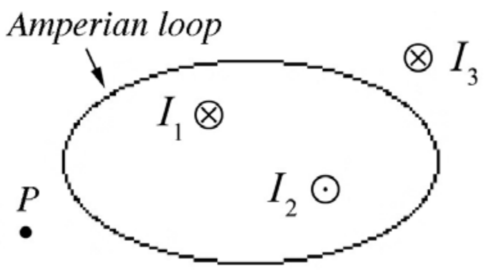
Creating Fields: Ampere's Law
Challenge Problem Solutions

Problem 1:

The sketch below shows three wires carrying currents I_1, I_2 and I_3, with an Ampèrian loop drawn around I_1 and I_2. The wires are all perpendicular to the plane of the paper.



Which currents produce the magnetic field at the point P shown in the sketch (circle one)?

a) I_3 only.

b) I_1 and I_2.

c) I_1, I_2 and I_3.

d) None of them.

e) It depends on the size and shape of the Amperian Loop.

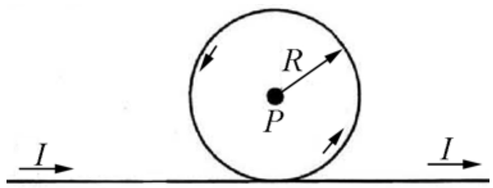
Problem 1 Solution:

c. All three currents I_1, I_2 and I_3 contribute to the magnetic field at the

at the point P .

Problem 2:

Find the magnitude and direction of the magnetic field at the point P generated by the current carrying wire and loop depicted in the figure.



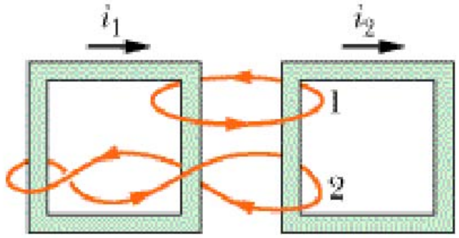
Problem 2 Solution:

The magnetic field is the superposition of the field of a very long wire and a circular current loop. At the point *P*, both fields point in the same direction, out of the page. The magnitude is given by

$$B(P) = B_{\infty-wire}(P) + B_{circular\ wire}(P) = \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2R}\left(\frac{1}{\pi} + 1\right)$$

Problem 3:

The figure below shows two closed paths wrapped around two conducting loops carrying currents i_1 and i_2. What is the value of the integral for **(a)** path 1 and **(b)** path 2?



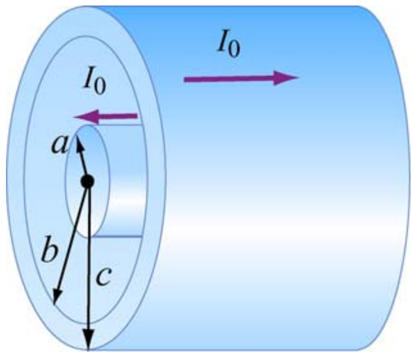
Problem 3 Solution:

To do this you have to use the right hand rule to check whether the currents are positive or negative relative to the path. On path 1 i_1 penetrates in the negative direction while i_2 penetrates in the positive direction, so $\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o \left(i_2 - i_1\right)}$.

On path 2 i_1 penetrates twice in the negative direction and i_2 once in the negative direction so $\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = -\mu_o \left(2i_1 + i_2\right)}$

Problem 4:

A coaxial cable consists of a solid inner conductor of radius *a*, surrounded by a concentric cylindrical tube of inner radius *b* and outer radius *c*. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections*. Determine the magnitude and direction of the magnetic field at a distance *r* from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance *r* from the axis.



(a) $r < a$;

(b) $a < r < b$;

(c) $b < r < c$;

(d) $r > c$.

Problem 4 Solutions:

(a) The enclosed current is $I_{enc} = I_0\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{I_0 r^2}{a^2}$. Applying Ampere's law, we have

$B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2}$ or $B = \frac{\mu_0 I_0}{2\pi a^2} r$, running counterclockwise when viewed from left

(b) The enclosed current is $I_{enc} = I_0$. Applying Ampere's law, we obtain

$B(2\pi r) = \mu_0 I_0$ or $B = \frac{\mu_0 I_0}{2\pi r}$, running counterclockwise when viewed from left

(c)

$$I_{enc} = I_0 - I_0\left(\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right) = \frac{I_0(c^2 - r^2)}{c^2 - b^2}$$

Applying Ampere’s law,

$$B(2\pi r) = \mu_0 \frac{I_0(c^2 - r^2)}{c^2 - b^2}$$

or $B = \dfrac{\mu_0 I_0(c^2 - r^2)}{2\pi(c^2 - b^2)r}$, running counterclockwise when viewed from left

(d)

$$B = 0 \text{ since } I_{enc} = 0$$

Problem 5:

Consider an infinitely long, cylindrical conductor of radius *R* carrying a current *I* with a *non-uniform* current density $J = \alpha r^2$, where α is a constant and *r* is the distance from the center of the cylinder.

