Module 20:
Sources of Magnetic Fields:
Ampere's Law

Module 20: Outline

Ampere’s Law

Concept Question: Question

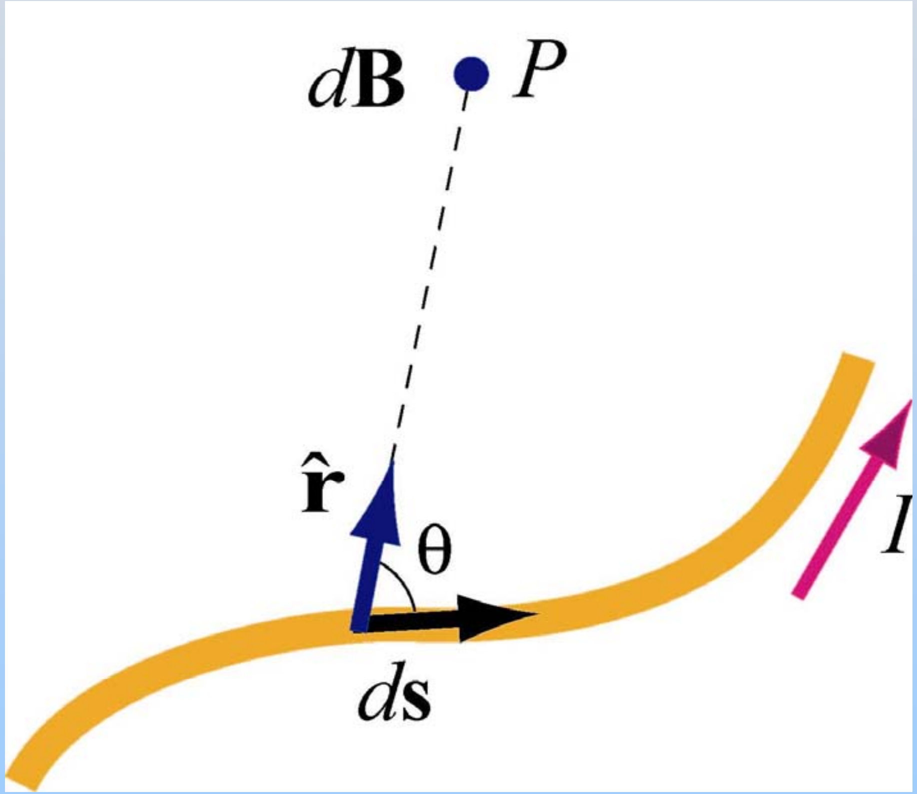
The integral expression $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$

1. is equal to the magnetic work done around a closed path
2. is equal to the current through an open surface bounded by the closed path.
3. is always zero.
4. is equal to the magnetic potential energy between two points.
5. None of the above.

Last Time:
Creating Magnetic Fields:
Biot-Savart

The Biot-Savart Law

Current element of length *d**s*** carrying current *I* produces a magnetic field:



