Module 08: Electric Potential and Gauss's Law; Configuration Energy

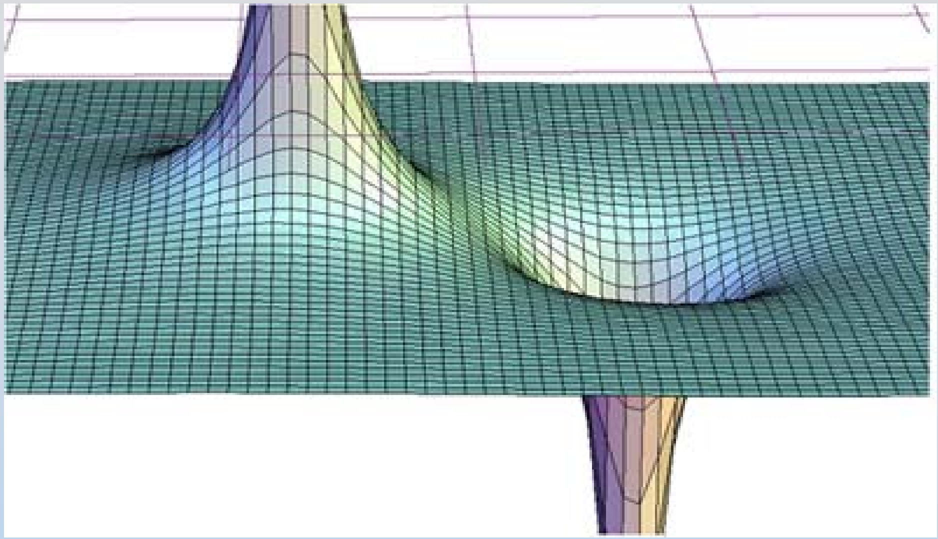
Module 08: Outline

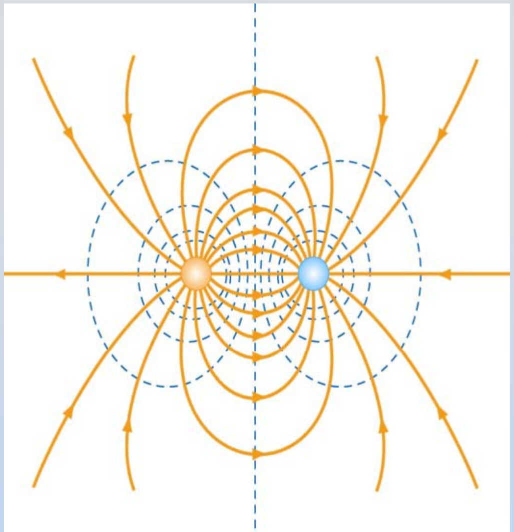
Deriving E from V

Using Gauss's Law to find V from E

Configuration Energy

E Field and Potential: Creating





A point charge *q* creates a field and potential around it:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}};\ V = k_e \frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}} = -\nabla V;\ \Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

E Field and Potential: Effects

If you put a charged particle, (charge *q*), in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

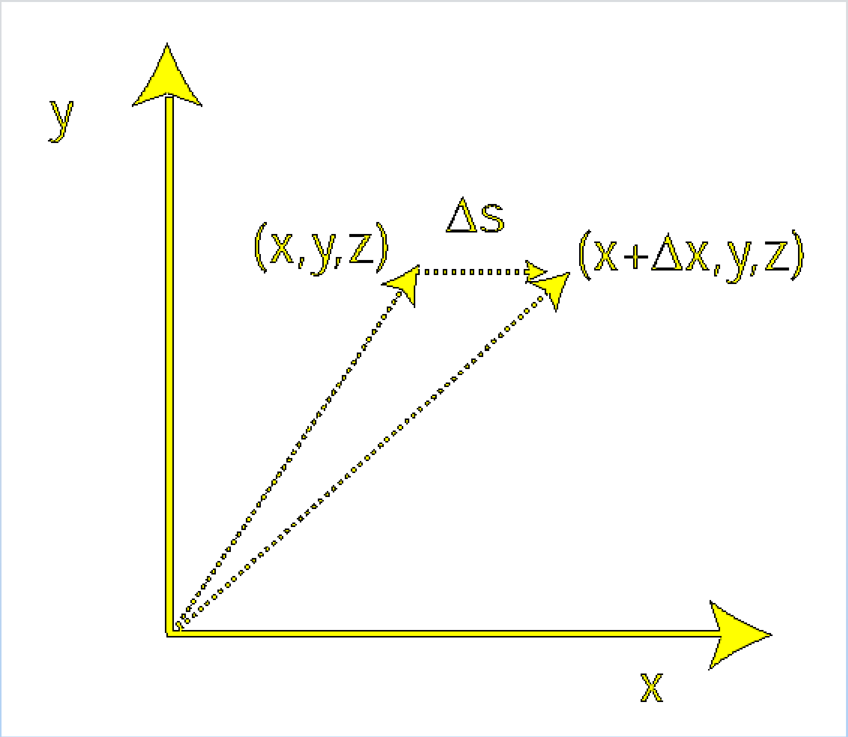
To move a charged particle, (charge *q*), in a field and the particle does not change its kinetic energy then:

$$W_{ext} - \Delta U - q\Delta V$$

Deriving E from V

Deriving E from V

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$



A = (x,y,z), B=(x+Δx,y,z)

$$\Delta \vec{\mathbf{s}} = \Delta x\,\hat{\mathbf{i}}$$

$$\Delta V = -\int_{(x,y,z)}^{(x+\Delta x,y,z)} \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} \;\cong -\vec{\mathbf{E}} \cdot \Delta \vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot (\Delta x\,\hat{\mathbf{i}}) = -E_x \Delta x$$

$$E_x \cong -\frac{\Delta V}{\Delta x} \;\text{—}\; -\frac{\partial V}{\partial x}$$

E_x = Rate of change in V with y and z held constant

Deriving E from V

If we do all coordinates:

$$\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}} \right)$$

$$= -\underbrace{\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}} \right)}V$$

Gradient (del) operator:

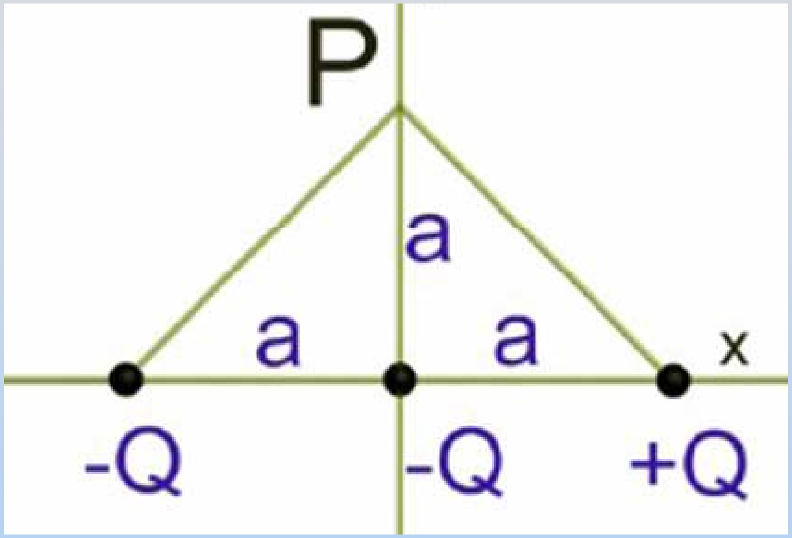
$$\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$$

$$\boxed{\vec{\mathbf{E}} = -\nabla V}$$

Concept Question Question: E from V

Concept Question: E from V

Consider the point charges you looked at earlier:

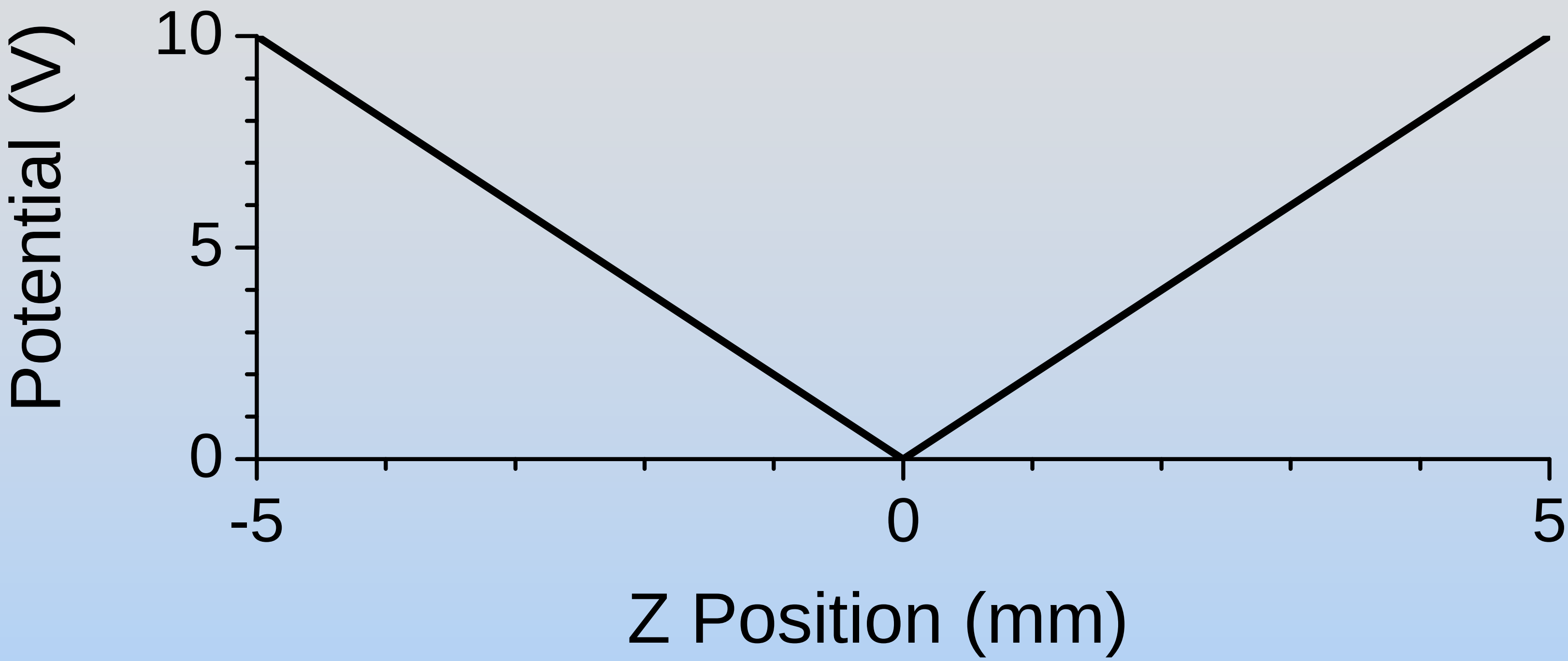


$$V\left(P\right) = -\ kQ/a$$

You calculated V(P). From that can you derive E(P)?

1. Yes, its kQ/a^2 (up)
2. Yes, its kQ/a^2 (down)
3. Yes in theory, but I don't know how to take a gradient
4. No, you can't get E(P) from V(P)
5. I don't know

Problem: E from V



A potential V(*x*,*y*,*z*) is plotted above. It does not depend on *x* or *y*.

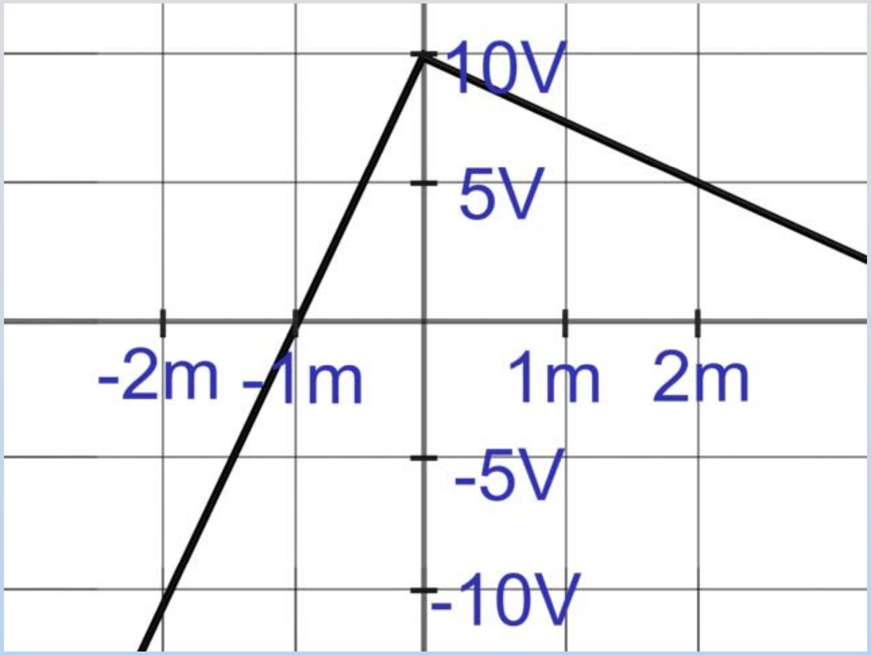
What is the electric field everywhere?