(a) Find the magnetic field everywhere.

(b) Plot the magnitude of the magnetic field as a function of *r*.

[Hint: See Example 9.11.6 for a similar problem.]

Problem 5 Solution:

The enclosed current is given by

$$I_{enc} = \int \vec{J} \cdot d\vec{A} = \int (\alpha r'^2)(2\pi r' dr') = \int 2\pi\alpha r'^3 \, dr'$$

For $r < R$,

$$I_{enc} = \int_0^r 2\pi\alpha \, r'^3 \, dr' = \frac{\pi\alpha \, r^4}{2}$$

Applying Ampere's law, the magnetic field is given by

$$B(2\pi r) = \frac{\mu_0 \pi \alpha r^4}{2}$$

or

$$B = \frac{\mu_0 \alpha}{4} r^3$$

For $r > R$,

$$I_{enc} = \int_0^R 2\pi\alpha r'^3 \, dr' = \frac{\pi\alpha R^4}{2}$$

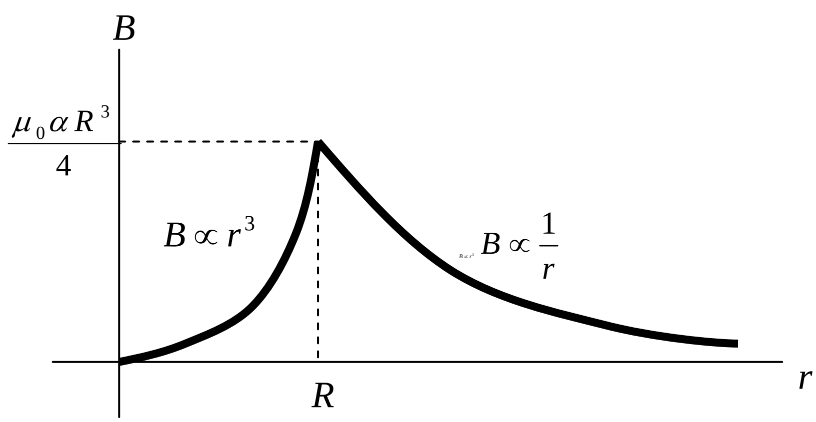
Applying Ampere's law, the magnetic field is given by

$$B(2\pi r) = \frac{\mu_0 \pi\alpha R^4}{2}$$

or

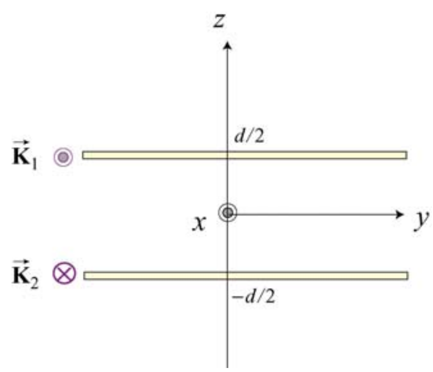
$$B = \frac{\mu_0 \alpha R^4}{4r}$$

(b) Plot the magnitude of the magnetic field as a function of r.



Problem 6:

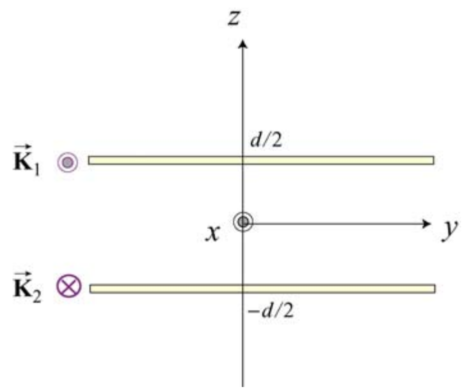
Consider two infinitely large sheets lying in the *xy*-plane separated by a distance *d* carrying surface current densities $\vec{\mathbf{K}}_1 = K\,\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\,\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the *y* direction is infinite.) Note that *K* is the current per unit width perpendicular to the flow.



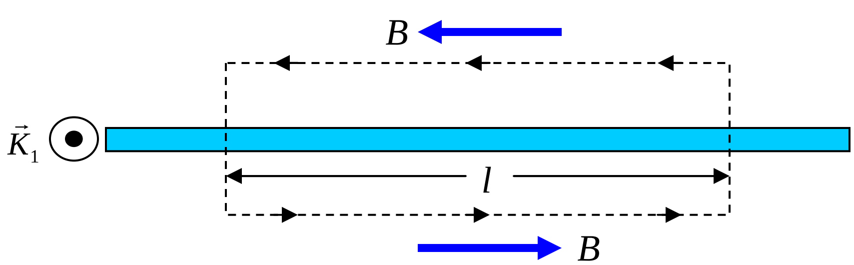
a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.
b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.
c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.
d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\,\hat{\mathbf{i}}$?

Problem 6 Solution:

Consider two infinitely large sheets lying in the *xy*-plane separated by a distance *d* carrying surface current densities $\vec{\mathbf{K}}_1 = K\,\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\,\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the *y* direction is infinite.) Note that *K* is the current per unit width perpendicular to the flow.