$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

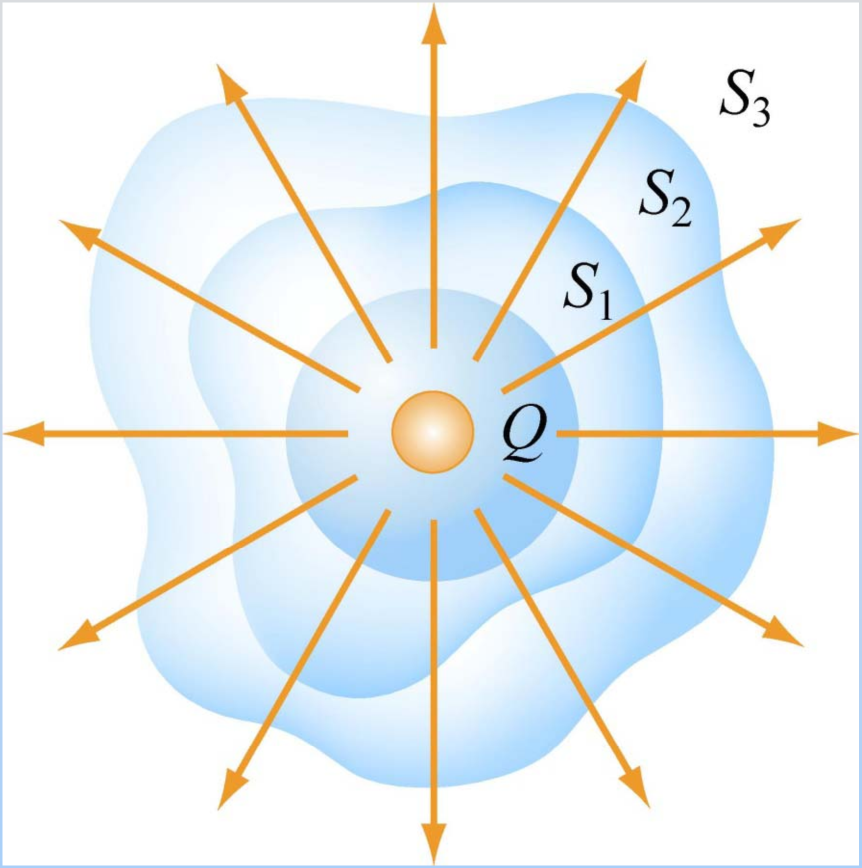
Moving charges are currents too…

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

Today:
3rd Maxwell Equation: Ampere's Law

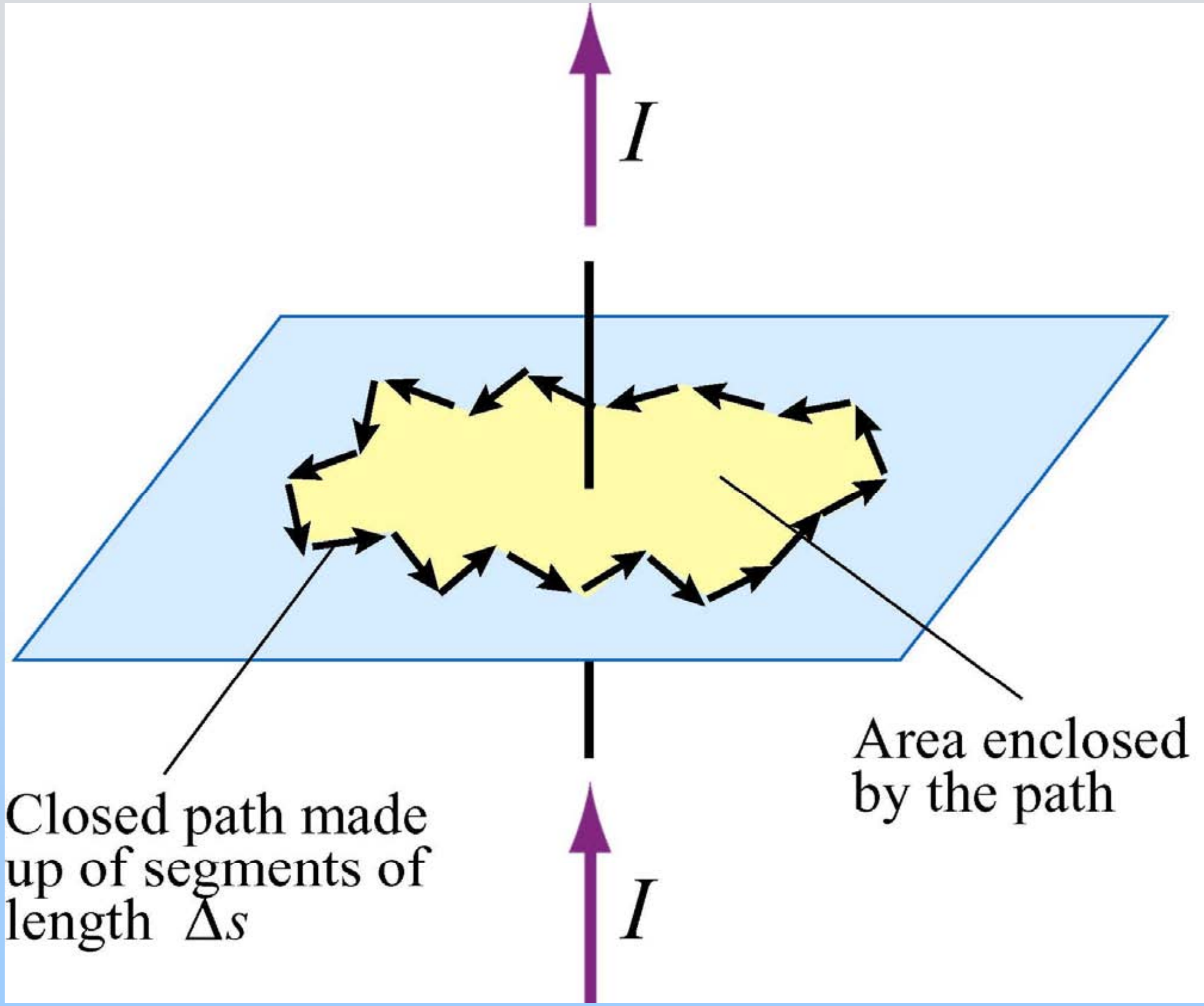
Analog (in use) to Gauss's Law

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

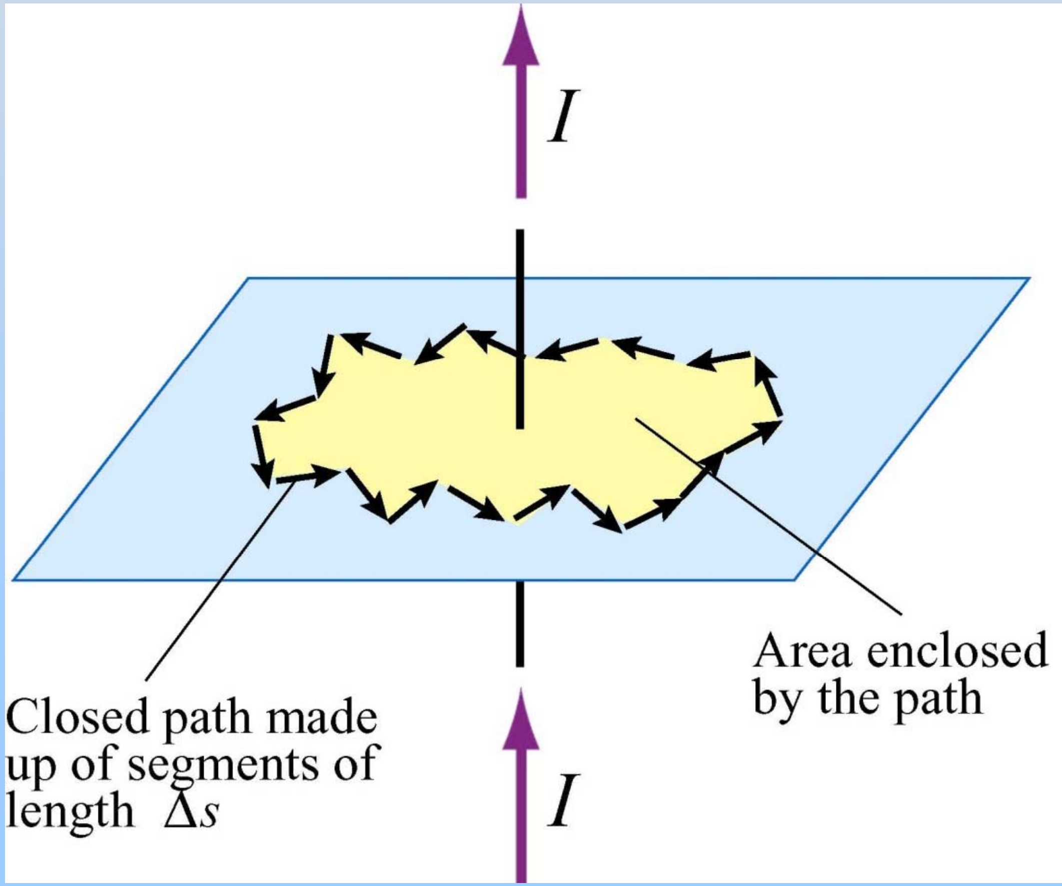
Ampere's Law: The Idea



In order to have a B field around a loop, there must be current punching through the loop

Ampere's Law: The Equation

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$



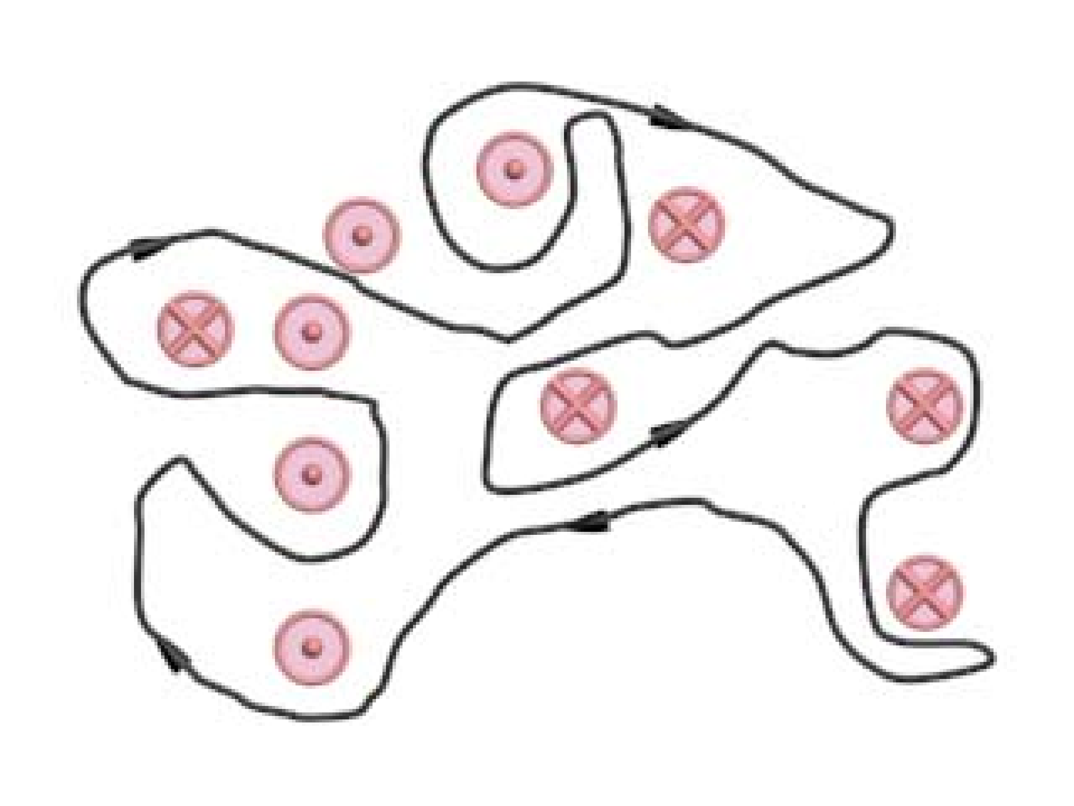
The line integral is around any closed contour bounding an open surface *S*.

I_{enc} is current **through** *S*:

$$I_{enc} = \iint_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

Concept Question Questions: Ampere's Law

Concept Question: Ampere's Law



Integrating B around the loop shown gives us:

1. a positive number
2. a negative number
3. zero

Biot-Savart vs. Ampere

Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\,\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

Applying Ampere's Law

1. Identify regions in which to calculate B field. Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

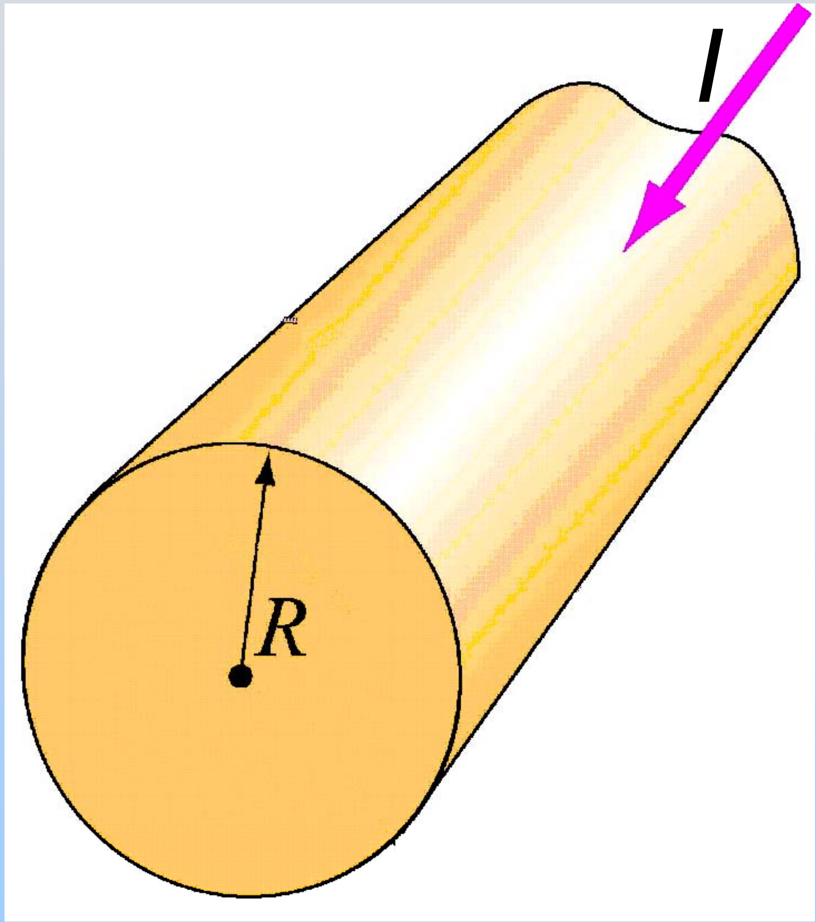
Qualifications

Ampere's Law is only useful for calculation in certain specific situations, involving highly symmetric currents.

Only holds for constant fields. We will need to introduce another term when the electric field is changing in time.

