Current, Current Density, Resistance and Ohm's Law
Challenge Problems

Problem 1:

A straight cylindrical wire lying along the x-axis has a length L and a diameter d. It is made of a material described by Ohm's law with a resistivity ρ. Assume that a potential V is maintained at $x = 0$, and that $V = 0$ at $x = L$. In terms of L, d, V, ρ, and physical constants, determine expressions for

(a) the electric field in the wire.

(b) the resistance of the wire.

(c) the electric current in the wire.

(d) the current density in the wire. Express vectors in vector notation.

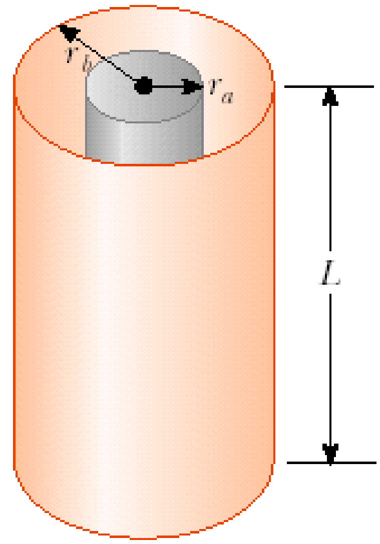
(e) Show that $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$.

Problem 2:

The first telegraphic messages crossed the Atlantic Ocean in 1858, by a cable 3000 km long laid between Newfoundland and Ireland. The conductor in this cable consisted of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the conductor. Use $3 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ for the resistivity of copper, which was of somewhat dubious purity.

Problem 3:

An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water (until completely submerged) a pair of concentric metallic cylinders (see figure) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a , outer radius r_b, and length L much larger than r_b. The scientist applies a potential difference ΔV between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water.



(a) Find the resistance of the water between the cylinders in terms of L, ρ, r_a, and r_b.

(b) Express the resistivity of the water in terms of the measured quantities L, r_a, r_b, ΔV, and I.

Problem 4:

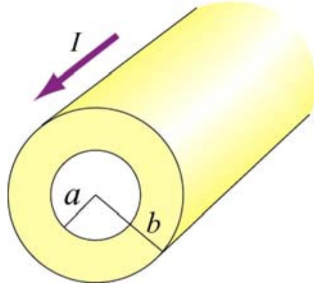
A cylindrical glass rod is heated with a torch until it conducts enough current to cause a light bulb to glow. The rod has a length L = 2 cm, a diameter d = 0.5 cm, and its ends, plated with material of infinite conductivity, are connected to the rest of the circuit. When red hot, the rod's conductivity varies with position x measured from the center of the rod as $\sigma(x) = \sigma_0 L^4 / x^4$, with $\sigma_0 = 4 \times 10^{-2} (\Omega - m)^{-1}$.

a) What is the resistance of the glass rod? Express your answer both symbolically and as a value in ohms.

b) When a voltage ΔV is applied between the two ends, what is the current density $\vec{\mathbf{J}}(x)$ and what is the steady-state electric field $\vec{\mathbf{E}}(x)$?

Problem 5:

Consider a cylindrical conductor with a hollow center and walls of inner and outer radii a and b respectively. The current I is ***non-uniformly*** spread over the cross section of the conductor, with a density that falls exponentially from the outer surface: $J_0 e^{\left(r^2-b^2\right)/\delta^2}$.



Find the constant J_0 in order to have total current I, given that δ, the "skin depth," is a distance much smaller than the wall thickness (b-a).