(a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.



Consider the Ampere's loop shown above. The enclosed current is given by

$$I_{\text{enc}} = \int \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = Kl$$

Applying Ampere's law, the magnetic field is given by

$$B(2l) = \mu_0 Kl \ \text{ or } \ B = \frac{\mu_0 K}{2}$$

Therefore,

$$\vec{\mathbf{B}}_1 = \begin{cases} -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > \dfrac{d}{2} \\ \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < \dfrac{d}{2} \end{cases}$$

(b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.

The result is the same as part (a) except for the direction of the current:

$$\vec{\mathbf{B}}_2 = \begin{cases} \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > -\dfrac{d}{2} \\ -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

(c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \begin{cases} \mu_0 K\,\hat{\mathbf{j}}, & -\dfrac{d}{2} < z < \dfrac{d}{2} \\ 0, & |z| > \dfrac{d}{2} \end{cases}$$

(d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\,\hat{\mathbf{i}}$?

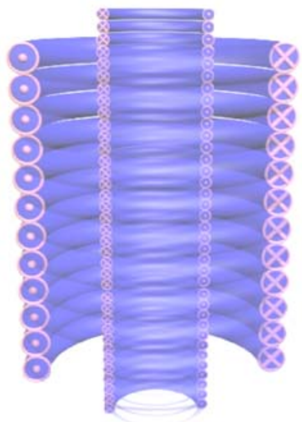
In this case, $\vec{\mathbf{B}}_1$ remains the same but

$$\vec{\mathbf{B}}_2 = \begin{cases} -\dfrac{\mu_0 K}{2}\,\hat{\mathbf{j}}, & z > -\dfrac{d}{2} \\ \dfrac{\mu_0 K}{2}\,\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

Therefore,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \begin{cases} -\mu_0 K\,\hat{\mathbf{j}}, & z > \dfrac{d}{2} \\ 0, & -\dfrac{d}{2} < z < \dfrac{d}{2} \\ \mu_0 K\,\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

Problem 7: Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.



Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all your calculations.

a) $0 < r < R_1$

b) $R_1 < r < R_2$

c) $R_2 < r$

Problem 7 Solution:

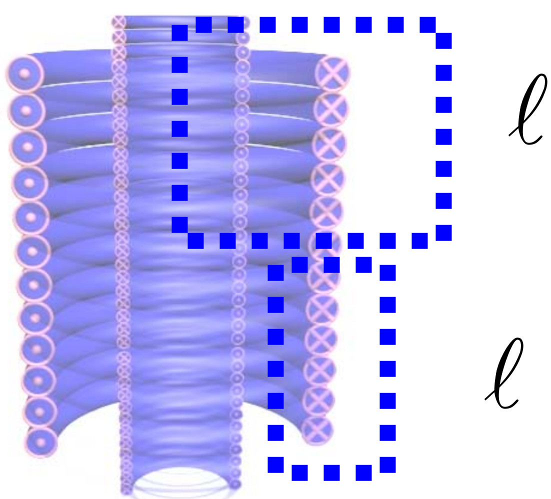
Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.

Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions:

(a) $0 < r < R_1$;

To solve for the magnetic field in this case, we take the top rectangular loop shown in the figure. The current through the loop is

$$I_{\text{enc}} = -n_1 \ell I + n_2 \ell I = (-n_1 + n_2) \ell I$$



The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$, and $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$, we have

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0 = B\ell = \mu_0\left(-n_1\ell I + n_2\ell I\right) \quad \Rightarrow \; \vec{\mathbf{B}} = \mu_0 I\left(-n_1 + n_2\right)\hat{\mathbf{k}}$$

(b) $R_1 < r < R_2$

To solve for the magnetic field in this case, we take the bottom rectangular loop shown in the figure. The current through the loop is

$$I_{\text{enc}} = n_2 \ell I$$

The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\,\vec{\mathbf{s}}$, and $\vec{\mathbf{B}}\cdot d\,\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$, we have

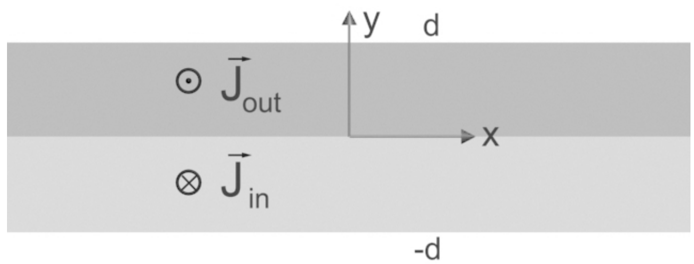
$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0 = B\ell = \mu_0 n_2 \ell I \quad \Rightarrow \; \vec{\mathbf{B}} = \mu_0 n_2 I\,\hat{\mathbf{k}}$$

(c) $R_2 < r$

Since the net current enclosed by the Amperian loop is zero, the magnetic field is zero in this region.

