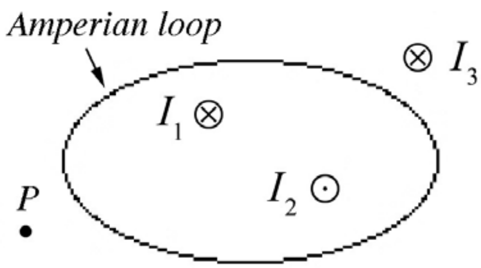
Creating Fields: Ampere's Law
Challenge Problems

Problem 1:

The sketch below shows three wires carrying currents I_1, I_2 and I_3, with an Ampèrian loop drawn around I_1 and I_2. The wires are all perpendicular to the plane of the paper.



Which currents produce the magnetic field at the point P shown in the sketch (circle one)?

a) I_3 only.

b) I_1 and I_2.

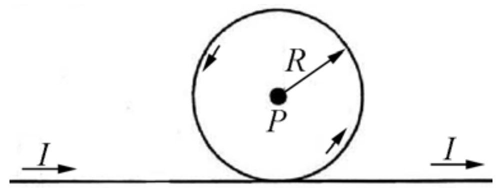
c) I_1, I_2 and I_3.

d) None of them.

e) It depends on the size and shape of the Amperian Loop.

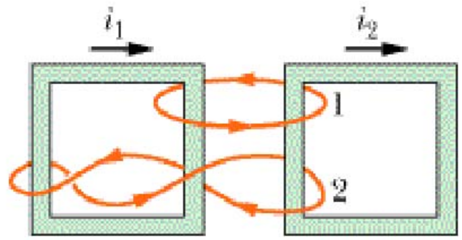
Problem 2:

Find the magnitude and direction of the magnetic field at the point P generated by the current carrying wire and loop depicted in the figure.



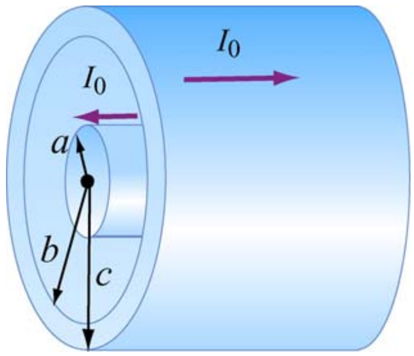
Problem 3:

The figure below shows two closed paths wrapped around two conducting loops carrying currents i_1 and i_2. What is the value of the integral for **(a)** path 1 and **(b)** path 2?



Problem 4:

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections*. Determine the magnitude and direction of the magnetic field at a distance r from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance r from the axis.



(a) $r < a$;

(b) $a < r < b$;

(c) $b < r < c$;

(d) $r > c$.

Problem 5:

Consider an infinitely long, cylindrical conductor of radius *R* carrying a current *I* with a *non-uniform* current density $J = \alpha r^2$, where α is a constant and *r* is the distance from the center of the cylinder.

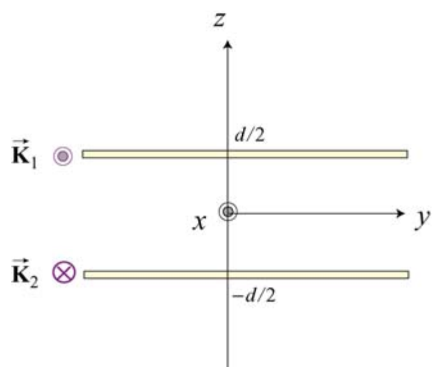
(a) Find the magnetic field everywhere.

(b) Plot the magnitude of the magnetic field as a function of *r*.

[Hint: See Example 9.11.6 for a similar problem.]

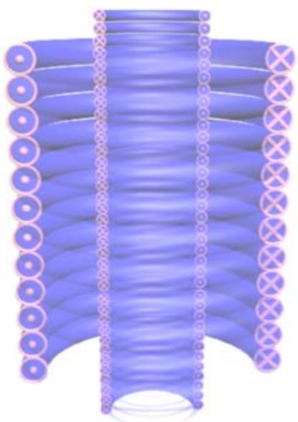
Problem 6:

Consider two infinitely large sheets lying in the *xy*-plane separated by a distance d carrying surface current densities $\vec{\mathbf{K}}_1 = K\,\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\,\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the y direction is infinite.) Note that K is the current per unit width perpendicular to the flow.



a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.
b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.
c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.
d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\,\hat{\mathbf{i}}$?

Problem 7: Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.



Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all your calculations.

