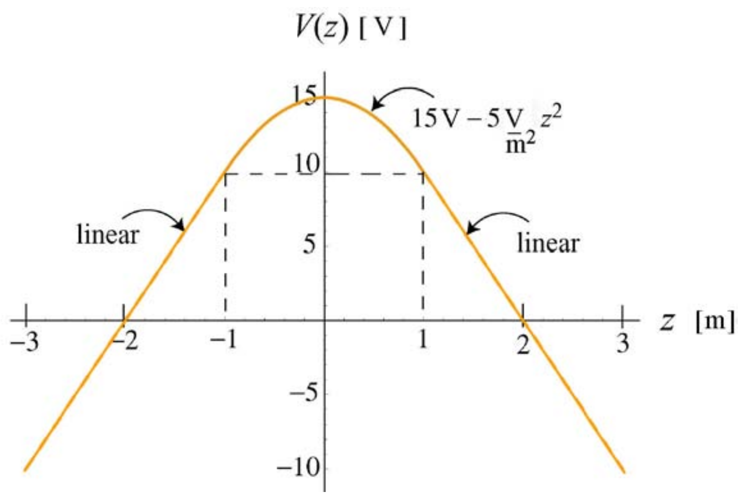
Electric Potential, Equipotential Lines and Electric Fields
Challenge Problem Solutions

Problem 1:

A graph of the electric potential $V(z)$ vs. z is shown in the figure below.



Which of the following statements about the z-component of the electric field E_z is true?

a) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

b) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

c) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

d) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

e) None of the above because E_z cannot be determined from information in the graph for the regions $-3\,\text{m} < z < 0$ and $0 < z < 3\,\text{m}$.

Problem 1 Solution:

b. For values of $-3\,\text{m} < z < 0$, the derivative $dV(z)/dz > 0$, and $E_z = -dV(z)/dz < 0$. For values of $0 < z < 3\,\text{m}$, the derivative $dV(z)/dz < 0$, and $E_z = -dV(z)/dz > 0$.

Problem 2:

Suppose an electrostatic potential has a maximum at point P and a minimum at point M.

(a) Are either (or both) of these points equilibrium points for a negative charge? If so are they stable?

(b) Are either (or both) of these points equilibrium points for a positive charge? If so are they stable?

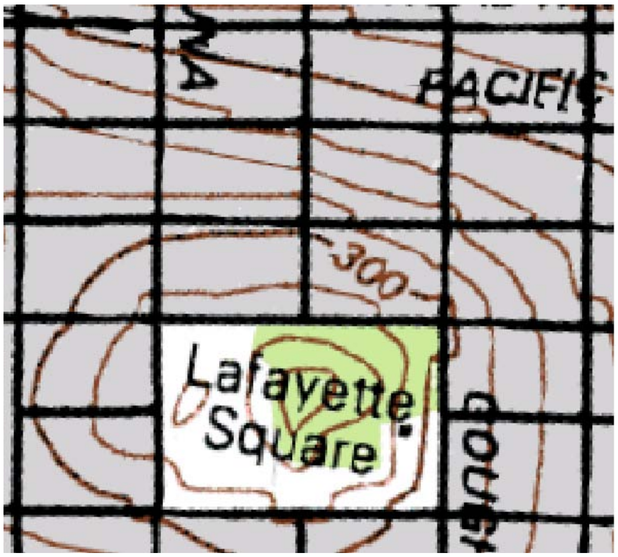
Problem 2 Solution:

(a)The electric field is the gradient of the potential, which is zero at both potential minima and maxima. So a negative charge is in equilibrium (feels no net force) at both P & M. However, only the maximum (P) is stable. If displaced slightly from P, a negative charge will roll back "up" hill, back to P. If displaced from M a negative charge will roll away from the potential minimum.

(b)Similarly, both P & M are equilibria for positive charges, but only M is a stable equilibrium because positive charges seek low potential (this is probably the case that seems more logical since it is like balls on mountains).