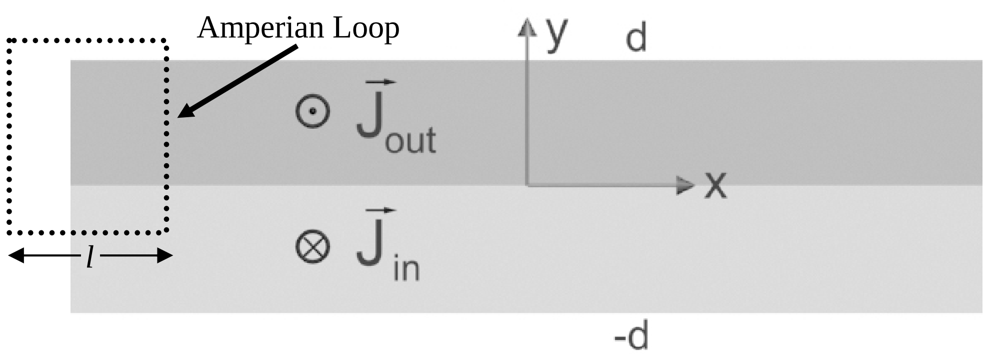
Problem 8:

The figure below shows two slabs of current. Both slabs of current are infinite in the x and z directions, and have thickness d in the y-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\vec{\mathbf{J}}_{out} = J\,\hat{\mathbf{z}}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\vec{\mathbf{J}}_{in} = -J\,\hat{\mathbf{z}}$ into the page.



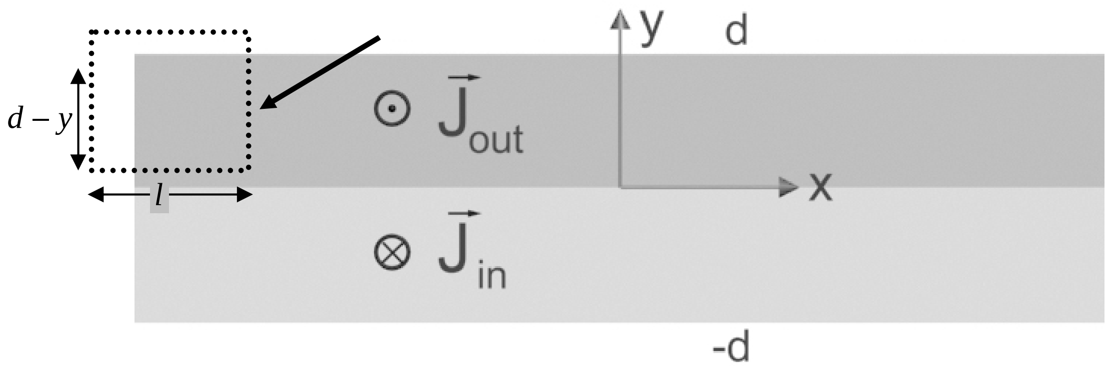
(a) What is the magnetic field for $|y| > d$? Justify your answer.

(b) Use Ampere's Law to find the magnetic field at $y = 0$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.



c) Use Ampere's Law to find the magnetic field for $0 < y < d$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

Amperian Loop



(d) Plot the *x*-component of the magnetic field as a function of the distance *y* on the graph below. Label your vertical axis.

Problem 8 Solution:

(a) Zero. The two parts of the slab create equal and opposite fields for $|y| > d$.

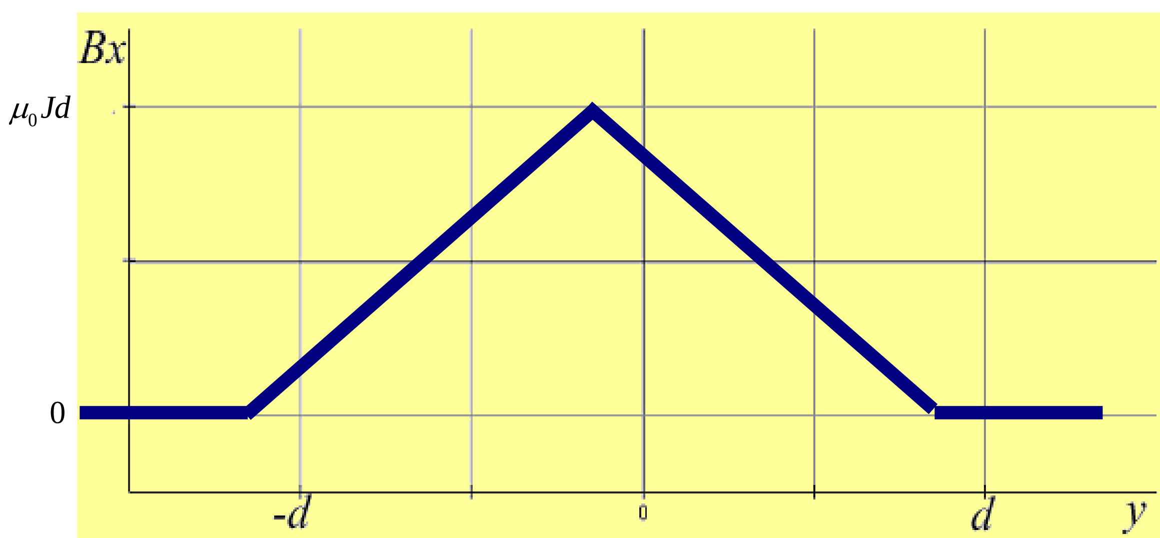
(b) The field at $y = 0$ points to the right (both slabs make it point that way). So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \frac{4\pi}{c} I_{enc} = \mu_0 (Jld) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 Jd\,\hat{\mathbf{i}} \text{ (to the right)}}$$

