Module 06: Electric Potential Discrete and Continuous Distributions of Charge

Summary:
Gravitational & Electric Fields

Summary: Gravity - Electricity

SOURCE: Mass M_s Charge q_s (±)

CREATE: $\vec{\mathbf{g}} = -G\frac{M_s}{r^2}\hat{\mathbf{r}}$ $\vec{\mathbf{E}} = k_e\frac{q_s}{r^2}\hat{\mathbf{r}}$

FEEL: $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ $\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to envisage field, by the forces they produce!

Potential Energy and Potential

Start with Gravity

Gravity: Force and Work

Gravitational force on m due to M:

$$\vec{\mathbf{F}}_g = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

Work done by gravity moving m from A to B:

$$\boxed{W_g = \int_A^B \vec{\mathbf{F}}_{\mathrm{g}} \cdot d\,\vec{\mathbf{s}}}$$

PATH INTEGRAL

Work Done by Earth's Gravity

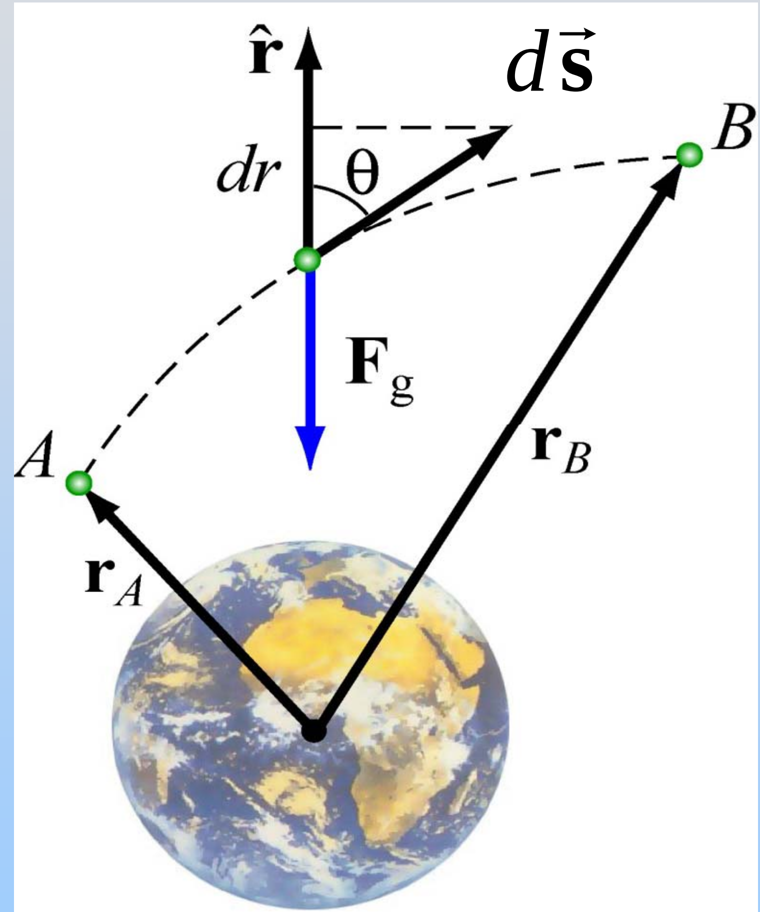
Work done by gravity moving m from *A* to *B:*

$$W_g = \int \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}}$$

$$= \int_A^B \left(-\frac{GMm}{r^2}\hat{\mathbf{r}} \right) \cdot \left(dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\theta}} \right)$$

$$= \int_{r_A}^{r_B} -\frac{GMm}{r^2}\,dr \quad = \left[\frac{GMm}{r} \right]_{r_A}^{r_B}$$