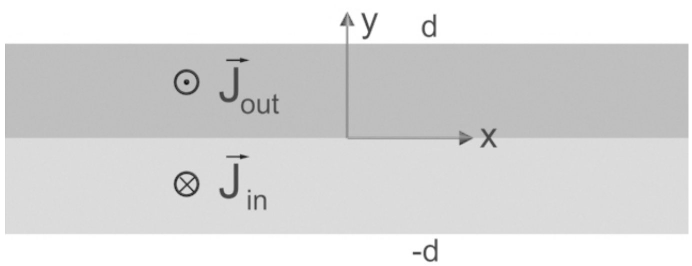
a) $0 < r < R_1$

b) $R_1 < r < R_2$

c) $R_2 < r$

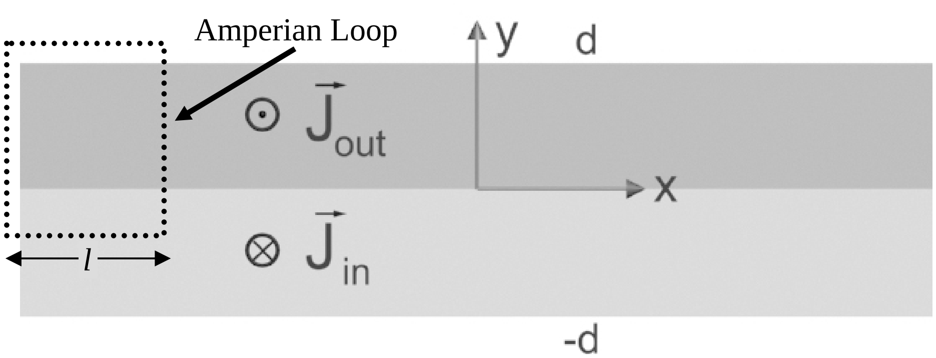
Problem 8:

The figure below shows two slabs of current. Both slabs of current are infinite in the x and z directions, and have thickness d in the y-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\vec{\mathbf{J}}_{out} = J\,\hat{\mathbf{z}}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\vec{\mathbf{J}}_{in} = -J\,\hat{\mathbf{z}}$ into the page.



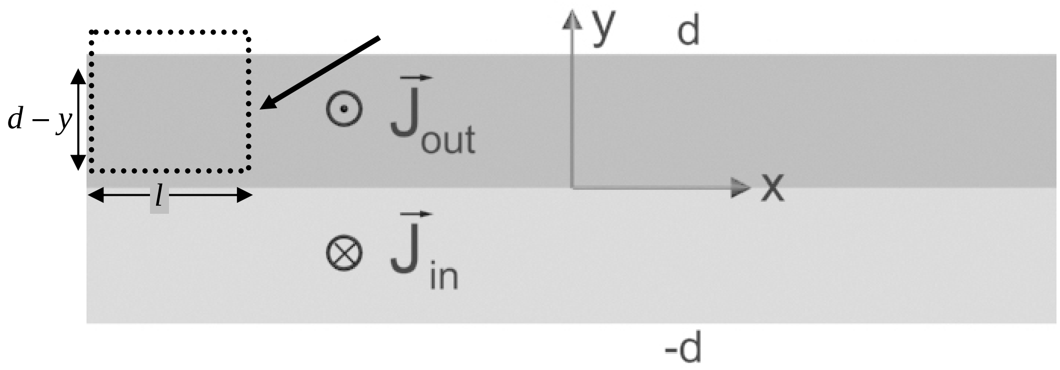
(a) What is the magnetic field for $|y| > d$? Justify your answer.

(b) Use Ampere's Law to find the magnetic field at $y = 0$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.



c) Use Ampere's Law to find the magnetic field for $0 < y < d$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

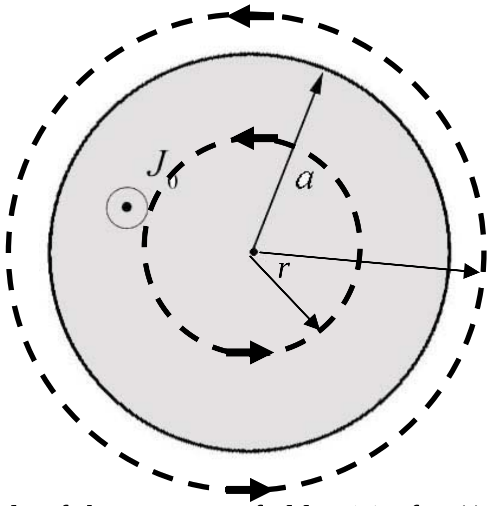
Amperian Loop



(d) Plot the *x*-component of the magnetic field as a function of the distance *y* on the graph below. Label your vertical axis.

Problem 9:

An infinitely long wire of radius a carries a current density J_0 which is uniform and constant. The current points "out of" the page, as shown in the figure.