(c) The field for $0 < y < d$ still points to the right. So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

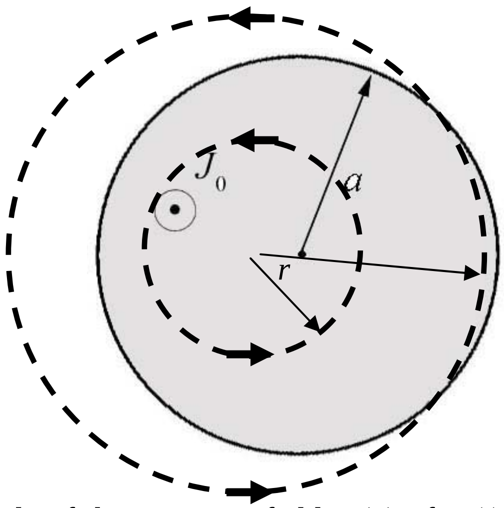
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \mu_o I_{enc} = \frac{4\pi}{c} Jl(d-y) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 J(d-y)\hat{\mathbf{i}} \text{ (to the right)}}$$

(d)



Problem 9:

An infinitely long wire of radius a carries a current density J_0 which is uniform and constant. The current points "out of" the page, as shown in the figure.

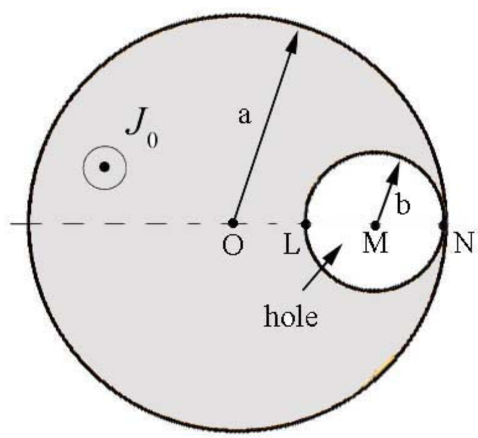


(a) Calculate the magnitude of the magnetic field $B(r)$ for (i) $r < a$ and (ii) $r > a$. For both cases show your Amperian loop and indicate (with arrows) the direction of the magnetic field.

(b) What happens to the answers above if the direction of the current is reversed so that it flows "into" the page ?

(c) Consider now the same wire but with a hole bored throughout. The hole has radius b (with $2b < a$) and is shown in the figure. We have also indicated four special points: O, L, M, and N. The point O is at the center of the original wire and the point M is at the center of the hole. In this new wire, the current density exists and remains equal to J_0 over the remainder of the cross section of the wire. Calculate the magnitude of the magnetic field at (i) the point M, (ii) at the point L, and (iii) at the point N. Show your work.

Hint: Try to represent the configuration as the "superposition" of two types of wires.



Problem 9 Solutions:

(a)The dashed lines above are the Amperian loops I will use for (i) and (ii). They both have a radius of *r*, and in both cases the paths are counterclockwise, as is the B field, due to a current out of the page (right hand rule).

(i) $r < a$.

From Ampere's Law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi r^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi r^2}{2\pi r} = \frac{\mu_0 J_0 r}{2} \text{ counterclockwise}}$$

(ii) $r > a$.

Now we just contain all of the current:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi a^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi a^2}{2\pi r} = \frac{\mu_0 J_0 a^2}{2r} \text{ counterclockwise}}$$

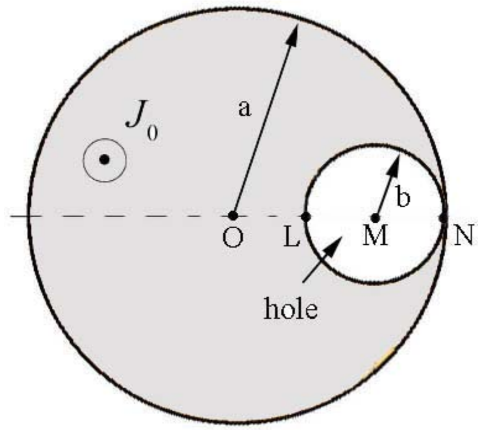
(b)If the direction of current flips then so does the direction of the magnetic field, so it is clockwise rather than counterclockwise. The magnitude of the field remains the same.