Are there charges anywhere? What sign?

Demonstration:
Making & Measuring Potential
(Lab Preview)

Two Concept Question Questions: Potential & E Field

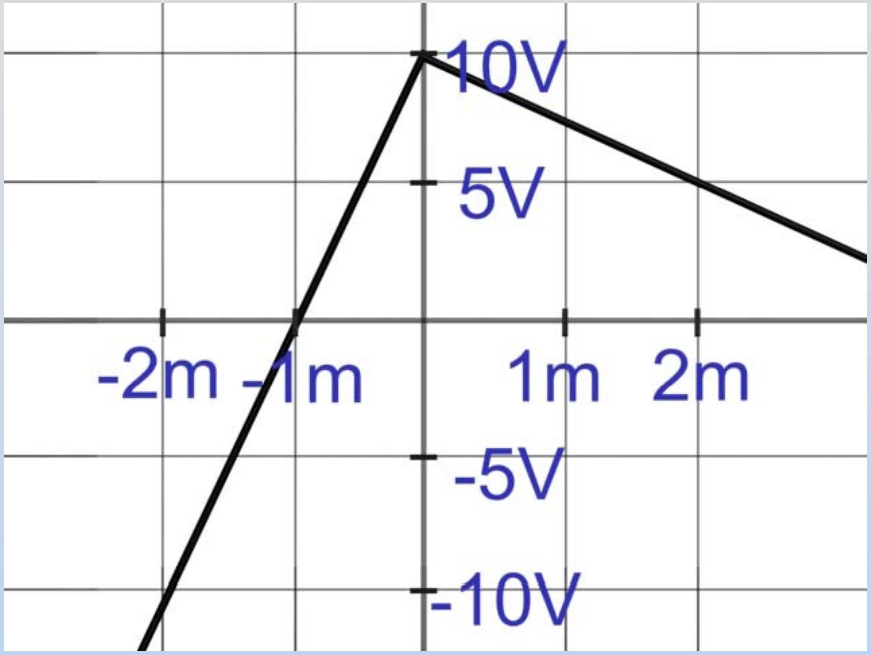
Concept Question: E from V



The graph above shows a potential V as a function of x. The *magnitude* of the electric field for x > 0 is

1. larger than that for x < 0
2. smaller than that for x < 0
3. equal to that for x < 0
4. I don't know

Concept Question: E from V



The above shows potential V(x). Which is true?

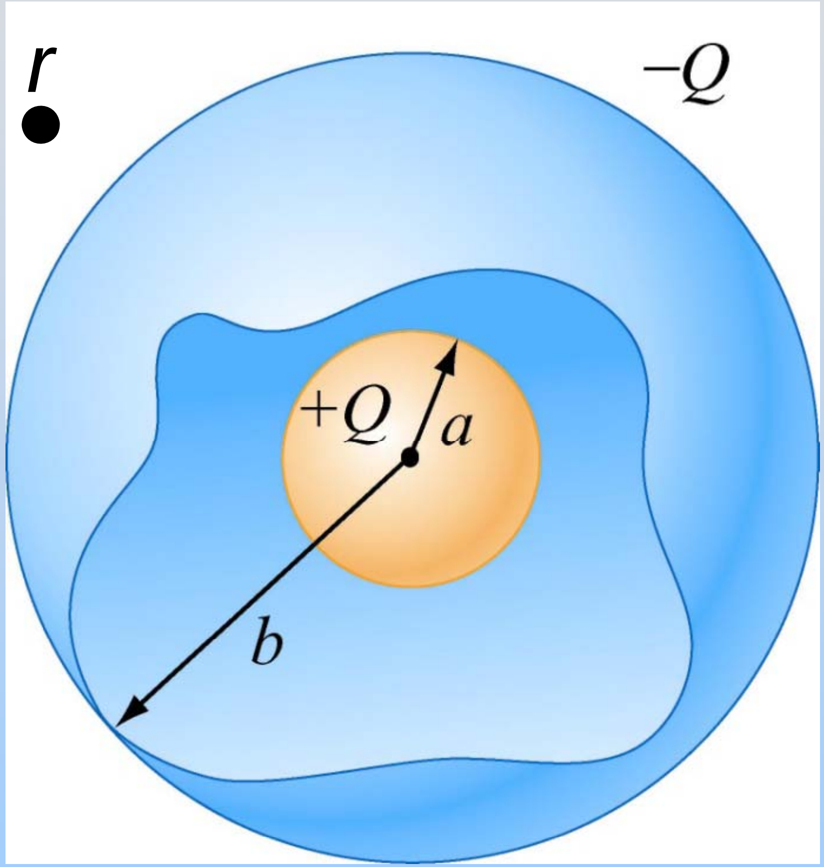
1. $E_{x>0}$ is > 0 and $E_{x<0}$ is > 0
2. $E_{x>0}$ is > 0 and $E_{x<0}$ is < 0
3. $E_{x>0}$ is < 0 and $E_{x<0}$ is < 0
4. $E_{x>0}$ is < 0 and $E_{x<0}$ is > 0
5. I don't know