Problem 3:

Below is a topographic map of a 0.4 mi square region of San Francisco. The contours shown are separated by heights of 25 feet (so from 375 feet to 175 feet above sea level for the region shown)



From left to right, the NS streets shown are Buchanan, Laguna, Octavia, Gough and Franklin. From top to bottom, the EW streets shown are Broadway, Pacific, Jackson, Washington, Clay (which stops on either side of the park) and Sacramento.

(a) In the part of town shown in the above map, which street(s) have the steepest runs? Which have the most level sections? How do you know?

(b) How steep is the steepest street at its steepest (what is its slope in ft/mi)?

(c) Which would take more work (in the physics sense): walking 3 blocks south from Laguna and Jackson or 1 block west from Clay and Franklin?

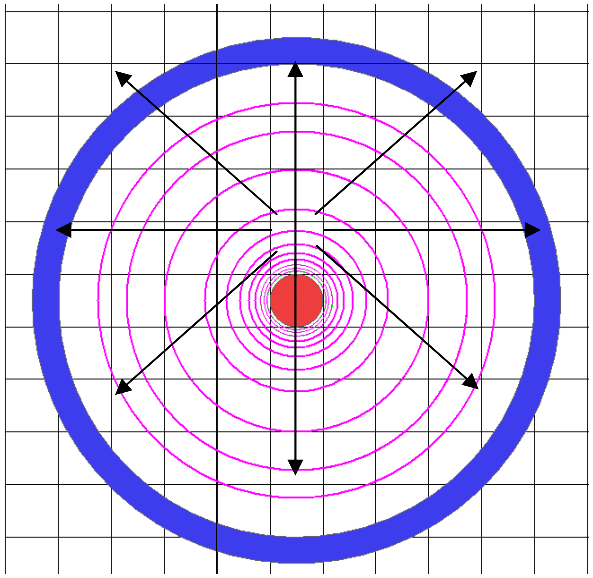
Problem 3 Solution:

(a)You can tell how steep something is by looking at how quickly it passes through constant height contours (~ equipotentials). The steepest section is along Octavia between Pacific and Washington. The most level street is Jackson between Buchanan and Octavia, which runs parallel to the 275 foot contour and hence is very flat.

(b)Looking at Octavia, it passes through 5 contours (125 feet) in two blocks (about 0.12 miles) so it has a slope of ~1000 ft/mi.

(c)Work is change in potential energy (and hence height). The change in height walking 3 blocks S on Laguna is almost nothing (you go up but come back down again). West on Clay from Franklin you rise 50 feet in the block, so that is more work.

Problem 4:



The equipotentials for a potential landscape (on a 1 cm grid) are shown in the figure.

The equipotential curves (the magenta circles) are marked at V = 0.25 V, 0.5V and then from V = 1 V to V = 10 V in 1 V increments.

Use the convention that red is the positive electrode (V = +10 V) and blue isground (V = 0 V).

(a) Copy the above figure and sketch eight electric field lines on it (equally spaced around the inner conductor).

(b) What, approximately, is the magnitude of the electric field at r = 1 cm, 2 cm, and 3 cm, where r is measured from the center of the inner conductor? You should express the field in V/cm. (HINT: The field is the local slope (derivative) of the potential. Also, if you choose to use a ruler realize that the above reproduction of this group's results is not the same size as the original, where the grid size was 1 cm).

(c) What is the relationship between the density of the equipotential lines, the density of the electric field lines, and the strength of the electric field?

(d) Plot the field strength vs. $1/r^2$ for the three points from part (a). If the field were created by a single point charge what shape should this sketch be? Is it?

(e) Approximately how much charge was on the inner conductor when the group made their measurements?