(c)The point here is that we have two wires superimposed on top of each other. The large (radius a) wire carries current out of the page while the smaller (radius b) wire carries current into the page (with the same current density). At all points L, M and N we are inside the large wire and on the right, so the counterclockwise B field is pointing up the page. What is happening from the small wire changes from place to place

(i) the point M:

Here we are at the center of the small wire, so it contributes nothing. We are at a radius r = a - b inside the big wire, so from part (a.i) of this problem we have:

$$\boxed{B = \frac{\mu_0 J_0 (a-b)}{2} \text{ up}}$$



(ii) at the point L:

Here we are to the left of the small wire (at a radius r = b), so the clockwise field (as we said in part b) is pointing up, just like the CCW field from the big wire We are at a radius r = a - 2b inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 (a-2b)}{2} + \frac{\mu_0 J_0 b}{2} \text{up} = \frac{\mu_0 J_0 (a-b)}{2} \text{up}}$$

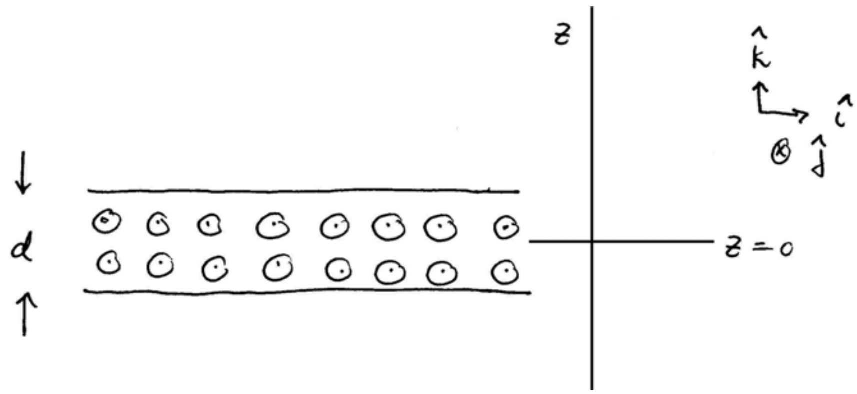
(iii) at the point N:
Here we are to the right of the small wire (at a radius r = b), so the clockwise field is pointing down, opposite the CCW field from the big wire so they subtract rather than add We are at a radius r = a inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 a}{2} - \frac{\mu_0 J_0 b}{2} \text{up} = \frac{\mu_0 J_0 (a-b)}{2} \text{up}}$$

A comment about people's work on this problem: I was stunned at how many people tried to do Ampere's law on the wire with a hole in it. Since the hole breaks the cylindrical symmetry of the problem you just can't do this. That is, since B is no longer constant around an Amperian centered on O, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} \neq 2\pi r B$. B isn't constant, so you can't just pull it out!

Problem 10:

An infinitely large (in the $x-$ and $y-$directions) conducting slab of thickness d is centered at $z=0$. The current density $\vec{\mathbf{J}} = -J_0\,\hat{\mathbf{j}}$ in the slab is uniform and points out of the page in the diagram below.



Calculate the direction and magnitude of the magnetic field of the slab

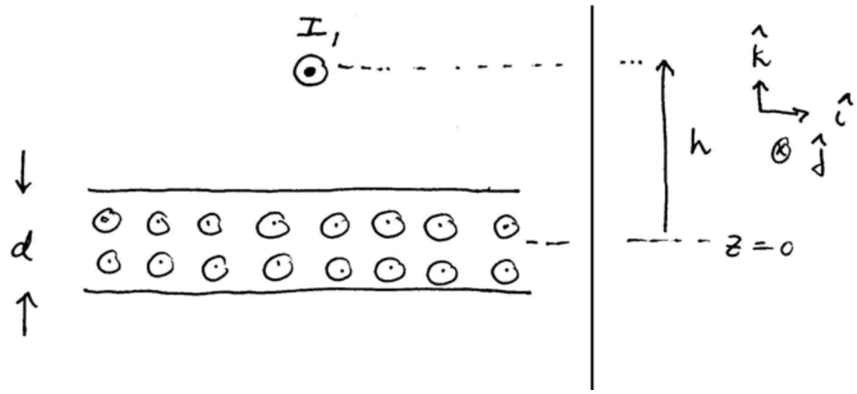
i) above the slab, $z > d/2$.

ii) $\vec{B} = \mu_0 J_0 d/2\hat{i}$ below the slab, $z < -d/2$.

iii) $\vec{B} = -\mu_0 J_0 z\hat{i}$ for $-d/2 < z < d/2$.

