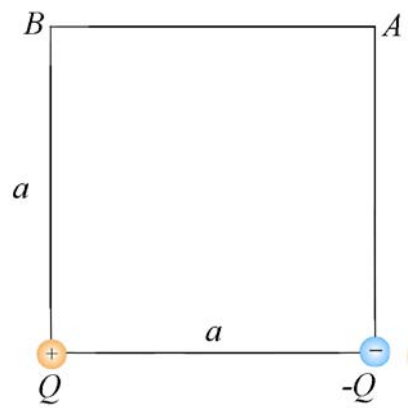
Electric Potential, Discrete and Continuous Distributions of Charge
Challenge Problems

Problem 1:

Two point-like charged objects with charges $+Q$ and $-Q$ are placed on the bottom corners of a square of side a, as shown in the figure.



You move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B. Which of the following statements is true?

a) You do a negative amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

b) You do a positive amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

c) You do a positive amount of work on the electron and the potential energy of the system of three charged objects increases.

d) You do a negative amount of work on the electron and the potential energy of the system of three charged objects decreases.

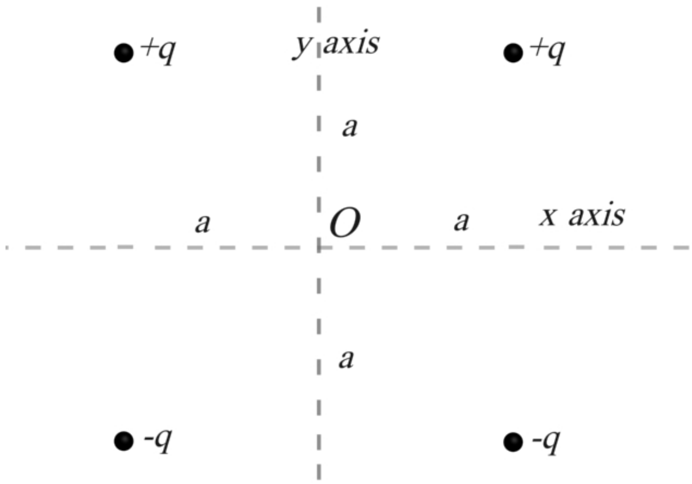
e) You do a negative amount of work on the electron and the potential energy of the system of three charged objects increases.

f) You do a positive amount of work on the electron and the potential energy of the system of three charged objects decreases.

Problem 2:

Four charged point-like objects, two of charge $+q$ and two of charge $-q$, are arranged on the vertices of a square with sides of length $2a$, as shown in the sketch.

a) What is the electric field at point O, which is at the center of the square? Indicate the direction and the magnitude.

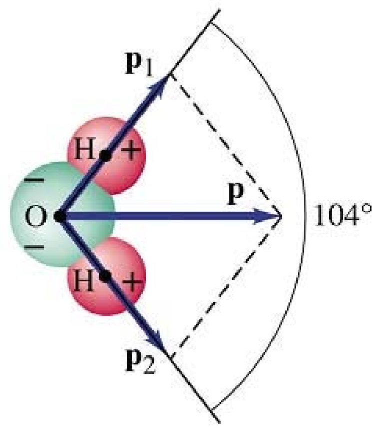


b) What is the electric potential V at point O, the center of the square, taking the potential at infinity to be zero?

c) Sketch on the figure below one path leading from infinity to the origin at O where the integral $\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is trivial to do by inspection. Does your answer here agree with your result in b)?