Here are the basic examples…

Problem: Infinite Wire



A cylindrical conductor has radius R and a uniform current density with total current I. For the

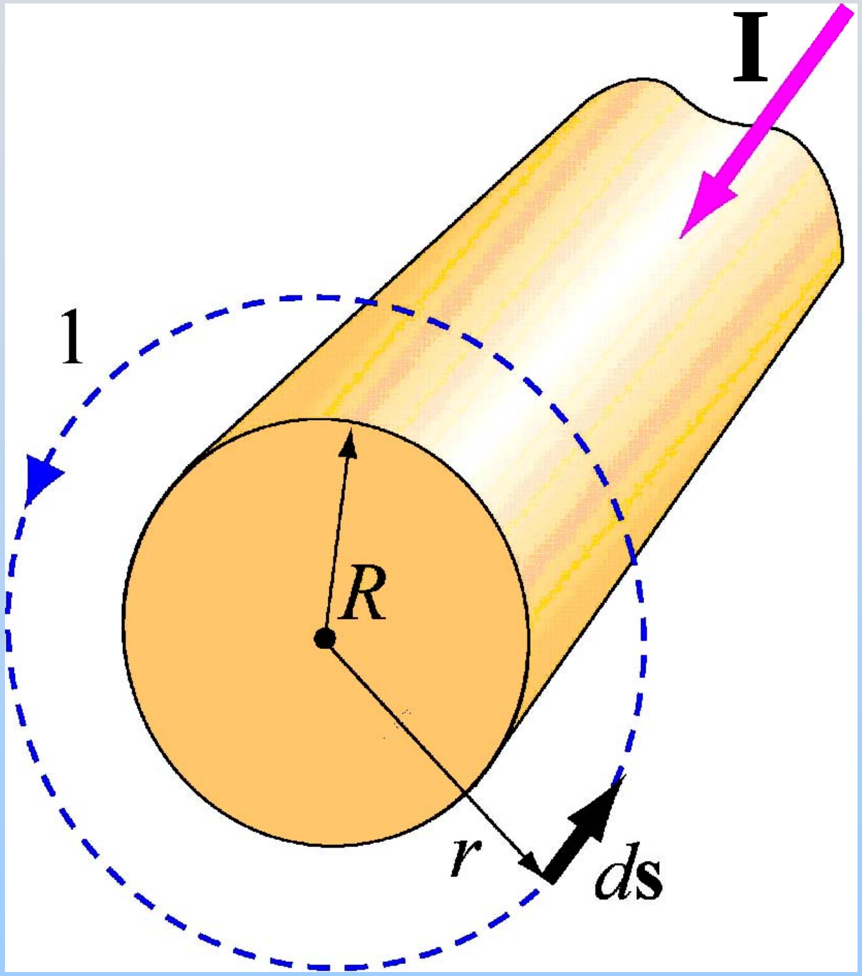
two regions:

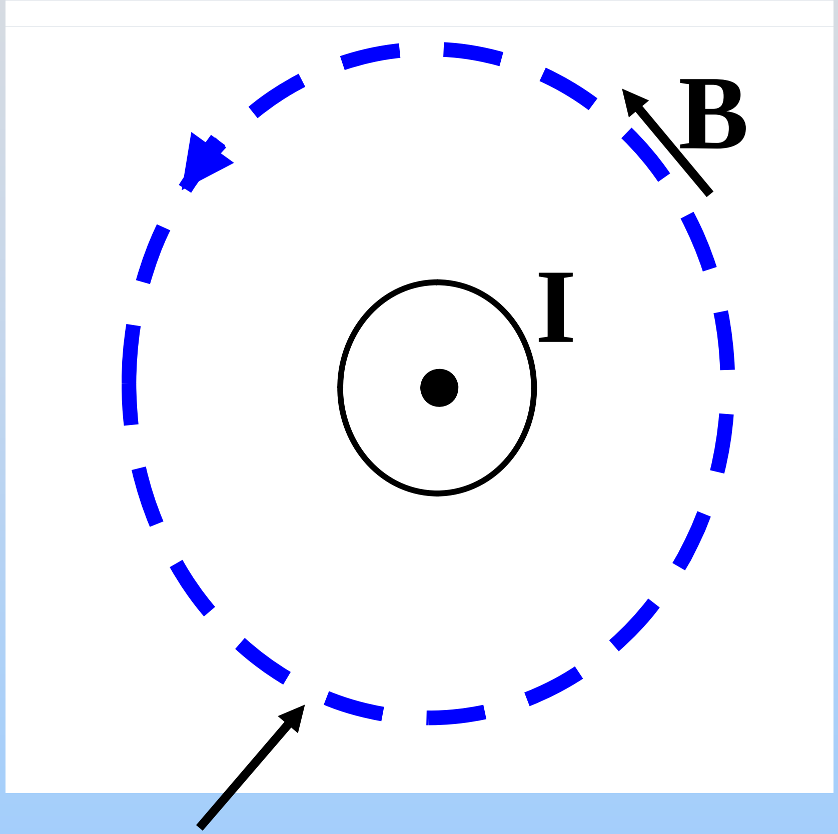
(1) outside wire ($r \geq R$)

(2) inside wire ($r < R$)

draw diagrams showing your choice of Amperian Loop and any parameters that you may need for that loop.

Solution: Ampere's Law Infinite Wire



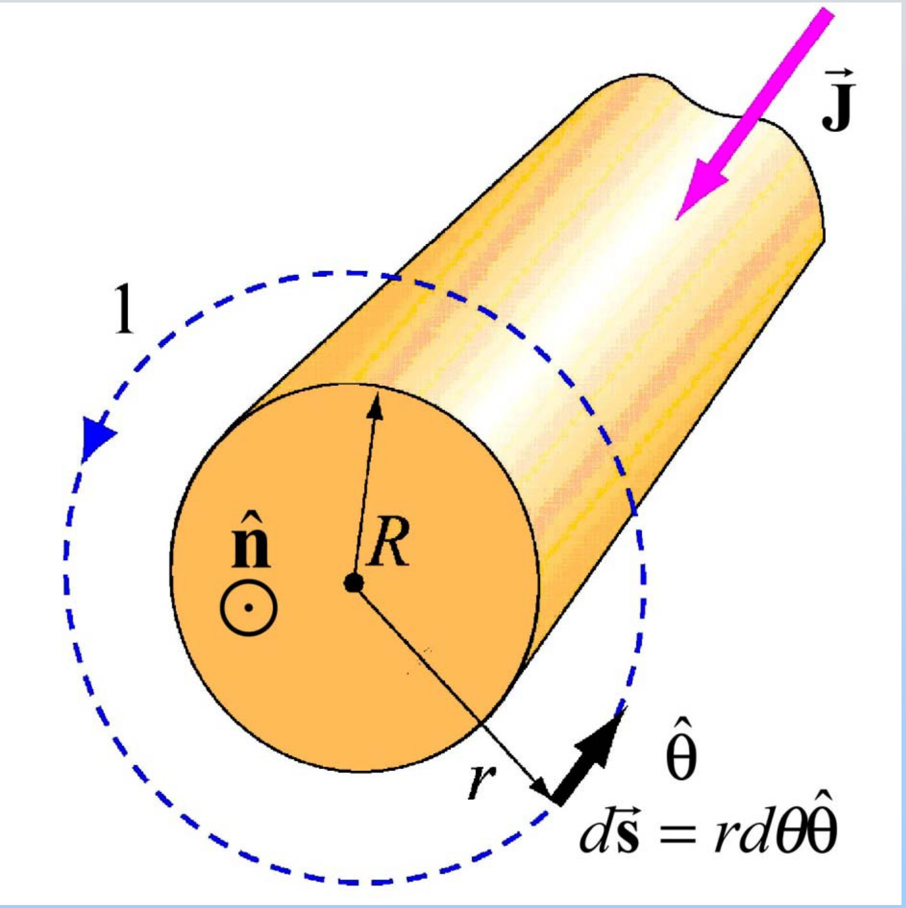


Amperian Loop:

B is Constant & Parallel

I Penetrates

Example: Infinite Wire



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry →

Amperian Circle

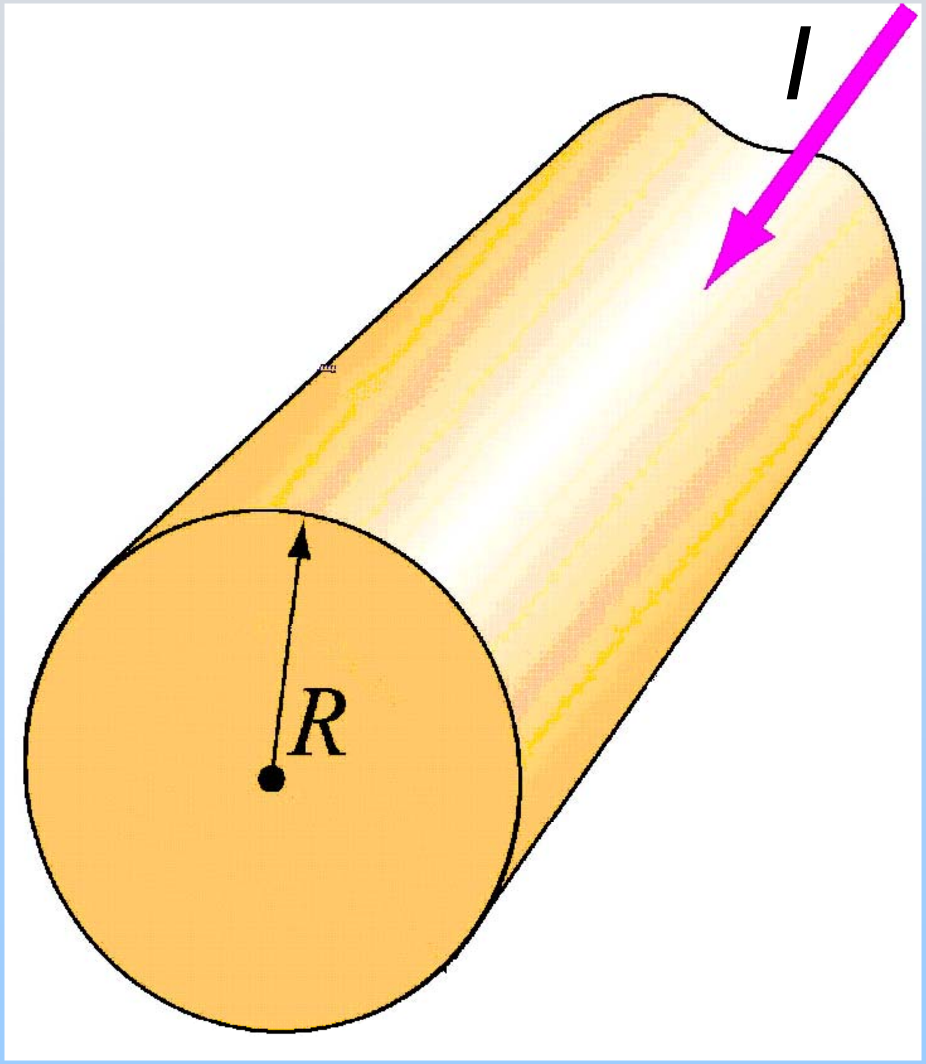
B-field counterclockwise

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = B\left(2\pi r\right)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