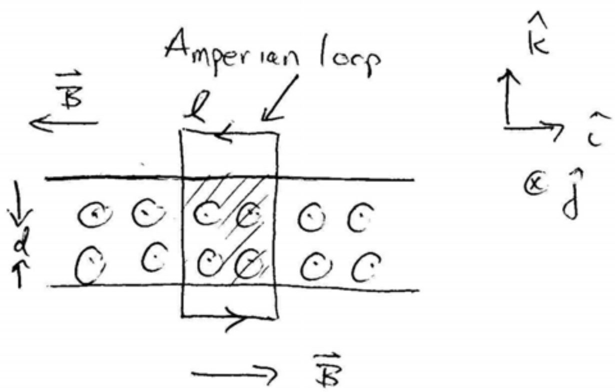
Make a carefully labeled graph showing your results for the dependence of the field components upon position.

A very long wire is now placed at a height $z = h$ above the slab. The wire carries a current I_1 pointing out of the page in the diagram below. What is the direction and magnitude of the force per unit length on the wire?



Problem 10 Solutions:

i) I choose an Amperian loop circulating counterclockwise as shown in the figure above.

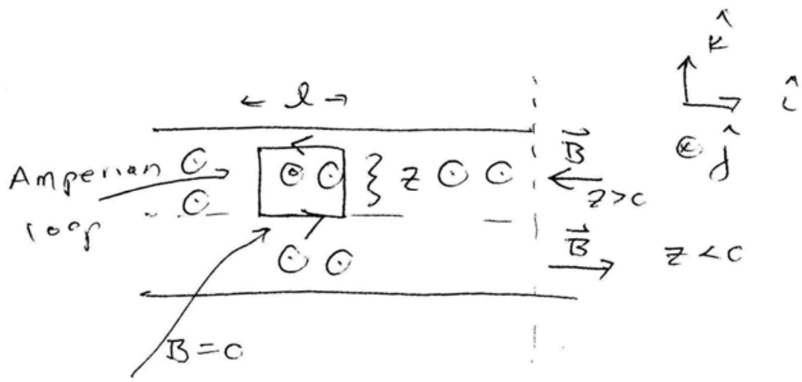


By symmetry, the magnitude of the magnetic field is the same on the upper and lower legs of the loop. Therefore with our choice of circulation direction the left-hand-side of Ampere's Law $\oint \vec{B}\cdot d\vec{s} = \mu_0 \iint \vec{J}\cdot d\vec{a}$ becomes $\oint \vec{B}\cdot d\vec{s} = 2Bl$. The current density is uniform and with the unit normal pointing out of the page ($-\hat{\mathbf{j}}$-direction) consistent with the choice of counterclockwise circulation direction, the right-hand side of Ampere's Law becomes $\mu_0 \iint \vec{J}\cdot d\vec{a} = \mu_0 J_0 l d$. Equate the two sides of Ampere's Law, we have that $2Bl = \mu_0 J_0 l d$ which we can solve for the magnitude of the magnetic field $B = \mu_0 J_0 d/2$. The direction of the magnetic field is the same as the circulation direction on the upper and lower legs. Thus

i) $\vec{B} = -\mu_0 J_0 d/2\hat{i}$ above the slab, $z > d/2$.

ii) $\vec{B} = \mu_0 J_0 d/2\hat{i}$ below the slab, $z < -d/2$.

Inside the slab, the magnetic field is zero at $z = 0$, so we choose an Amperian loop with one leg at $z = 0$ as shown in the figure below.

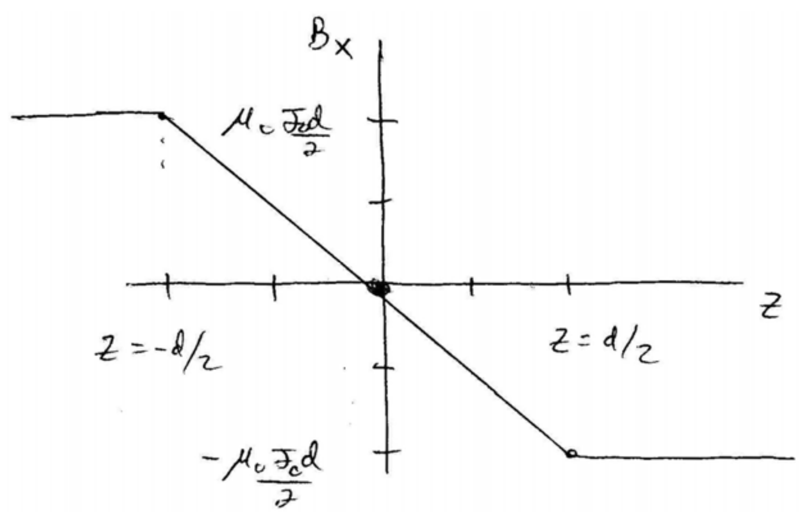


Therefore with our choice of circulation direction the left-hand-side of Ampere's Law $\oint \vec{B}\cdot d\vec{s} = \mu_0 \iint \vec{J}\cdot d\vec{a}$ is now $\oint \vec{B}\cdot d\vec{s} = Bl$. The right-hand side of Ampere's Law becomes $\mu_0 \iint \vec{J}\cdot d\vec{a} = \mu_0 J_0 lz$. Equate the two sides of Ampere's Law, we have that $Bl = \mu_0 J_0 lz$ which we can solve for the magnitude of the magnetic field $B = \mu_0 J_0 |z|$.