Problem 4 Solutions:

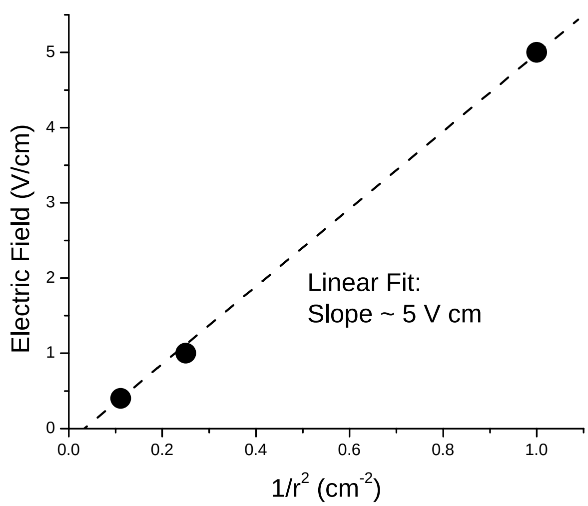
(a)See black arrows

(b)

At r = 1 cm, V ~ 4 V and we move 1 V in about 1/5 cm.	E ~ 5 V/cm
At r = 2 cm, V ~ 1.5 V and we move about 1/2 V in 1/2 cm.	E ~ 1 V/cm
At r = 3 cm, V ~ 0.7 V and we move about 0.2 V in 1/2 cm.	E ~ 0.4 V/cm

(c)The denser the equipotential lines and hence electric field lines, the stronger the field.

(d)It should be (and is!) a straight line

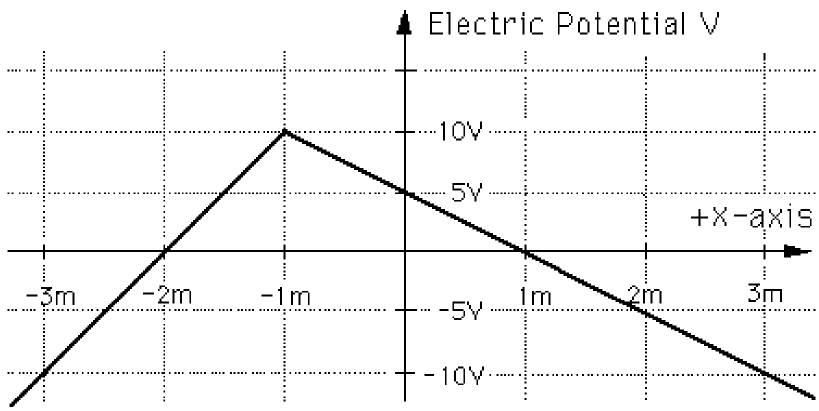


(e)

$E = k_e \frac{q}{r^2}$, so slope is $k_e q = 5$ V cm. $q \approx 5\times10^{-12}\,\mathrm{C}$

Problem 5:

The graph shows the variation of an electric potential *V* with distance *x*. The potential does not vary in the *y* or *z* directions. Be sure to include units as appropriate.



(a) What is E_x in the region $x > -1\,\text{m}$? (Be careful to indicate the sign of E_x.)

(b) What is E_x in the region $x < -1\,\text{m}$? (Be careful to indicate the sign of E_x.)

(c) A negatively charged dust particle with mass $m_q = 1\times 10^{-13}\ \text{kg}$ and charge $q = -1\times 10^{-12}\ \text{C}$ is released from rest at $x = +2\ \text{m}$. Will it move to the left or to the right?

Problem 5 Solutions:

(a) In the region $x > -1\,\text{m}$, $V(x) = 5\ \text{V} - (5\ \text{V}\cdot\text{m}^{-1})\,x$. So

