Electric Potential and Gauss's Law, Configuration Energy
Challenge Problems

Problem 1:

Consider a very long rod, radius R and charged to a uniform linear charge density λ.

a) Calculate the electric field everywhere outside of this rod (i.e. find $\vec{\mathbf{E}}(\vec{\mathbf{r}})$).

b) Calculate the electric potential everywhere outside, where the potential is defined to be zero at a radius $R_0 > R$ (i.e. $V(R_0) \equiv 0$)

Problem 2:

Estimate the largest voltage at which it's reasonable to hold high voltage power lines. Then check out this video, care of a Boulder City, Nevada power company. Air ionizes when electric fields are on the order of $3\times10^{6}\ \mathrm{V\cdot m^{-1}}$.

Problem 3:
Consider a uniformly charged sphere of radius R and charge Q. Find the electric potential difference between any point lying on a sphere of radius r and the point at the origin, i.e. $V(r)-V(0)$. Choose the zero reference point for the potential at $r=0$, i.e. $V(0)=0$. How does your expression for $V(r)$ change if you chose the zero reference point for the potential at $r=\infty$, i.e. $V(\infty)=0$.

Problem 4:

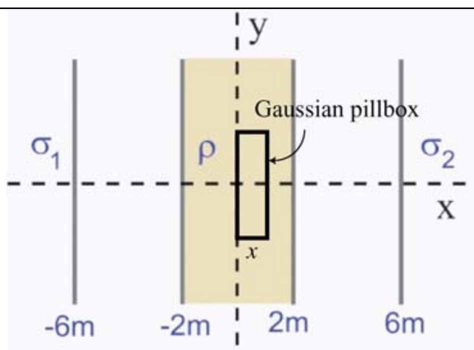
An infinite slab of charge carrying a charge per unit volume ρ has a charged sheet carrying charge per unit area σ_1 to its left and a charged sheet carrying charge per unit area σ_2 to its right (see top part of sketch). The lower plot in the sketch shows the electric potential $V(x)$ in volts due to this slab of charge and the two charged sheets as a function of horizontal distance x from the center of the slab. The slab is 4 meters wide in the x-direction, and its boundaries are located at $x = -2$ m and $x = +2$ m, as indicated. The slab is infinite in the y direction and in the z direction (out of the page). The charge sheets are located at $x = -6$ m and $x = +6$ m, as indicated.

(a) The potential $V(x)$ is a linear function of x in the region $-6\text{ m} < x < -2\text{ m}$. What is the electric field in this region?

(b) The potential $V(x)$ is a linear function of x in the region $2\text{ m} < x < 6\text{ m}$. What is the electric field in this region?

(c) In the region $-2\text{m} < x < 2\text{ m}$, the potential $V(x)$ is a quadratic function of x given by the equation $V(x) = \frac{5}{16} x^2 \frac{\text{V}}{m^2} - \frac{25}{4}\text{V}$. What is the electric field in this region?

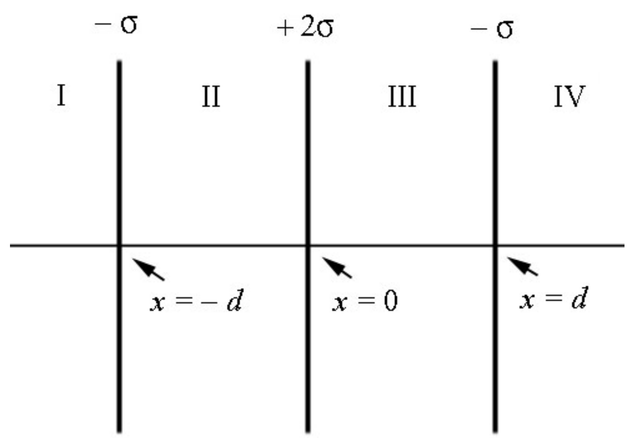
(d) Use Gauss's Law and your answers above to find an expression for the charge density ρ of the slab. Indicate the Gaussian surface you use on a figure.



(e) Use Gauss's Law and your answers above to find the two surface charge densities of the left and right charged sheets. Indicate the Gaussian surface you use on a figure.

Problem 5:

Three infinite sheets of charge are located at $x = -d$, $x = 0$, and $x = d$, as shown in the sketch. The sheet at $x = 0$ has a charge per unit area of 2σ, and the other two sheets have charge per unit area of $-\sigma$.



a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the *x* component of the electric field , E_x, on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.

b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of *x* on the graph on the bottom of the next page. Indicate units on the vertical axis.

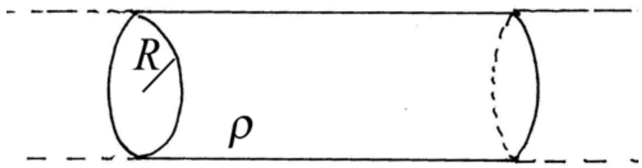
c) How much work must you do to bring a point-like object with charge $+Q$ in from infinity to the origin $x = 0$?

Problem 6:

You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b} r^n dr = \frac{1}{n+1}\left(r_b^{\,n+1} - r_a^{\,n+1}\right);\ n \neq 1\ , \qquad \int_{r_a}^{r_b} \frac{dr}{r} = \ln(r_b / r_a)\ .$$

Consider a charged infinite cylinder of radius R.



The charge density is non-uniform and given by

$$\rho(r) = br\ ;\ \ r < R\ ,$$

where r is the distance from the central axis and b is a constant.

a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder.