DC Circuits
Challenge Problems

Problem 1:
A battery of emf $\mathcal{E}$ has internal resistance R_i, and let us suppose that it can provide the emf to a total charge Q before it expires. Suppose that it is connected by wires with negligible resistance to an external (load) with resistance R_L.

a) What is the current in the circuit?

b) What value of R_L maximizes the current extracted from the battery, and how much chemical energy is generated in the battery before it expires?

c) What value of R_L maximizes the total power delivered to the load, and how much energy is delivered to the load before it expires? How does this compare to the energy generated in the battery before it expires?

d) What value for the resistance in the load R_L would you need if you want to deliver 90% of the chemical energy generated in the battery to the load? What current should flow? How does the power delivered to the load now compare to the maximum power output you found in part c)?

Problem 2:

AAA, AA, … D batteries have an open circuit voltage (EMF) of 1.5 V. The difference between different sizes is in their lifetime (total energy storage). A AAA battery has a life of about 0.5 A-hr while a D battery has a life of about 10 A-hr. Of course these numbers depend on how quickly you discharge them and on the manufacturer, but these numbers are roughly correct. One important difference between batteries is their internal resistance – alkaline (now the standard) D cells are about 0.1Ω.

Suppose that you have a multi-speed winch that is 50% efficient (50% of energy used does useful work) run off a D cell, and that you are trying to lift a mass of 60 kg (hmmm, I wonder what mass that would be). The winch acts as load with a variable resistance R_L that is speed dependent.

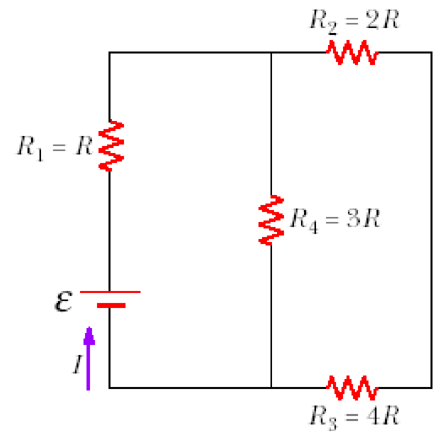
a) Suppose the winch is set to super-slow speed. Then the load (winch motor) resistance is much greater than the battery's internal resistance and you can assume that there is no loss of energy to internal resistance. How high can the winch lift the mass before discharging the battery?

b) To what resistance R_L should the winch be set in order to have the battery lift the mass at the fastest rate? What is this fastest rate (m/sec)? HINT: You want to maximize the power delivery to the winch (power dissipated by R_L).

c) At this fastest lift rate how high can the winch lift the mass before discharging the battery?

d) Compare the cost of powering a desk light with D cells as opposed to plugging it into the wall. Does it make sense to use rechargeable batteries? Residential electricity costs about $0.1/kwh.

Problem 3:



Four resistors are connected to a battery as shown in the figure. The current in the battery is I, the battery emf is $\mathcal{E}$, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.

(a) Rank the resistors according to the potential difference across them, from largest to smallest. Note any cases of equal potential differences.

(b) Determine the potential difference across each resistor in terms of $\mathcal{E}$.

(c) Rank the resistors according to the current in them, from largest to smallest. Note any cases of equal currents.

(d) Determine the current in each resistor in terms of I.

(e) If R_3 is increased, what happens to the current in each of the resistors?

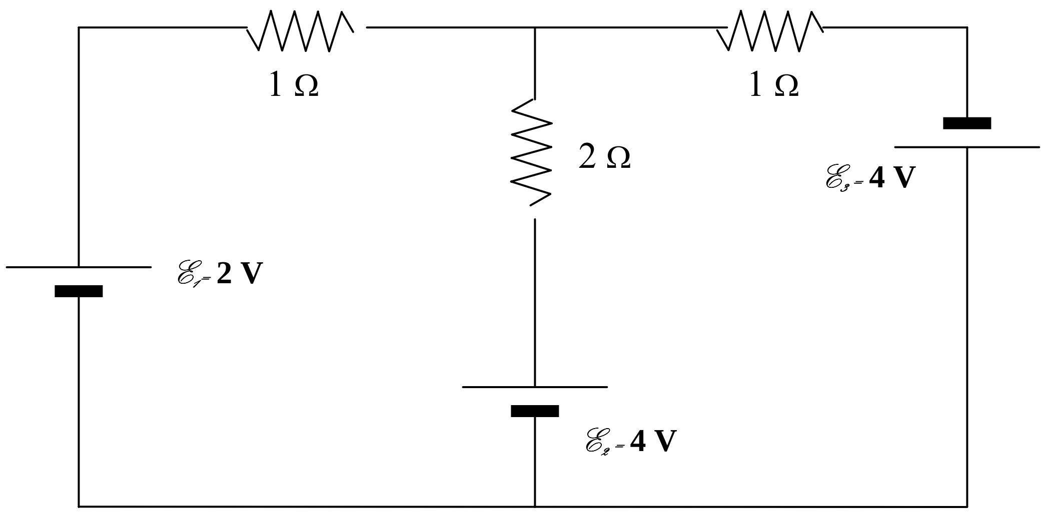
(f) In the limit that $R_3 \rightarrow \infty$, what are the new values of the current in each resistor in terms of I, the original current in the battery?

Problem 4:

In the circuit below, you can neglect the internal resistance of all batteries.

(a) Calculate the current through each battery

(b) Calculate the power delivered or used (specify which case) by each battery



8.02SC Physics II: Electricity and Magnetism
Fall 2010