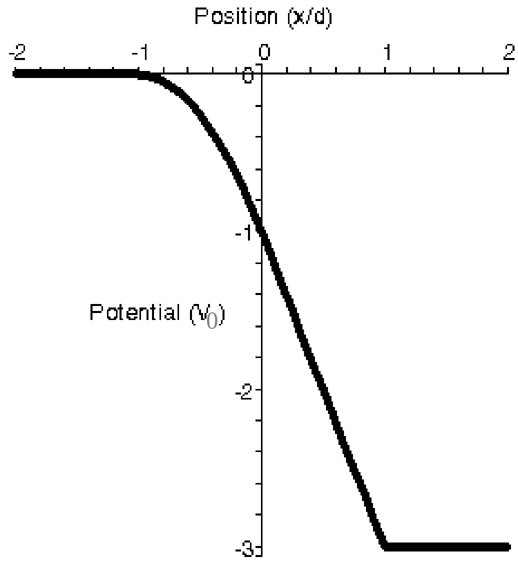
$$E_x = -\frac{d}{dx}V(x) = 5\ \text{V}\cdot\text{m}^{-1}$$

(b) In the region $x < -1\,\text{m}$, $V(x) = 20\ \text{V} + (10\ \text{V}\cdot\text{m}^{-1})\,x$. So

$$E_x = -\frac{d}{dx}V(x) = -10\ \text{V}\cdot\text{m}^{-1}\ .$$

(c) For $x > -1\,\text{m}$, the electric field is pointing in the positive *x*-direction, so a negatively charged particle will experience a force pointing in the negative *x*-direction, hence it will move to the left.

Problem 6:



The electric potential *V(x,y,z)* for a planar charge distribution is given by:

$$V(x,y,z)=\begin{cases} 0 & \text{for } x<-d \\ -V_0\left(1+\frac{x}{d}\right)^2 & \text{for } -d\le x<0 \\ -V_0\left(1+2\frac{x}{d}\right) & \text{for } 0\le x<d \\ -3V_0 & \text{for } x>d \end{cases}$$

where $-V_0$ is the potential at the origin and *d* is a distance.

This function is plotted to the right, with the x-axis in units of *d* and the y-axis in units of V_0.

(a) What is the electric field $\vec{\mathbf{E}}(x)$ for this problem?

Region I: -d > x

Region II: $-d\le x<0$

Region III: $0\le x<d$

Region IV: $x>d$

(b) Plot the electric field that you just calculated on the graph below. Be sure to properly label the y-axis (top and bottom) to indicate the limits of the magnitude of the E field!

Problem 6 Solutions:

(a) Region I: -d > x

$$\vec{\mathbf{E}}=-\nabla V=-\frac{\partial V}{\partial x}\hat{\mathbf{i}}=0$$

Region II: $-d\le x<0$

$$\vec{\mathbf{E}}=-\frac{\partial}{\partial x}\left(-V_0\left(1+\frac{x}{d}\right)^2\right)\hat{\mathbf{i}}=2\frac{V_0}{d}\left(1+\frac{x}{d}\right)\hat{\mathbf{i}}$$

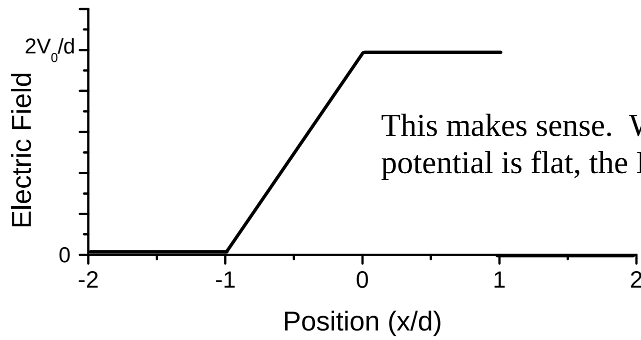
Region III: $0\le x<d$

$$\vec{\mathbf{E}}=-\frac{\partial}{\partial x}\left(-V_0\left(1+2\frac{x}{d}\right)\right)\hat{\mathbf{i}}=2\frac{V_0}{d}\hat{\mathbf{i}}$$

Region IV: $x>d$

$$\vec{\mathbf{E}} = -\nabla V = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} = 0$$

(b)



This makes sense. Where the potential is flat, the E field is 0.