Potential from E

Potential for Nested Shells

From Gauss's Law

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, & a < r < b \\ 0, & \text{elsewhere} \end{cases}$$



Use $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

Region 1: $r > b$

$$V(r) - \underbrace{V(\infty)}_{=0} = -\int_\infty^r 0\, dr = 0$$

No field →
No change in V!

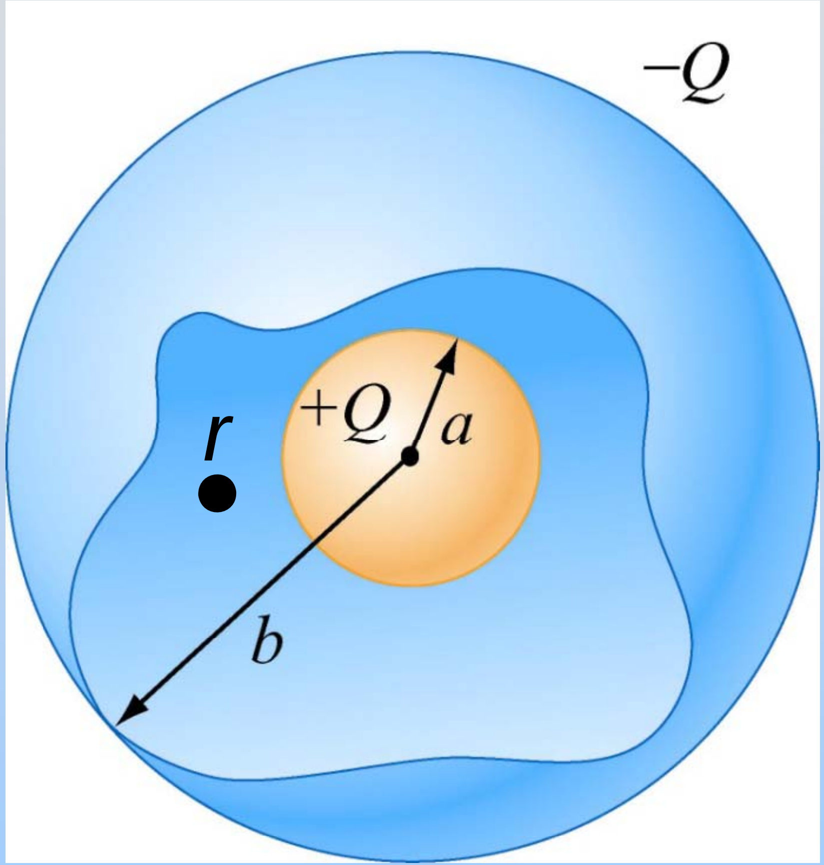
Potential for Nested Shells

Region 2: $a < r < b$

$$V(r) - \underbrace{V(r=b)}_{=0} = -\int_b^r dr \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\bigg|_{r=b}^{r}$$

$$= \frac{1}{4\pi\varepsilon_0} Q\left(\frac{1}{r} - \frac{1}{b}\right)$$



Electric field is just a point charge.

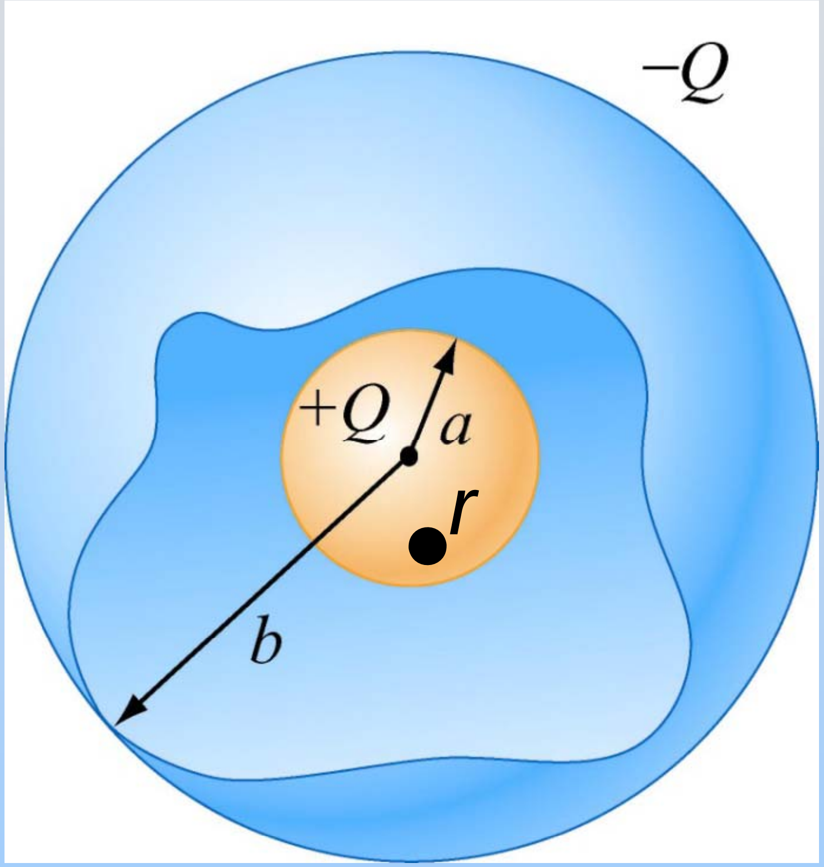
Electric potential is DIFFERENT – surroundings matter

Potential for Nested Shells

Region 3: $r < a$

$$V(r) - \underbrace{V(r=a)}_{=kQ\left(\frac{1}{a}-\frac{1}{b}\right)} = -\int_a^r dr\ 0 = 0$$

$$V(r) = V(a) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$



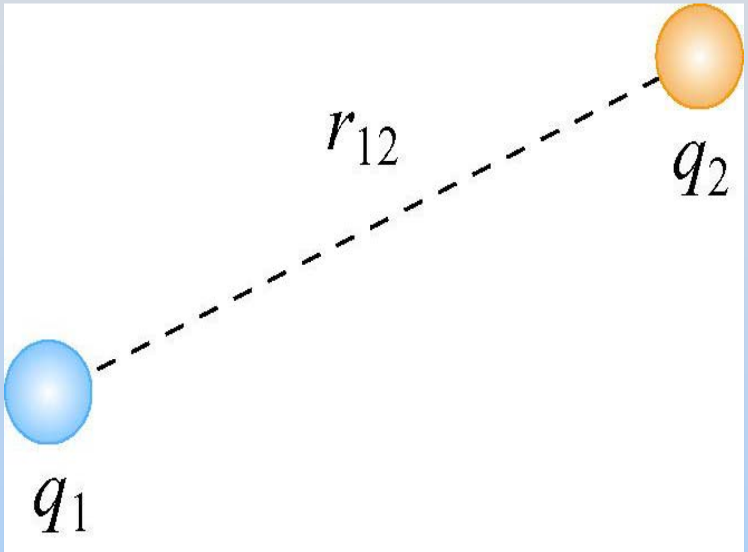
Again, potential is CONSTANT since E = 0.

Configuration Energy

Configuration Energy

How much energy to put two charges as pictured?

