Math Review
Challenge Problems

Problem 1:

Triangle Identity

Two sides of the triangle in Figure 1 form an angle θ. The sides have lengths a and b.

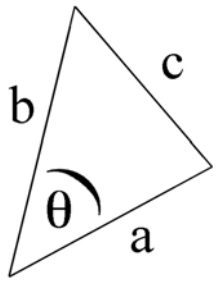

Figure 1: Law of cosines

The length of the opposite side is given by the relation triangle identity

$$c^2 = a^2 + b^2 - 2ab\cos\theta .$$

Suppose we describe the two given sides of the triangles by the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, with $\left|\vec{\mathbf{A}}\right| = a$ and $\left|\vec{\mathbf{B}}\right| = b$.

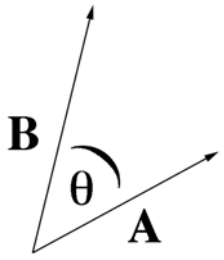

Figure 2: Vector construction

1) What is the geometric meaning of the vector $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$?
2) The square root of the dot product $\left|\vec{\mathbf{C}}\right| = \sqrt{\vec{\mathbf{C}} \cdot \vec{\mathbf{C}}}$ is the magnitude of the difference of the vectors. Show that the magnitude of the difference is the length of the opposite side of the triangle shown in figure 1, $\left|\vec{\mathbf{C}}\right| = c$.

Problem 2:

Dot and Cross products

Three vectors $\vec{\mathbf{A}}$ $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ form a geometric solid as shown in Figure 3. Show that the volume of the solid is equal to $\vec{\mathbf{C}} \cdot (\vec{\mathbf{A}} \times \vec{\mathbf{B}})$.

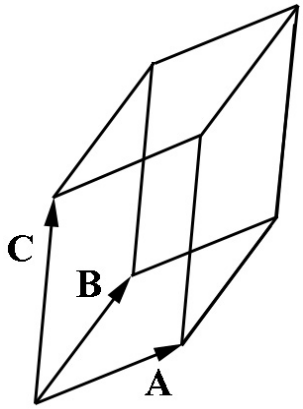

Figure 3: Volume

Problem 3:

Two Vectors

Given two vectors, $\vec{\mathbf{A}} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ and $\vec{\mathbf{B}} = (5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$, evaluate the following:

(a) $3\vec{\mathbf{A}} + \vec{\mathbf{B}}$;
(b) $\vec{\mathbf{A}} - 4\vec{\mathbf{B}}$;
(c) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$;
(d) $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$.
(e) What is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$?
(f) Find a unit vector perpendicular to $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$?

8.02SC Physics II: Electricity and Magnetism

Fall 2010