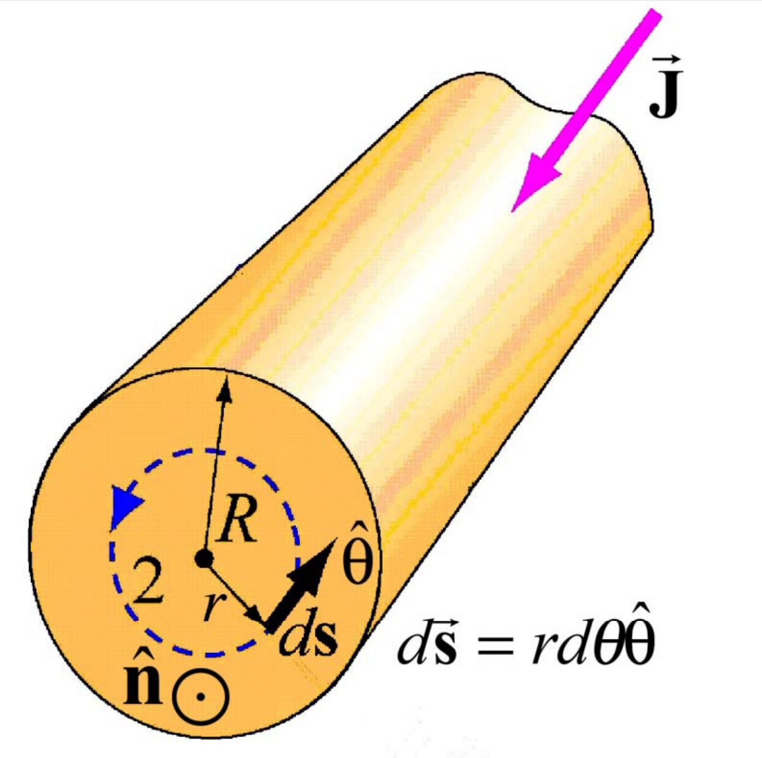
Problem: Inside



We just found B(r>R)

Now you find B(r<R)

Solution: Infinite Wire



Region 2: Inside wire (r < R)

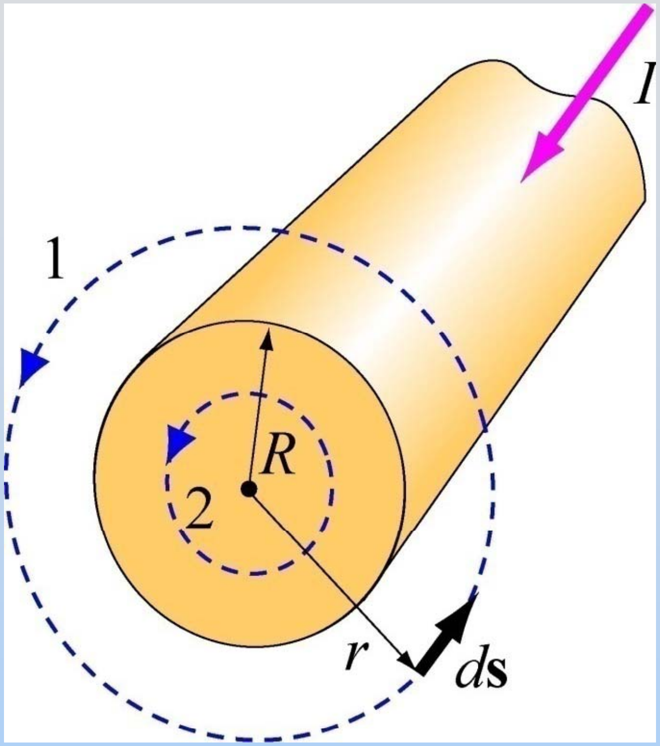
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B\left(2\pi r\right)$$

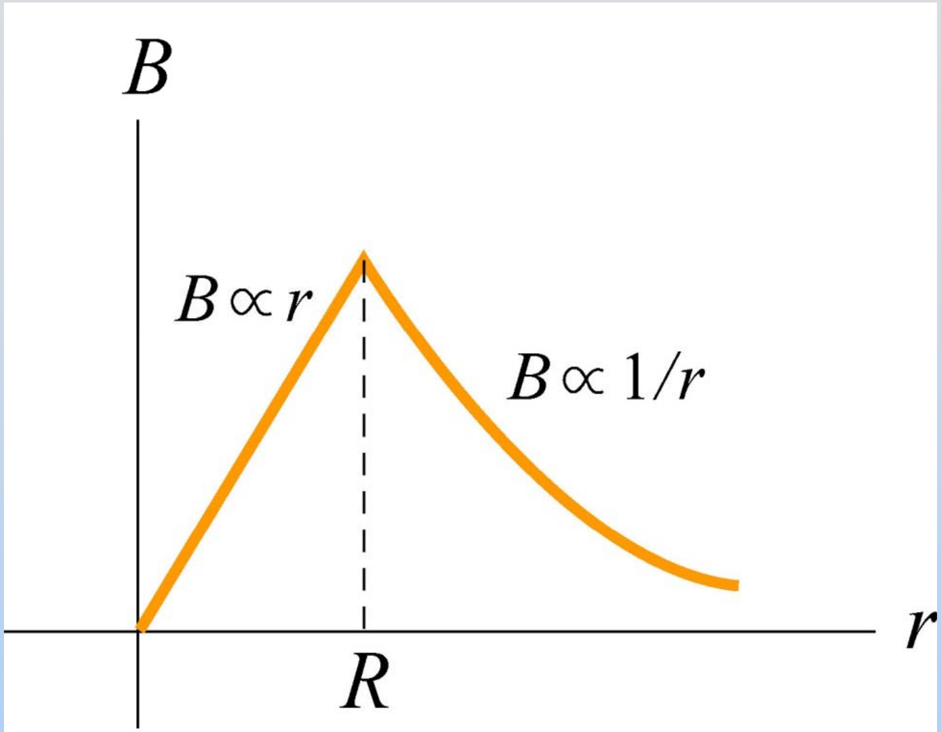
$$- \mu_0 I_{enc} - \mu_0 I \left(\frac{\pi r^2}{\pi R^2} \right)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \hat{\boldsymbol{\theta}}$$

Could also say: $J = \frac{I}{A} = \frac{I}{\pi R^2}; \; I_{enc} = J A_{enc} = \frac{I}{\pi R^2}(\pi r^2)$

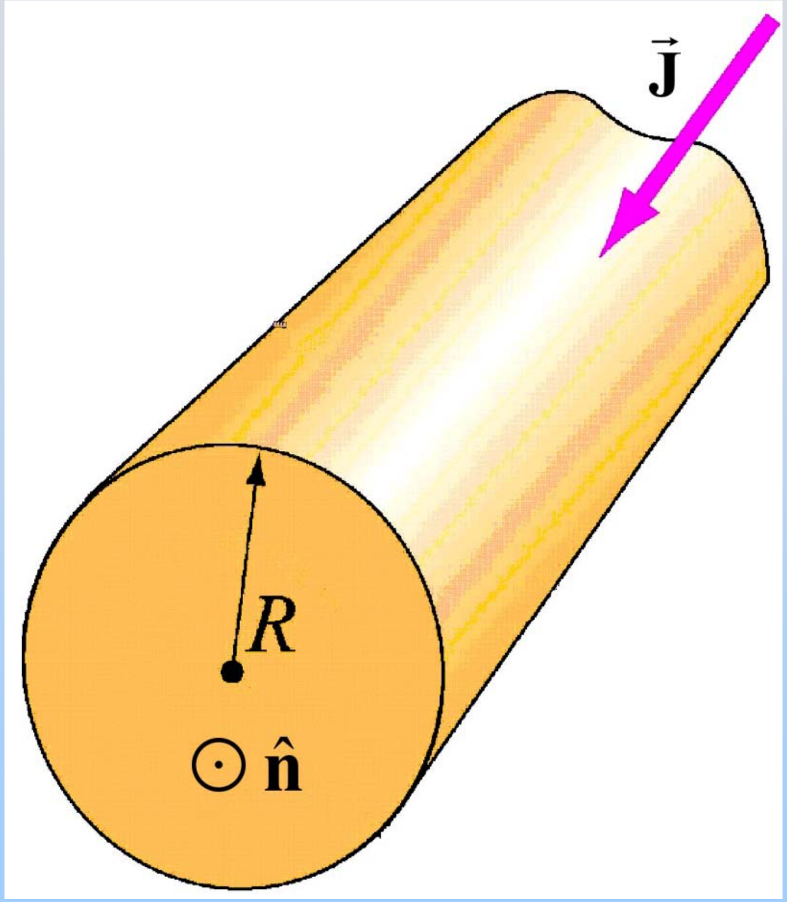
Example: Infinite Wire





$$B_{in} - \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

Problem: Non-Uniform Cylindrical Wire



A cylindrical conductor has radius R and a non-uniform current density with total current:

$$\vec{\mathbf{J}} = J_0 \frac{R}{r} \hat{\mathbf{n}}$$

Find B everywhere

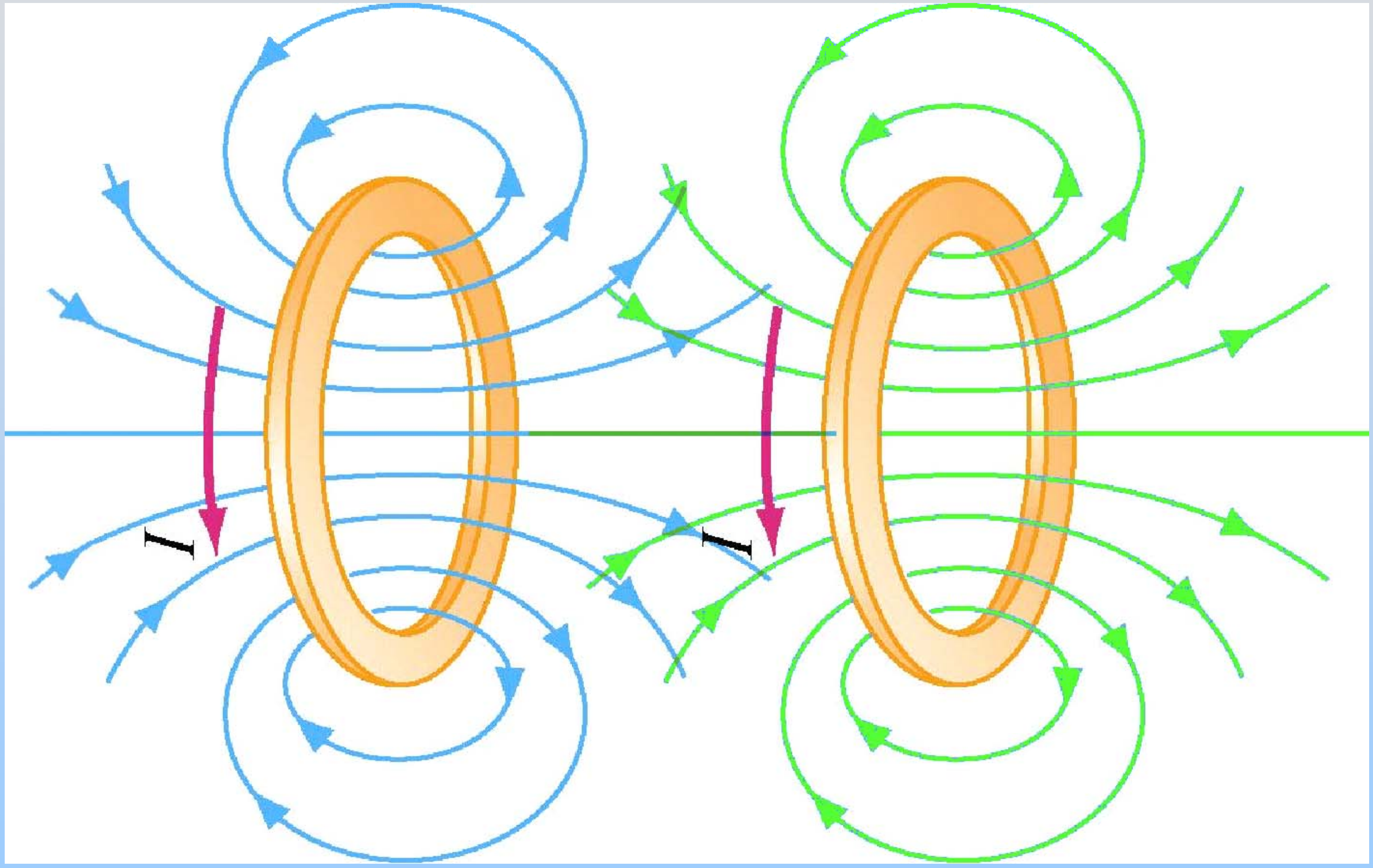
Applying Ampere's Law

In Choosing Amperian Loop:

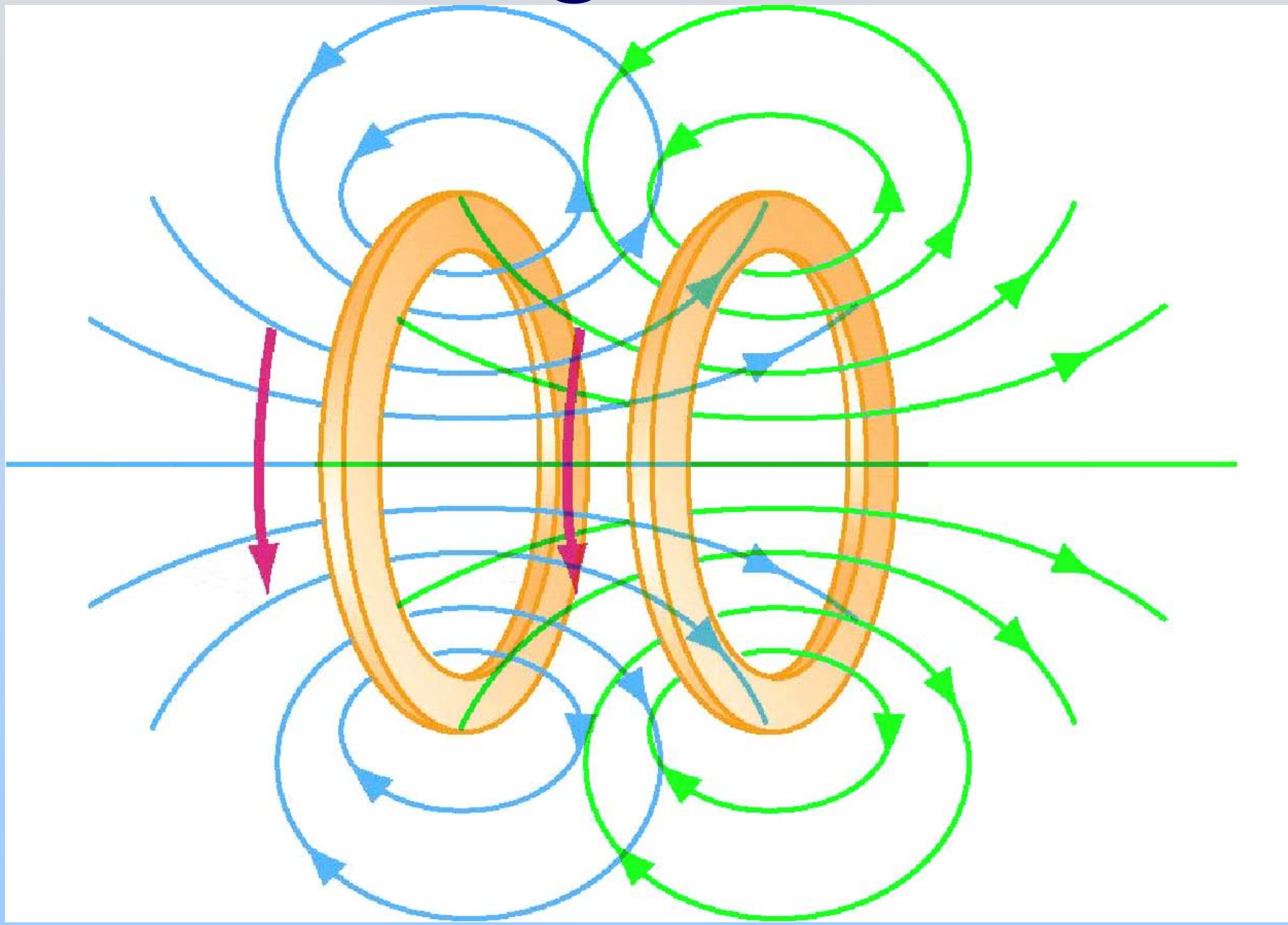
- Study & Follow Symmetry
- Determine Field Directions First
- Think About Where Field is Zero
- Loop Must
  - Be Parallel to (Constant) Desired Field
  - Be Perpendicular to Unknown Fields
  - Or Be Located in Zero Field

Other Geometries

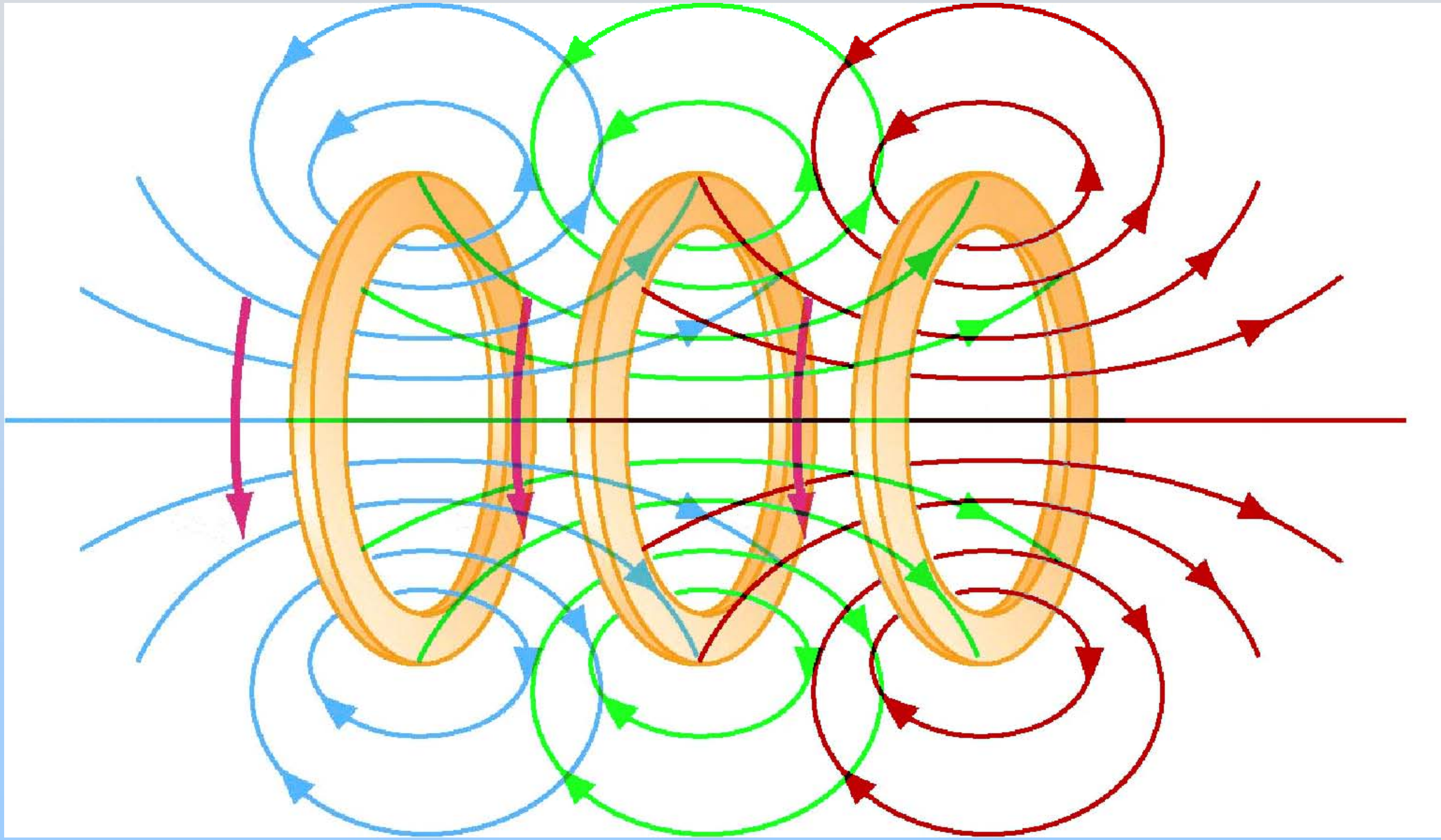
Two Loops



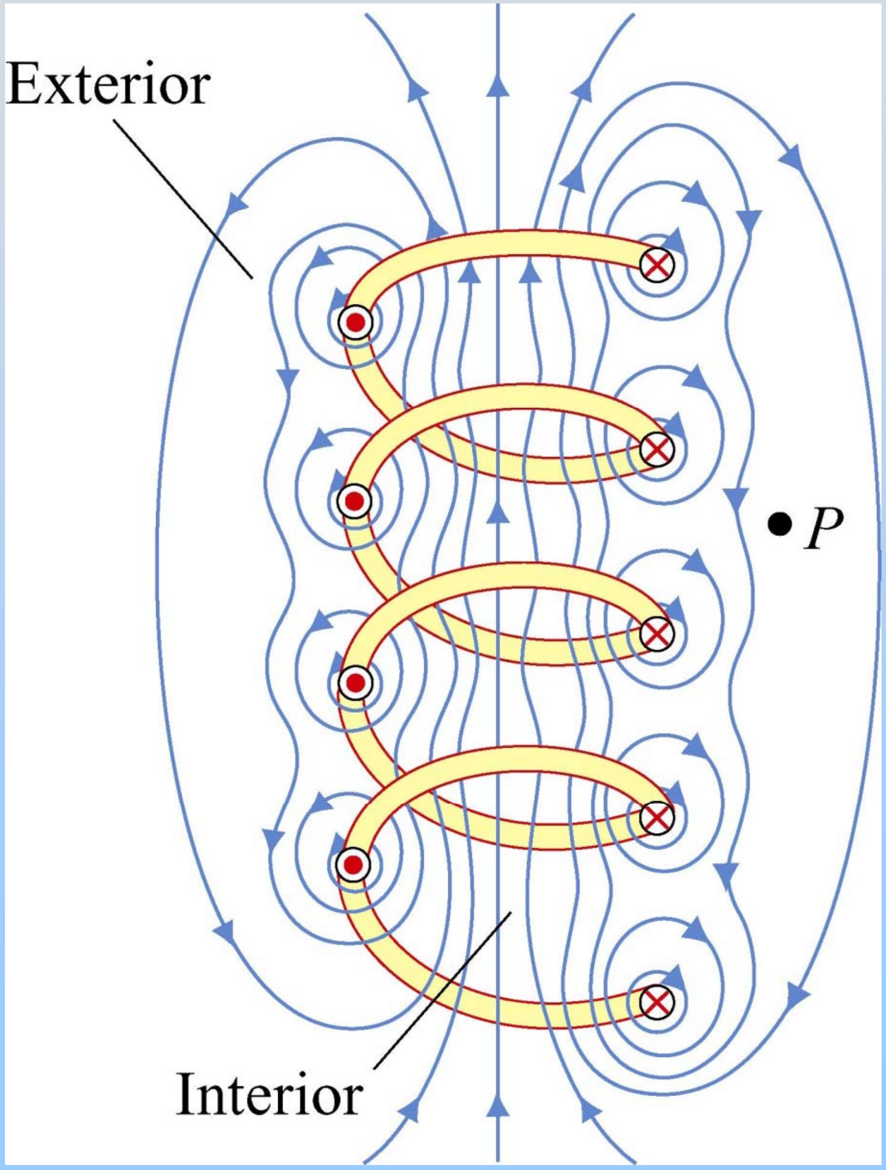
Two Loops Moved Closer Together



Multiple Wire Loops



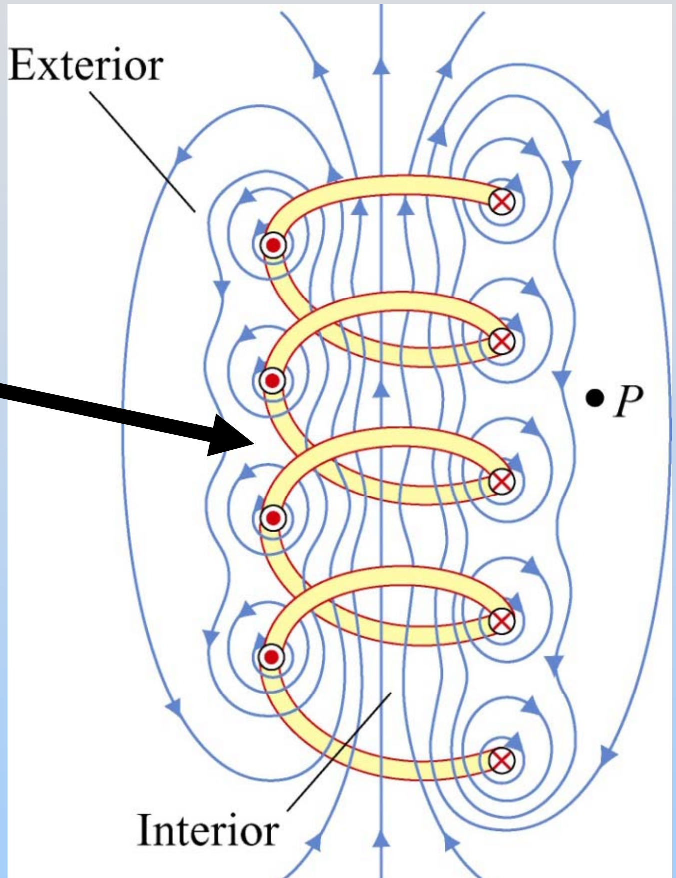
Multiple Wire Loops – Solenoid



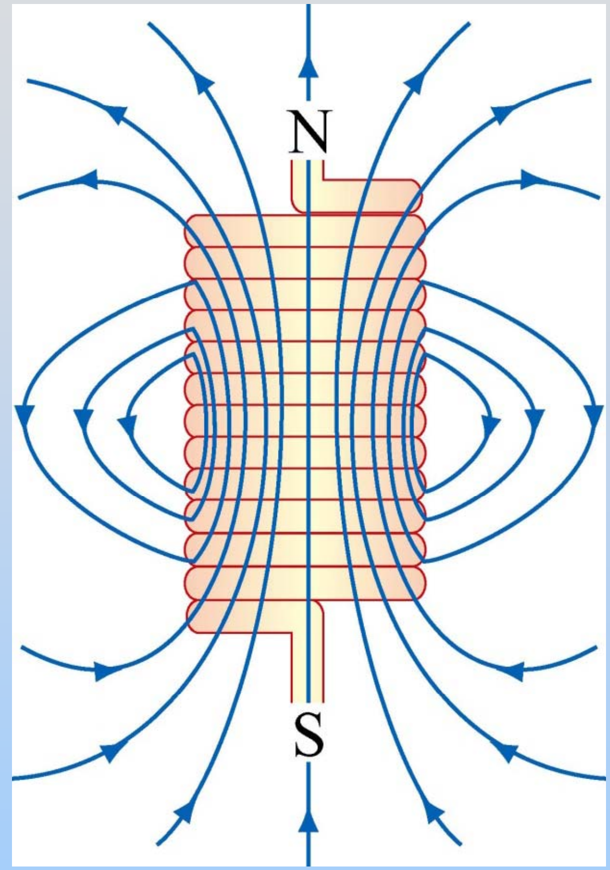
Demonstration:
Long Solenoid

Magnetic Field of Solenoid

Horiz.
comp.
cancel



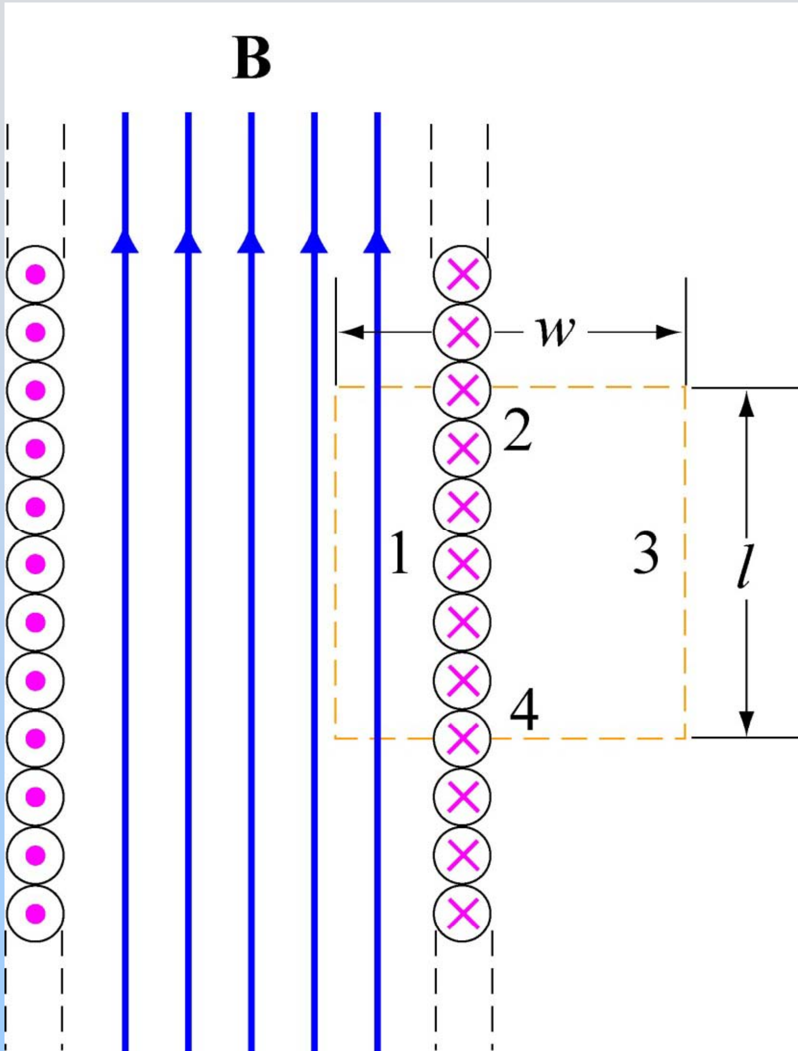
loosely wound



tightly wound

For ideal solenoid, B is uniform inside & zero outside

Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\,\vec{\mathbf{s}} \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \text{ along side 3} \end{cases}$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \int_{\mathbf{1}} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{\mathbf{2}} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{\mathbf{3}} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{\mathbf{4}} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}}$$

$$= \quad Bl \quad + \quad 0 \quad + \quad 0 \quad + \quad 0$$

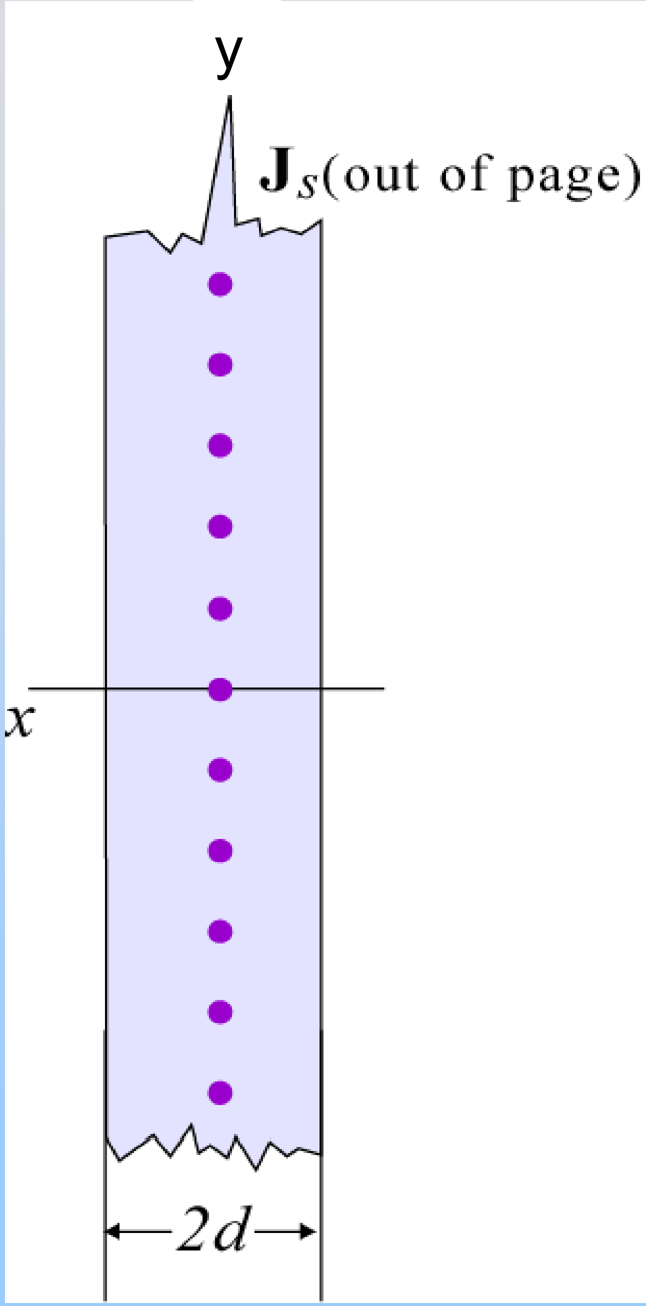
$I_{enc} = nlI$ n: turn density

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N / L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