1) First charge is free
2) Second charge sees first:



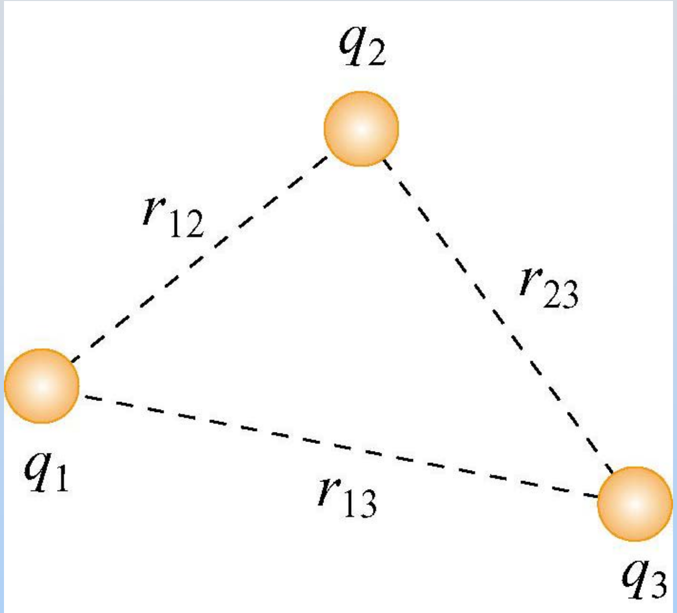
$$U_{12} = W_2 = q_2 V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

Configuration Energy

How much energy to put three charges as pictured?

1) Know how to do first two
2) Bring in third:

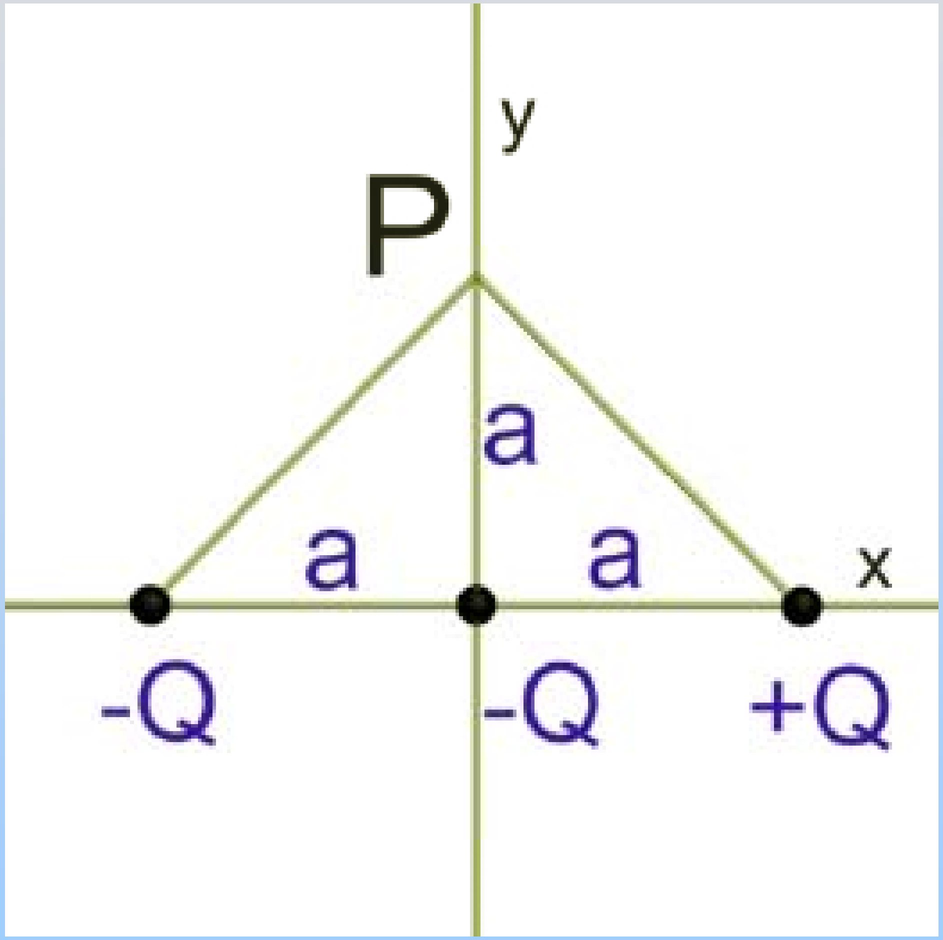
$$W_3 - q_3\left(V_1 + V_2\right) - \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$



Total configuration energy:

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) = U_{12} + U_{13} + U_{23}$$

Problem: Build It



1) How much energy did it take to assemble the charges at left?

2) How much energy would it take to add a 4^{th} charge +3Q at P?

8.02SC Physics II: Electricity and Magnetism
Fall 2010