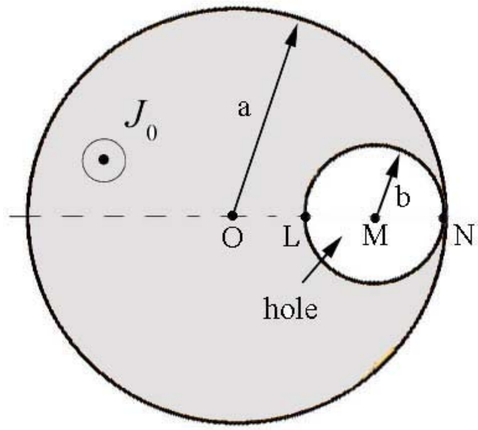


(a) Calculate the magnitude of the magnetic field $B(r)$ for (i) $r < a$ and (ii) $r > a$. For both cases show your Amperian loop and indicate (with arrows) the direction of the magnetic field.

(b) What happens to the answers above if the direction of the current is reversed so that it flows "into" the page ?

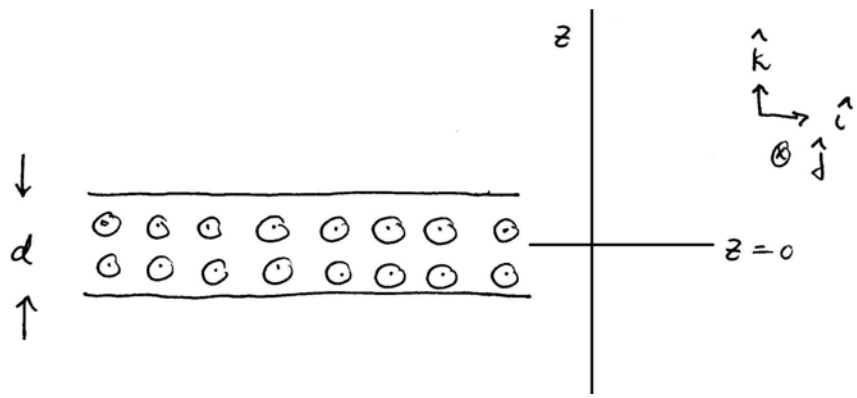
(c) Consider now the same wire but with a hole bored throughout. The hole has radius b (with $2b < a$) and is shown in the figure. We have also indicated four special points: O, L, M, and N. The point O is at the center of the original wire and the point M is at the center of the hole. In this new wire, the current density exists and remains equal to J_0 over the remainder of the cross section of the wire. Calculate the magnitude of the magnetic field at (i) the point M, (ii) at the point L, and (iii) at the point N. Show your work.

Hint: Try to represent the configuration as the "superposition" of two types of wires.



Problem 10:

An infinitely large (in the $x-$ and $y-$directions) conducting slab of thickness d is centered at $z = 0$. The current density $\vec{\mathbf{J}} = -J_0\,\hat{\mathbf{j}}$ in the slab is uniform and points out of the page in the diagram below.



Calculate the direction and magnitude of the magnetic field of the slab

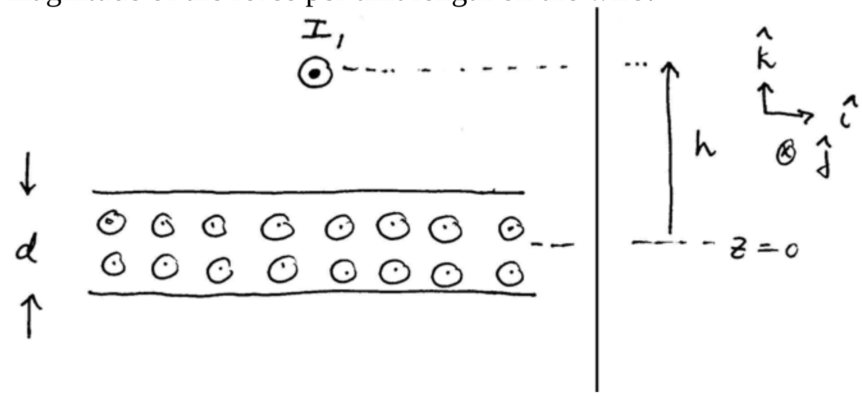
i) above the slab, $z > d/2$.

ii) $\vec{B} = \mu_0 J_0 d/2\hat{i}$ below the slab, $z < -d/2$.

iii) $\vec{B} = -\mu_0 J_0 z\hat{i}$ for $-d/2 < z < d/2$.

Make a carefully labeled graph showing your results for the dependence of the field components upon position.

A very long wire is now placed at a height $z = h$ above the slab. The wire carries a current I_1 pointing out of the page in the diagram below. What is the direction and magnitude of the force per unit length on the wire?



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Problem 11:
In the figure at right a long circular pipe with outside radius R carries a (uniformly distributed) current i into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the current in the wire such that the ratio of the magnitude of the net magnetic field at point P to the magnitude of the net magnetic field at the center of the pipe is x, but it has the opposite direction.