$$= GMm \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

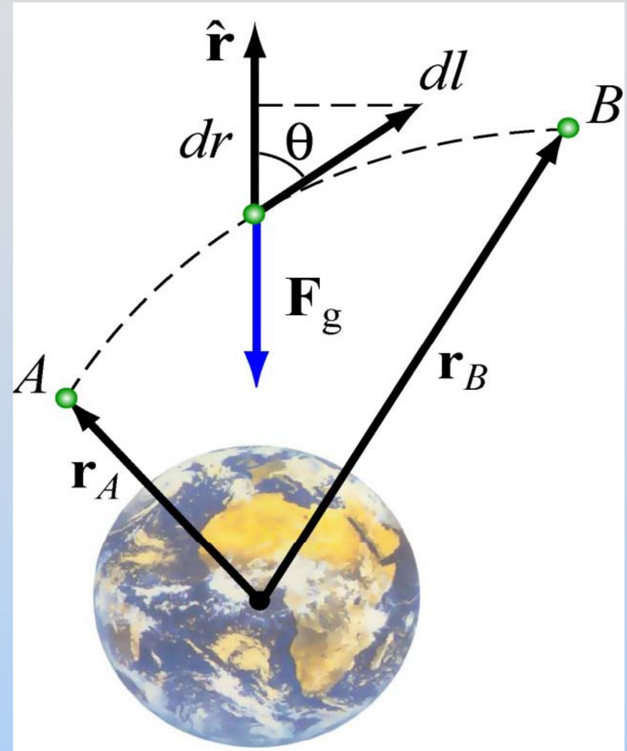


Concept Question Question: Sign of W_g

Concept Question: Sign of W_g

Thinking about the sign and meaning of this…

$$W_g - GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



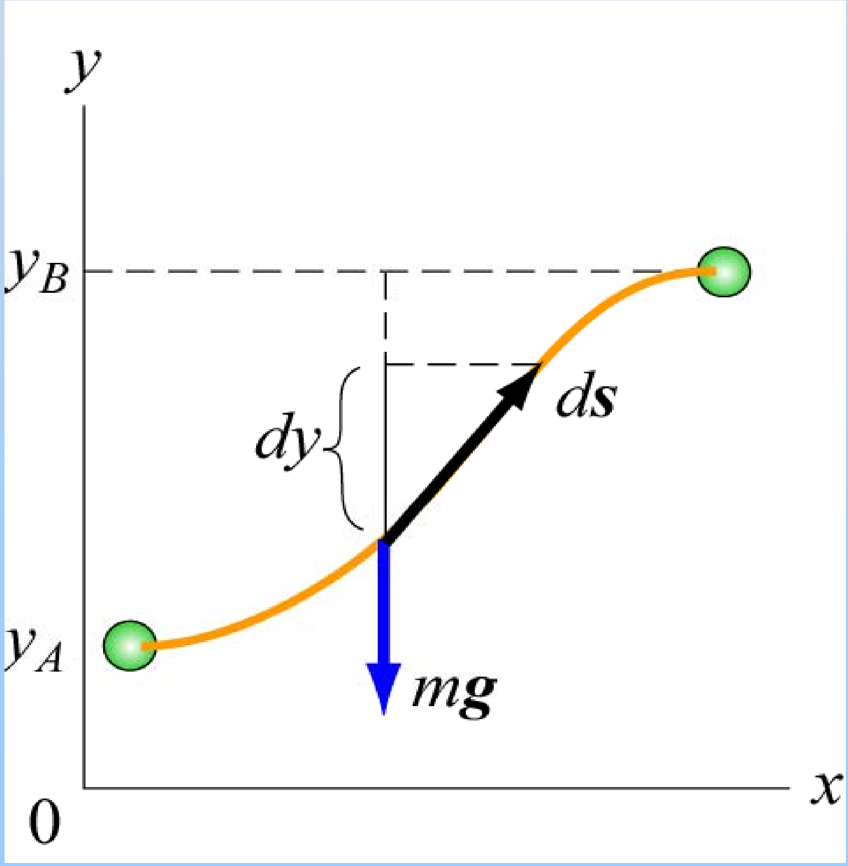
Moving from r_A to r_B:

1. W_g is positive – we do work
2. W_g is positive – gravity does work
3. W_g is negative – we do work
4. W_g is negative – gravity does work
5. I don’t know

Work Near Earth's Surface

G roughly constant: $\vec{\mathbf{g}} \approx -\frac{GM}{r_E^{\ 2}}\hat{\mathbf{y}} = -g\hat{\mathbf{y}}$

Work done by gravity moving m from *A* to *B*:



$$W_g = \int \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}} = \int_A^B \left(-mg\hat{\mathbf{y}}\right)\cdot d\vec{\mathbf{s}}$$

$$= - \int_{yA}^{y_B} mg\,dy = - mg(y_B - y_A)$$

W_g depends ***only*** on endpoints

– ***not*** on path taken –

Conservative Force

Potential Energy (Joules)

$$\Delta U_g = U_B - U_A = -\int_A^B \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}} = -W_g$$

$$(1)\quad \vec{\mathbf{F}}_g = -\frac{GMm}{r^2}\hat{\mathbf{r}} \quad \rightarrow \quad U_g = -\frac{GMm}{r} + U_0$$

$$(2)\quad \vec{\mathbf{F}}_g = -mg\hat{\mathbf{y}} \quad \rightarrow \quad U_g = mgy + U_0$$

- U_0: constant depending on reference point
- Only potential difference ΔU has physical significance

Gravitational Potential (Joules/kilogram)

Define gravitational potential difference:

$$\Delta V_g = \frac{\Delta U_g}{m} = -\int_A^B (\vec{\mathbf{F}}_{\mathrm{g}} / m) \cdot d\,\vec{\mathbf{s}} = -\int_A^B \vec{\mathbf{g}} \cdot d\,\vec{\mathbf{s}}$$

Just as $\underbrace{\vec{\mathbf{F}}_{\mathrm{g}}}_{\text{Force}} \rightarrow \underbrace{\vec{\mathbf{g}}}_{\text{Field}}$, $\underbrace{\Delta U_g}_{\text{Energy}} \rightarrow \underbrace{\Delta V_g}_{\text{Potential}}$

That is, two particle interaction → single particle effect

Concept Question Question: Masses in Potentials

Concept Question: Masses in Potentials

Consider 3 equal masses sitting in different gravitational potentials:

A) Constant, zero potential

B) Constant, non-zero potential

C) Linear potential ($V \propto x$) but sitting at $V = 0$

Which statement is true?

1. None of the masses accelerate
2. Only B accelerates
3. Only C accelerates
4. All masses accelerate, B has largest acceleration
5. All masses accelerate, C has largest acceleration
6. I don't know

Move to Electrostatics

Gravity - Electrostatics

Mass *M*

$$\vec{\mathbf{g}} \overset{(\pm)}{=} -G\frac{M}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_g - m\vec{\mathbf{g}}$$

Charge *q*

$$\vec{\mathbf{E}} = k_e\frac{q}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_E - q\vec{\mathbf{E}}$$

Both forces are conservative, so…

$$\Delta V_g - -\int_A^B \vec{\mathbf{g}} \cdot d\,\vec{\mathbf{s}}$$

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

$$\Delta U_g = -\int_A^B \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}}$$

$$\Delta U = -\int_A^B \vec{\mathbf{F}}_E \cdot d\,\vec{\mathbf{s}}$$

Potential & Potential Energy

$$\Delta V \equiv -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

Units: Joules/Coulomb = Volts

Change in potential energy in moving the charged object (charge *q*) from A to B:

$$\Delta U = U_B - U_A = q\Delta V \qquad \text{Joules}$$

Potential & External Work

Change in potential energy in moving the charged object (charge *q*) from A to B:

$$\Delta U - U_B - U_A - q\Delta V \quad \text{Joules}$$

The external work is

$$W_{ext} = \Delta K + \Delta U$$

If the kinetic energy of the charged object does not change, $\Delta K = 0$

then the external work equals the change in potential energy

$$W_{ext} = \Delta U = q\Delta V$$

Know These!

How Big is a Volt?

- AA, C, D Batteries 1.5 V
- Car Battery 12 V
- US Outlet 120 V (AC)
- Residential Power Line
- Our Van de Graaf
- Big Tesla Coil

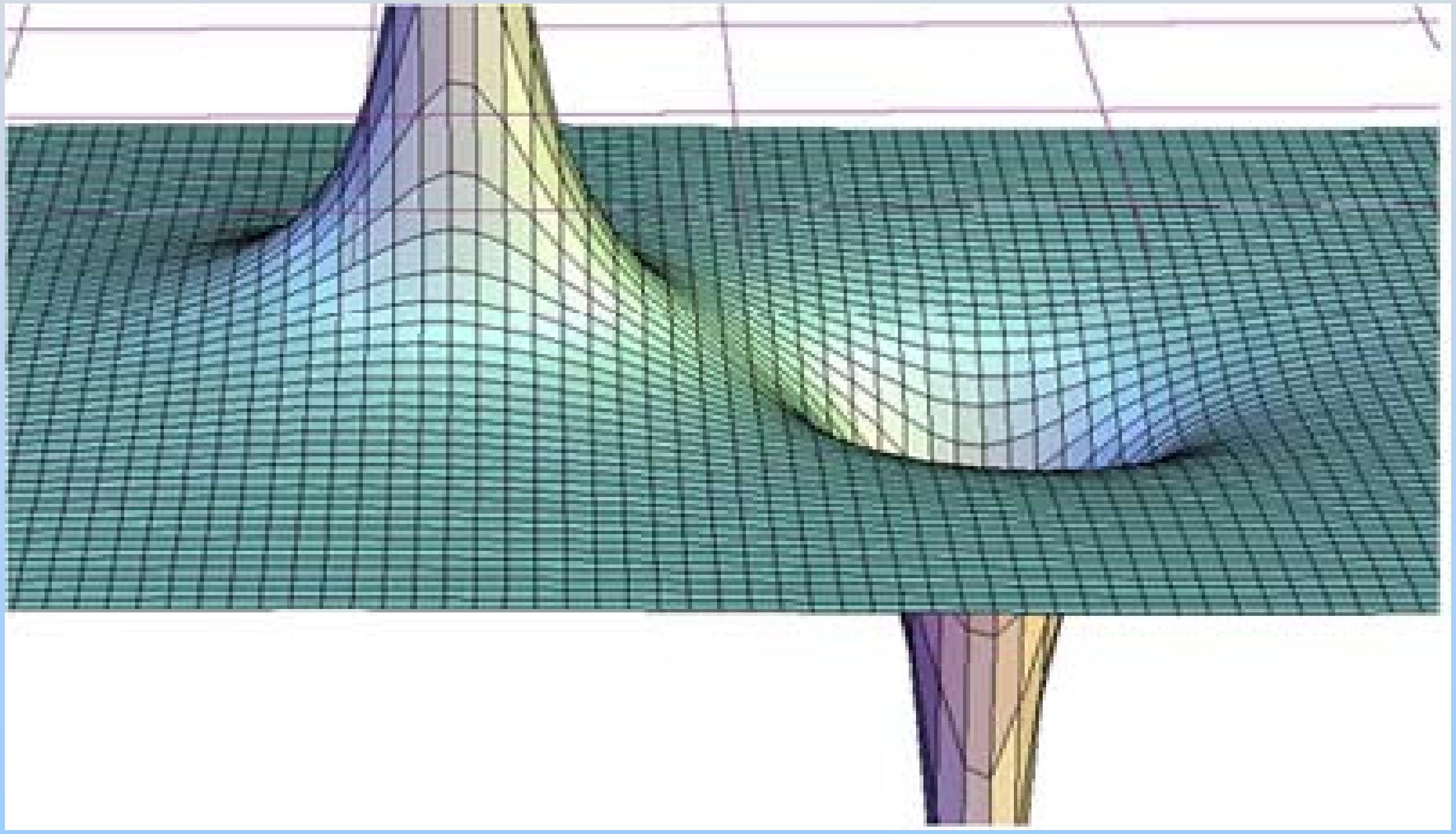
Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

$$V(\vec{\mathbf{r}}) - V_0 + \Delta V \equiv V_{"0"} - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

Potential Landscape

Positive Charge



Negative Charge

Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

$$V(\vec{\mathbf{r}}) - V_0 + \Delta V \equiv V_{"0"} - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

Charges *FEEL* Potential Landscapes

$$U(\vec{\mathbf{r}}) = qV(\vec{\mathbf{r}})$$

We work with ΔU (ΔV) because
only changes matter

2 Concept Question Questions: Potential & Potential Energy

Concept Question: Positive Charge

Place a positive charge in an electric field. It will accelerate from

1. higher to lower *electric potential*; lower to higher *potential energy*
2. higher to lower *electric potential*; higher to lower *potential energy*
3. lower to higher *electric potential*; lower to higher *potential energy*
4. lower to higher *electric potential*; higher to lower *potential energy*

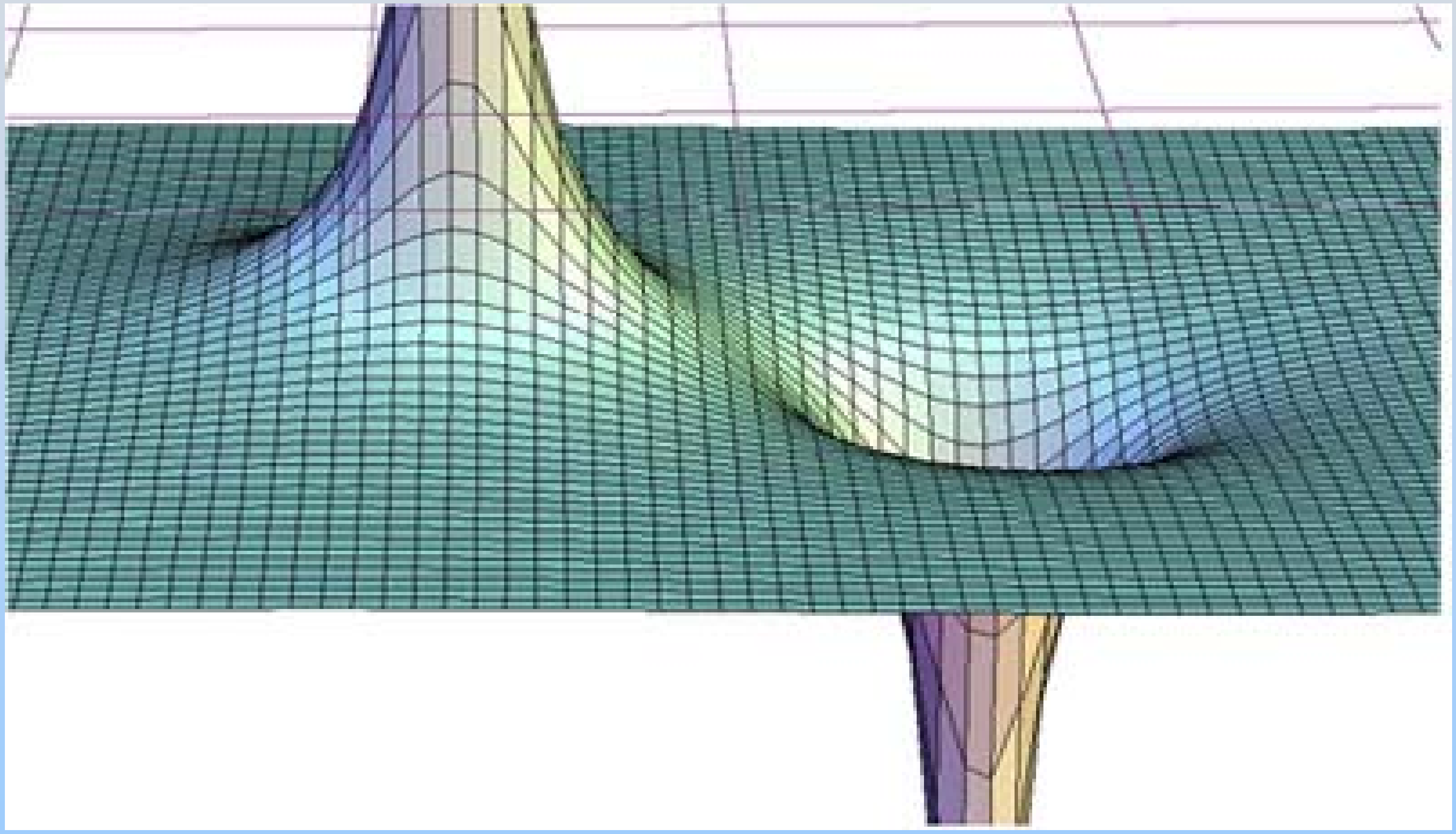
Concept Question: Negative Charge

Place a negative charge in an electric field. It will accelerate from

1. higher to lower *electric potential*; lower to higher *potential energy*
2. higher to lower *electric potential*; higher to lower *potential energy*
3. lower to higher *electric potential*; lower to higher *potential energy*
4. lower to higher *electric potential*; higher to lower *potential energy*

Potential Landscape

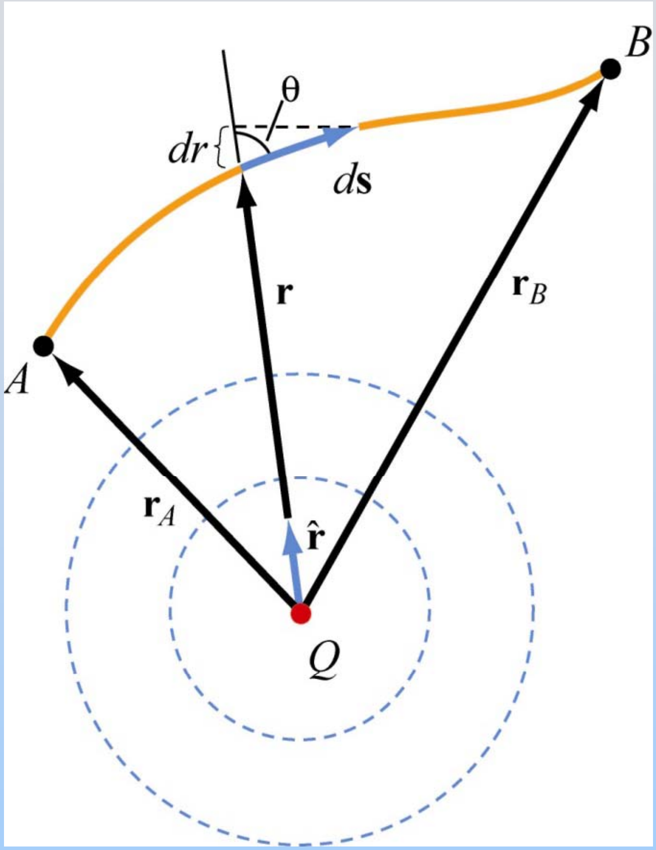
Positive Charge



Negative Charge

Creating Potentials: Calculating from E, Two Examples

Problem: Pt Charge Potential



Consider a SINGLE point charge Q.

What potential difference

$$\Delta V = V_B - V_A$$

does it create between point B and point A?

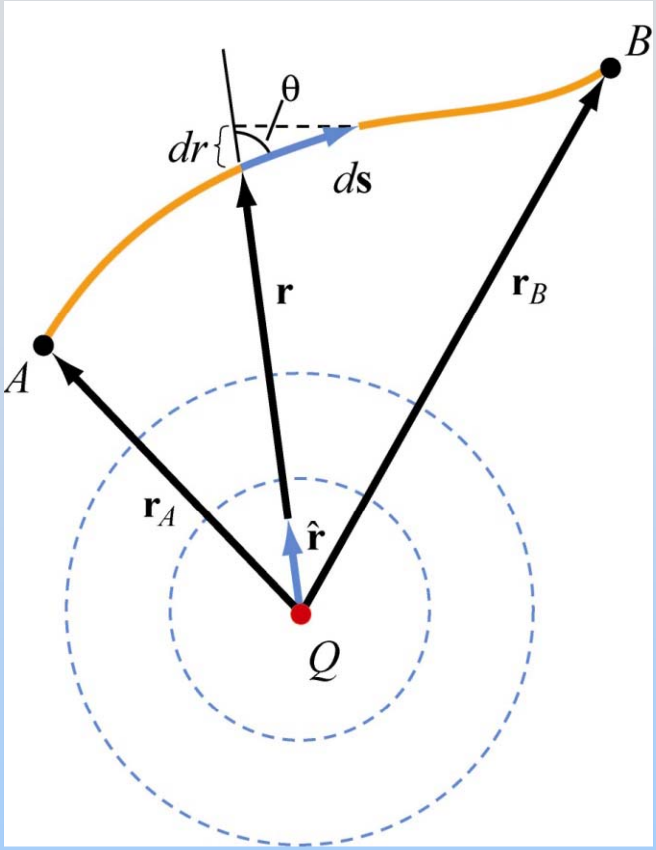
If $V_A \equiv 0$ for $r_A = \propto$, what is $V(r)$?

Potential Created by Pt Charge

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B kQ \frac{\hat{\mathbf{r}}}{r^2} \cdot d\vec{\mathbf{s}} = -kQ \int_A^B \frac{dr}{r^2}$$

$$= kQ \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$



Take $V = 0$ at $r = \infty$:

$$\boxed{V_{\text{Point Charge}}(r) = \frac{kQ}{r}}$$

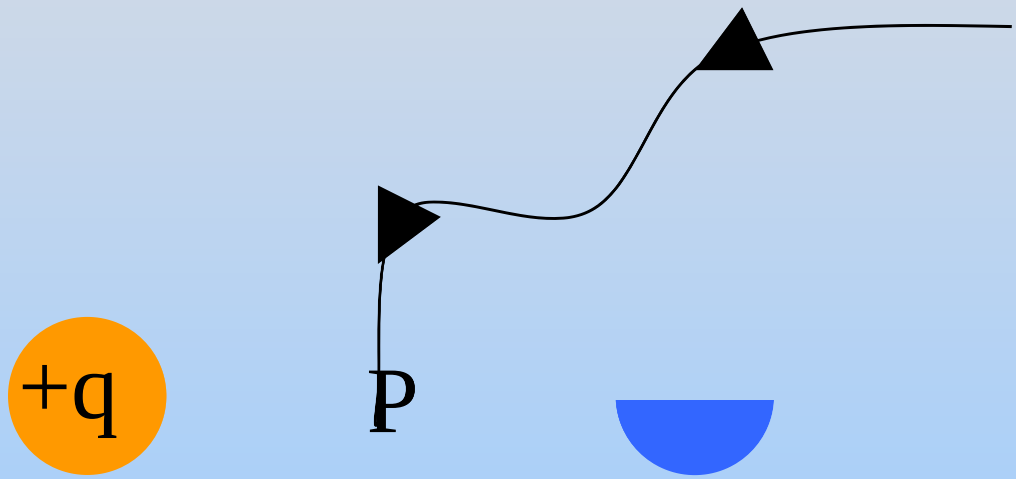
$$\vec{\mathbf{E}} = kQ \frac{\hat{\mathbf{r}}}{r^2}$$

$$d\vec{\mathbf{s}} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\theta}}$$

Concept Question Question: Point Charge Potential

Concept Question: Two Point Charges

The work done in moving a positive test charge from infinity to the point P midway between two charges of magnitude +q and –q:



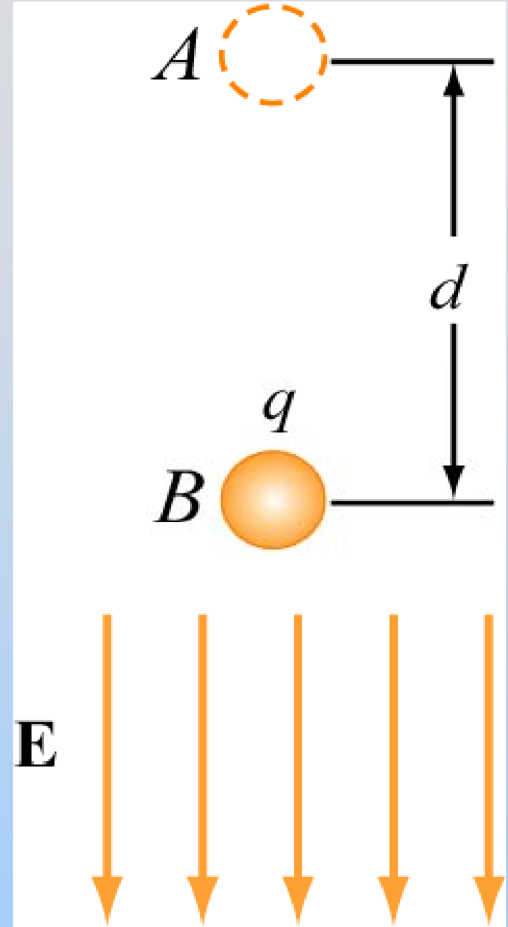
1. is positive.
2. is negative.
3. is zero.
4. can not be determined – not enough info is given.
5. I don’t know

Potential in a Uniform Field

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B -E\hat{\mathbf{j}} \cdot d\vec{\mathbf{s}} = E\int_A^B dy$$

$$= -\ Ed$$



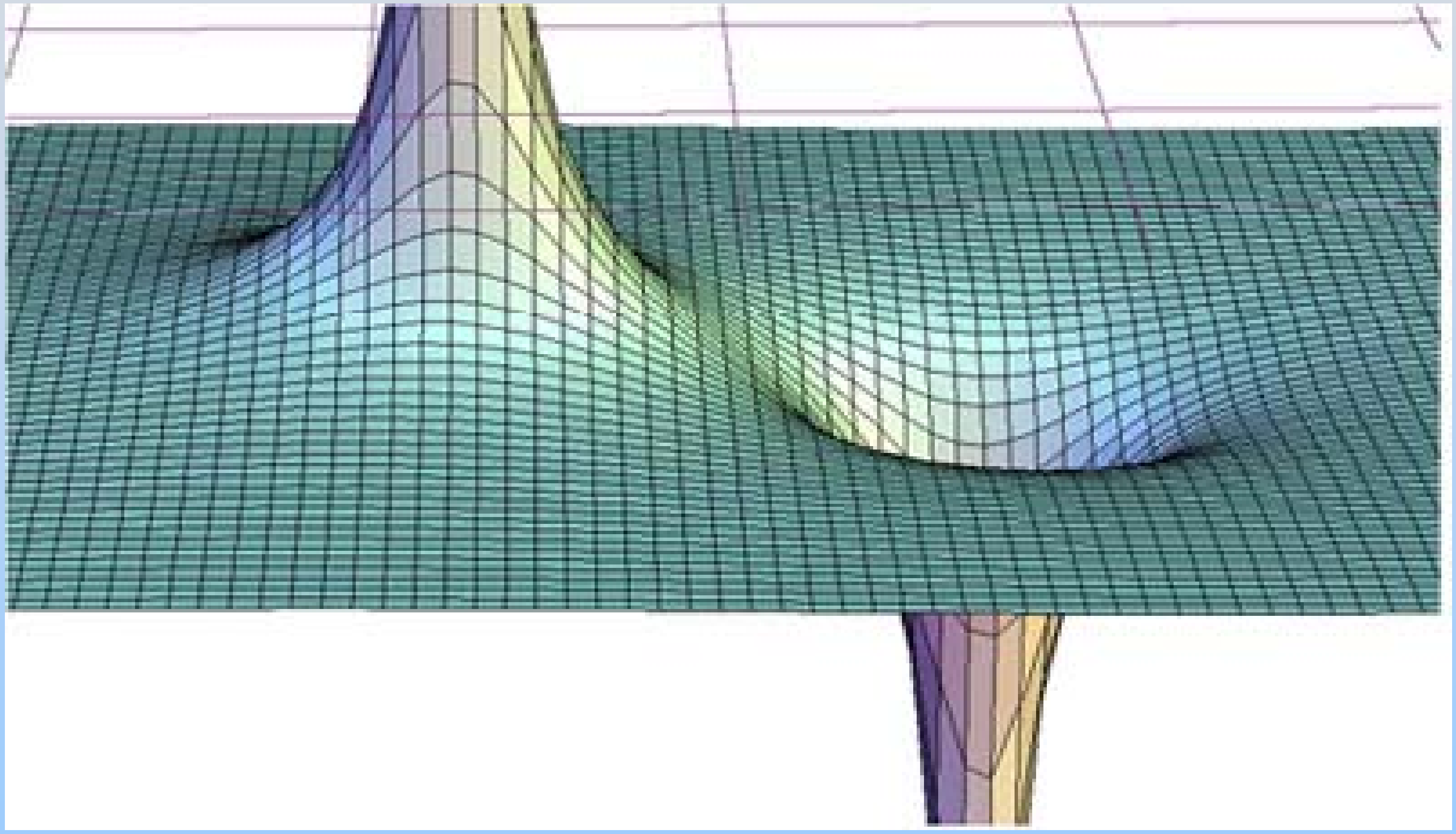
Just like gravity, moving in field direction reduces potential

$$\vec{\mathbf{E}} = -E\hat{\mathbf{j}}$$

$$\mathbf{d}\vec{\mathbf{s}} = dy\hat{\mathbf{j}}$$

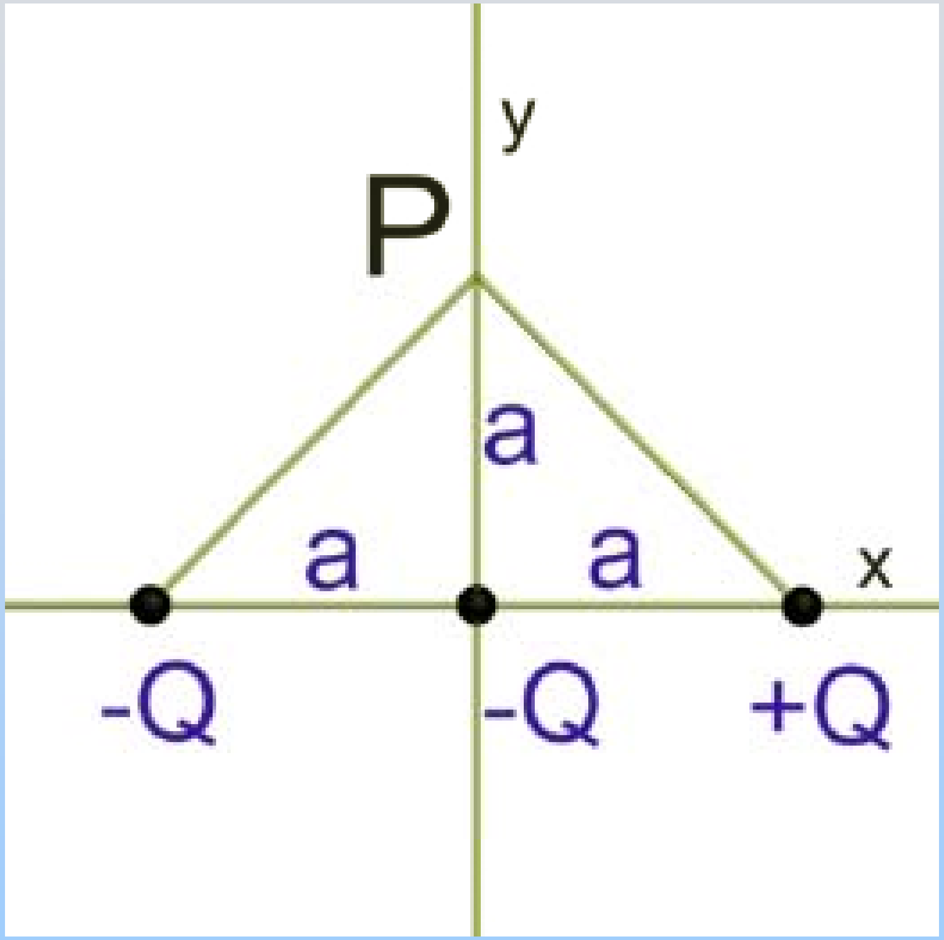
Potential Landscape

Positive Charge



Negative Charge

Problem: Superposition



Consider the 3 point charges at left.

What total electric potential do they create at point P (assuming $V_{\propto} = 0$)