Problem 3:

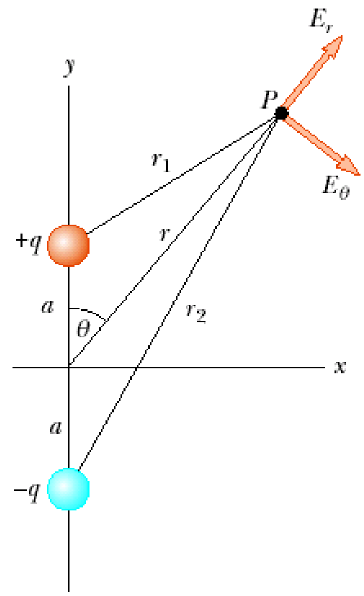
The water molecule, (see figure below), consists of two hydrogen atoms and one oxygen arranged such that the lines joining the center of the O atom with each H atom make an angle of 104^0. A distance $d = 9.6\times10^{-11} m$ separates the centers of each hydrogen and oxygen pair. Each pair of hydrogen and oxygen atoms have dipole moments $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$. The entire molecule has a dipole moment $\vec{\mathbf{p}}$, which is the sum of the dipole moments $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$. The magnitude of the total dipole moment is $|\vec{\mathbf{p}}| = 6.1\times10^{-30} C-m$.



a) What is the effective charge on each hydrogen atom?

b) Set the electric potential at infinity to zero, $V(\infty) = 0$. Find an expression for the electric potential at a point P, a distance $r >> d$, from the center of the water molecule, due to each dipole, $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$. The line connecting the origin at the center of the water molecule with the point P makes an angle θ with the direction of the total dipole moment $\vec{\mathbf{p}}$.

Problem 4:



An electric dipole is located along the y axis as shown in the figure. The magnitude of its electric dipole moment is defined as $p = 2qa$.

(a) At a point P, which is far from the dipole ($r >> a$), show that the electric potential is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

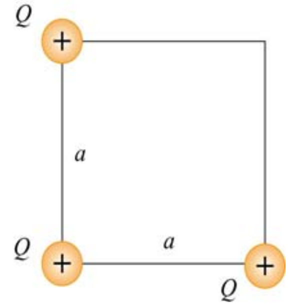
(b) Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

Do these results seem reasonable for $\theta = 90°$ and $0°$? for $r = 0$?

(c) For the dipole arrangement shown, express V in terms of Cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and $\cos\theta = \dfrac{y}{(x^2 + y^2)^{1/2}}$. Using these results and again taking $r >> a$, calculate the field components E_x and E_y.

Problem 5:



Three identical charges $+Q$ are placed on the corners of a square of side a, as shown in the figure.

(a) What is the electric field at the fourth corner (the one missing a charge) due to the first three charges?

(b) What is the electric potential at that corner?

(c) How much work does it take to bring another charge, $+Q$, from infinity and place it at that corner?

(d) How much energy did it take to assemble the pictured configuration of three charges?

Problem 6:

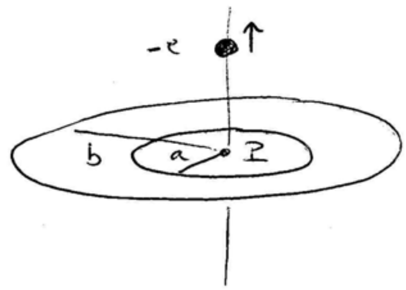
A thin rod extends along the z-axis from $z = -d$ to $z = d$. The rod carries a charge Q uniformly distributed along its length $2d$ with linear charge density $\lambda = Q/2d$.

a) Find the electric potential at a point $z > d$ along the z-axis. Indicate clearly where you have chosen your zero reference point for your potential.

b) Use the result that $\vec{\mathbf{E}} = -\vec{\nabla} V$ to find the electric field at a point $z > d$ along the z-axis.

c) How much work is done to move a particle of mass m and positive charge q from the point $z = 4d$ to the point $z = 3d$?

Problem 7:
A thin washer of outer radius b and inner radius a has a uniform negative surface charge density $-\sigma$ on the washer (note that $\sigma > 0$).



a) If we set $V(\infty) = 0$, what is the electric potential difference between a point at the center of the washer and infinity, $V(P) - V(\infty)$?

b) An electron of mass m and charge $q = -e$ is released with an initial speed v_0 from the center of the hole (at the origin) in the upward direction (along the perpendicular axis to the washer) experiencing no forces except repulsion by the charges on the washer. What speed does it ultimately obtain when it is very far away from the washer (i.e. at infinity)?