport.