Energy and Momentum in EM Waves
Challenge Problems

Problem 1:

As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ….) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations!)

Design a solar observatory. Specifically, we want an observatory that does not have to orbit but rather can just sit still, hovering over the sun. We will balance out gravity with radiation pressure. You need to estimate the mass of the observatory and choose suitable dimensions for it.

Some possibly useful numbers:

$G = 6.67\times10^{-11}\text{ N m}^2\text{ kg}^{-2}$ $\quad c = 3\times10^{8}\text{ m/s}$

Sail materials have to be very lightweight. Current materials have areal mass densities of about 1 g/m^2, but proposed materials are projected to be as low as 0.05 g/m^2.

The sun has mass 2 x 10^{30} kg and radius 7 x 10^{8} m. It radiates power at a rate of 3.9 x 10^{26} Watts and has a rotation period of about 30 days.

Problem 2:

You have designed a solar space craft of mass *m* that is accelerated by the force due to the 'radiation pressure' from the sun's light that fall on a perfectly reflective circular sail that it is oriented face-on to the sun. The time averaged radiative power of the sun is P_{sun}. The gravitational constant is *G*. The mass of the sun is m_s. The speed of light is c. Model the sun's light as a plane electromagnetic wave, traveling in the +z direction with the electric field given by

$$\vec{\mathbf{E}}(z,t) = E_{x,0}\cos(kz - \omega t)\,\hat{\mathbf{i}}\,.$$

You may express your answer in terms of the symbols m, $\langle P \rangle$, c, m_s, G, k, and ω as necessary.

a) What is the magnetic field $\vec{\mathbf{B}}$ associated with this electric field?

b) What is the Poynting vector $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ associated with this wave? What is the time averaged Poynting vector $\langle \vec{\mathbf{S}} \rangle = \frac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt$ associated with this superposition, where T is the period of oscillation. What is the amplitude of the electric field at your starting point?

c) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun?

Problem 3:

Consider a plane electromagnetic wave with the electric and magnetic fields given by

$$\vec{\mathbf{E}}(x,t) = E_z(x,t)\hat{\mathbf{k}}, \quad \vec{\mathbf{B}}(x,t) = B_y(x,t)\hat{\mathbf{j}}$$

Applying arguments similar to that presented in Section 13.4 of the *Course Notes*, show that the fields satisfy the following relationships:

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}, \quad \frac{\partial B_y}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t}$$

Problem 4:

The magnetic field of a plane electromagnetic wave is described as follows:

$$\vec{\mathbf{B}} = B_0 \sin(kx - \omega t)\,\hat{\mathbf{j}} \qquad (0.1)$$

a) What is the wavelength λ of the wave?

b) Write an expression for the electric field $\vec{\mathbf{E}}$ associated to this magnetic field. Be sure to indicate the direction with a unit vector and an appropriate sign (+ or –).

c) What direction is this wave moving?

d) What is the direction and magnitude Poynting vector associated with this wave? Give appropriate units, as well as magnitude.

e) This wave is totally reflected by the thin perfectly conducting sheet lying in the *y*-*z* plane at $x = 0$. What is the resulting radiation pressure on the sheet? Give appropriate units, as well as magnitude.

f) The component of an electric field parallel to the surface of an ideal conductor must be zero. Using this fact, find expressions for the electric and magnetic fields for the reflected wave. Check that the sum of your transmitted and reflected wave must satisfies the condition that the electric field is zero at the conducting sheet (located at $x = 0$).

Problem 5:

The electric field of an electromagnetic wave is given by the superposition of two waves

$$\vec{\mathbf{E}} = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}}\,. \tag{0.2}$$

You may find the following identities useful

$$\cos(kz + \omega t) = \cos(kz)\cos(\omega t) - \sin(kz)\sin(\omega t) \tag{0.3}$$

$$\sin(kz + \omega t) = \sin(kz)\cos(\omega t) + \cos(kz)\sin(\omega t)\,. \tag{0.4}$$

a) What is the associated magnetic field $\vec{\mathbf{B}}(x, y, z, t)$?

b) What is the energy per unit area per unit time (the Poynting vector $\vec{\mathbf{S}}$) transported by this wave?

c) What is the time average of the Poynting $\langle \vec{\mathbf{S}} \rangle$ vector? Explain your answer, (note: you may be surprised by your answer, but try to explain it). Recall that the time average of the Poynting vector is given by

$$\langle \vec{\mathbf{S}} \rangle \equiv \frac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt\,. \tag{0.5}$$

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