Problem: Current Sheet

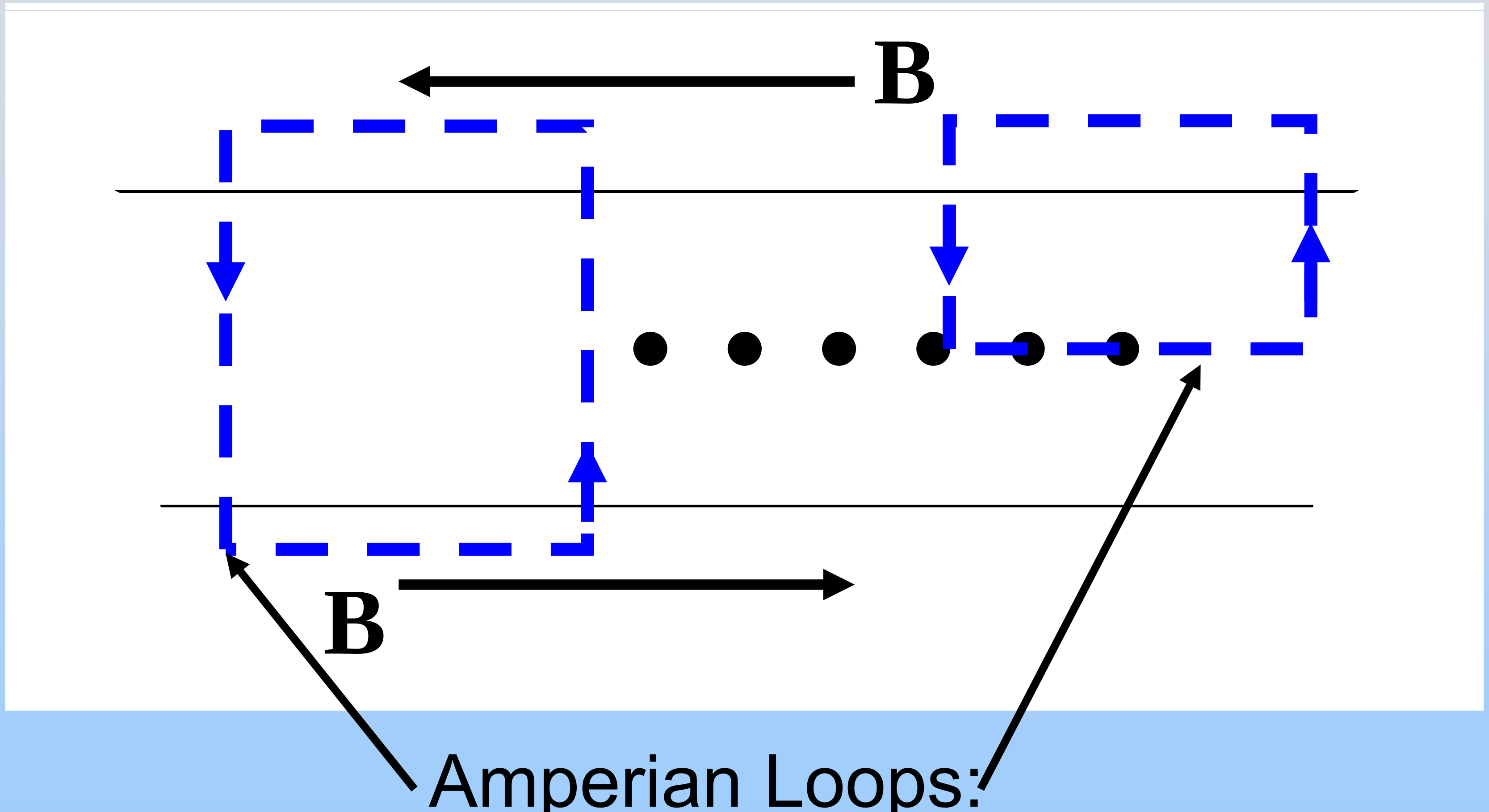


A sheet of current (infinite in the y & z directions, of thickness 2d in the x direction) carries a uniform current density:

$$\vec{\mathbf{J}}_s = J\hat{\mathbf{k}}$$

Find B for x > 0

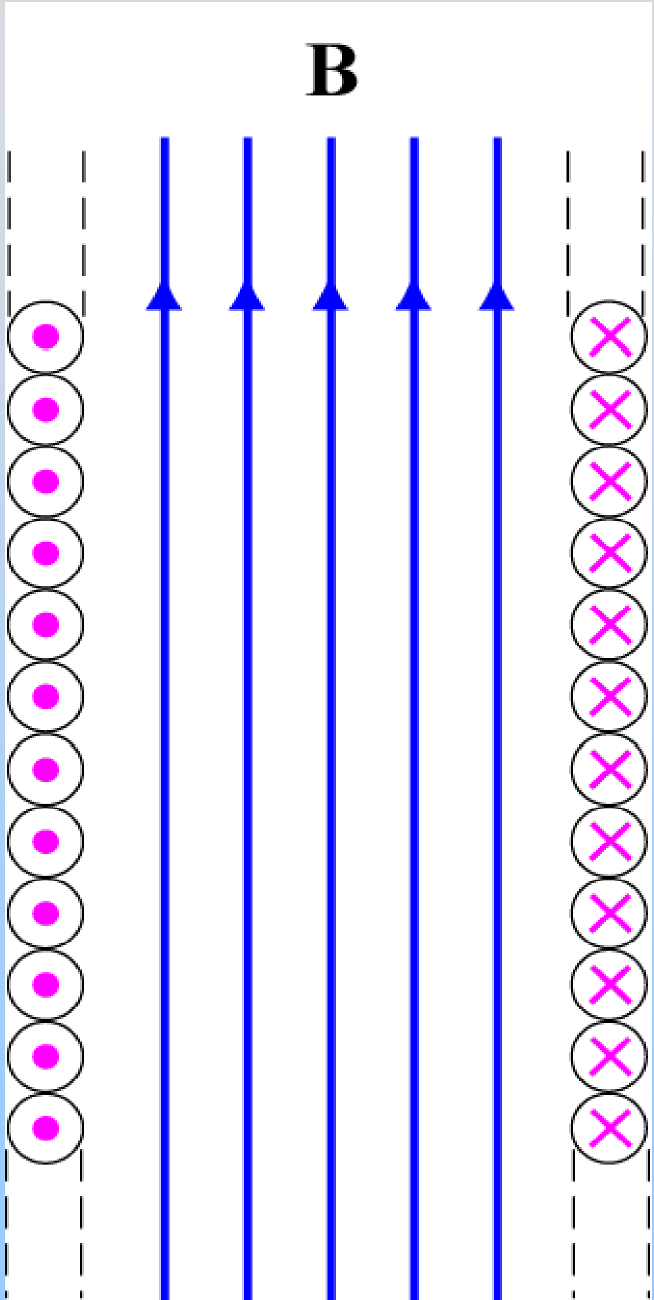
Ampere's Law: Infinite Current Sheet



B is Constant & Parallel OR Perpendicular OR Zero

I Penetrates

Solenoid is Two Current Sheets



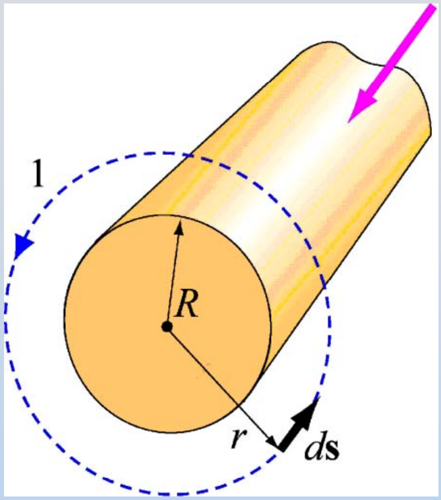
Field outside current sheet should be half of solenoid, with the substitution:

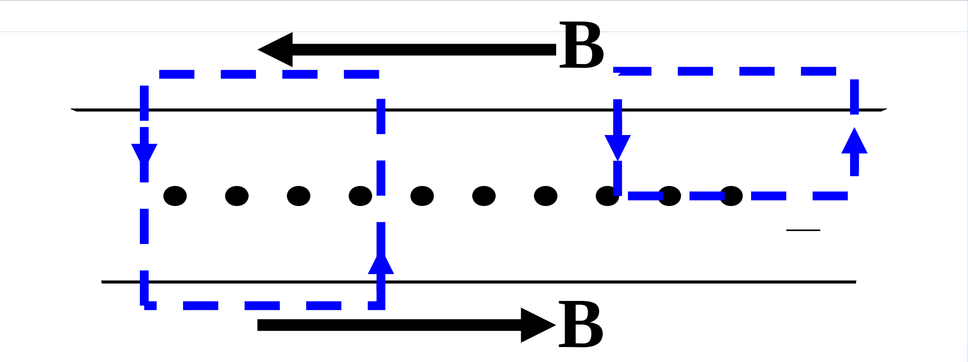
$$nI = 2dJ$$

This is current per unit length (equivalent of λ, but we don't have a symbol for it)

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

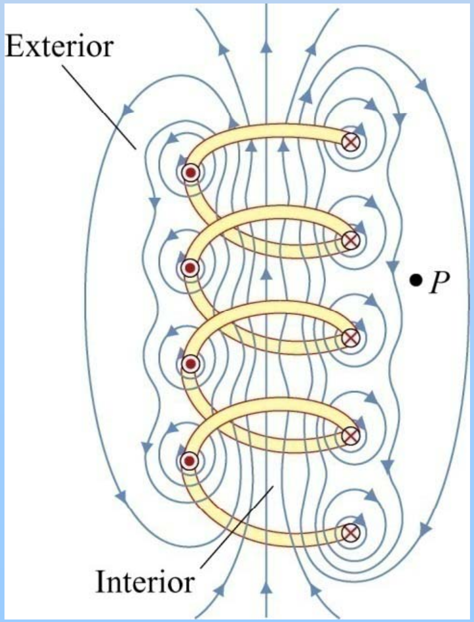
Long Circular Symmetry

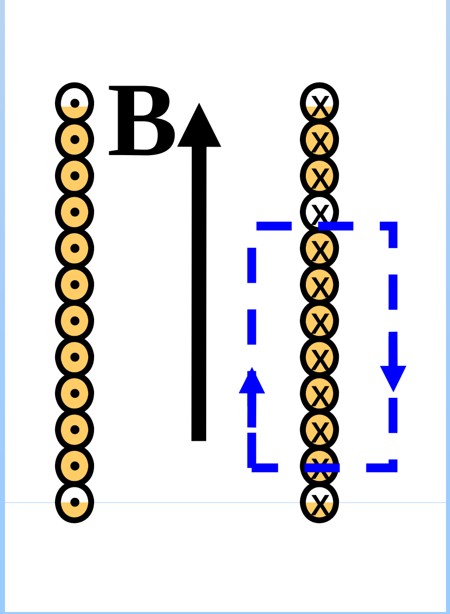


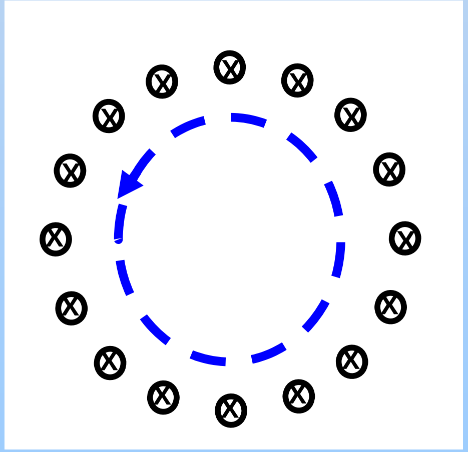


(Infinite) Current Sheet

Solenoid = 2 Current Sheets







Torus

Brief Review Thus Far…

Maxwell's Equations (So Far)

Gauss's Law: $$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: $$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

No Magnetic Monopoles! (No diverging B Fields)

Ampere's Law: $$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc}$$

Currents make curling Magnetic Fields