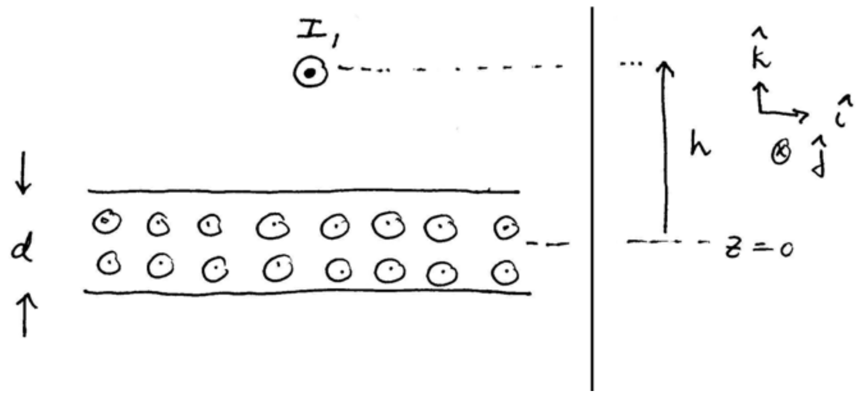
For positive z such that $0 < z < d/2$, the direction of the magnetic field is in the $-\hat{\mathbf{j}}$-direction and for negative z such that $-d/2 < z < 0$, the direction of the magnetic field is in the $+\hat{\mathbf{j}}$-direction. Thus

iii) $\vec{B} = -\mu_0 J_0 z\hat{i}$ for $-d/2 < z < d/2$.

Make a carefully labeled graph showing your results for the dependence of the field components upon position.



A very long wire is now placed at a height $z = h$ above the slab. The wire carries a current I_1 pointing out of the page in the diagram below. What is the direction and magnitude of the force per unit length on the wire?



Solution: The force on a small length ds **of the wire is given by**

$$d\vec{F} = I_1 d\vec{s} \times \vec{B} = -I_1 \hat{j} \times -\frac{\mu_0 J_0 d}{2}\hat{i} = -\frac{(ds) I_1 \mu_0 J_0 d}{2}\hat{k}$$

Therefore the direction and magnitude of the force per unit length on the wire is

$$\frac{d\vec{F}}{ds} = -\frac{I_1 \mu_0 J_0 d}{2}\hat{k}\,.$$

The current is the wire and the current in the slab are in the same direction so the force is attractive.

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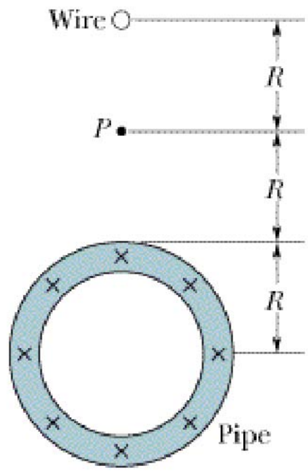
Problem 11:
In the figure at right a long circular pipe with outside radius *R* carries a (uniformly distributed) current *i* into the page. A wire runs parallel to the pipe at a distance of 3.00*R* from center to center. Find the current in the wire such that the ratio of the magnitude of the net magnetic field at point *P* to the magnitude of the net magnetic field at the center of the pipe is *x*, but it has the opposite direction.



Problem 11 Solution:
The field at point P is due both to the pipe and the wire. The field at the center of the pipe is ONLY due to the wire. Since the direction of these two is opposite the current in the wire must create an opposite direction field from the pipe at point P and hence it must also be *into* the page. The magnitude of the field at point P then is just the difference of the two, and realizing that we are outside of both, they both just look like long straight wires and hence:

$\vec{\mathbf{B}}(P) = \dfrac{\mu_o i_{\text{pipe}}}{2\pi(2R)} - \dfrac{\mu_o i_{\text{Wire}}}{2\pi R}$ to the right; $\vec{\mathbf{B}}(\text{Pipe Ctr.}) = \dfrac{\mu_o i_{\text{Wire}}}{2\pi(3R)}$ to the left

So the ratio

$$\frac{B(P)}{B(\text{Pipe Ctr.})} = x = \left(\frac{\mu_o i_{\text{pipe}}}{2\pi(2R)} - \frac{\mu_o i_{\text{Wire}}}{2\pi R}\right) \Bigg/ \frac{\mu_o i_{\text{Wire}}}{6\pi R} = 3\left(\frac{i_{\text{pipe}}}{2} - i_{\text{Wire}}\right) \Big/ i_{\text{Wire}} = 3\left(\frac{i_{\text{pipe}}}{2 i_{\text{Wire}}} - 1\right)$$

$$\Rightarrow \boxed{i_{\text{Wire}} = \frac{i_{\text{pipe}}}{2(x/3+1)} \text{ into the page}}$$