8.022 Lecture Notes Class 43 - 12/7/2006

Topics for Next Week (options)

1. More on E/M waves

2. Potentials (V,$\vec{A}$)

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{A} = 0 \quad \text{(Coulomb gauge)}$$

$$\vec{\nabla} \times \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \quad \text{(Lorentz gauge)}$$

3. Relativity
   - Maxwell ok
   - gravity

4. Something else
   - quantum

Relativity

Position 4-vector $x^\mu = (ct, x, y, z)$

Momentum 4-vector $p^\mu = (\frac{E}{c}\ , p_x, p_y, p_z)$

$$\Delta s^2 = -c\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -\tau^2$$

$$\Delta s^2 = g_{\mu\nu}\Delta x^\mu \delta^\nu$$

(Einstein Summation notation: $\Sigma_{\mu,\nu}$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Δx^μ - contravariant

Covariant

$$\Delta x_\mu = g_{\mu\nu}\Delta x^\nu$$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu$$

Derivative with respect with respect to contravariant is covariant?

$$\frac{\partial}{\partial x^\mu} x^\mu = \partial_\mu x^\mu \cong 1$$

Magnetic Potential for 4-vector

$$A^\mu = (V, A_x, A_y, A_z)$$

Let $c = 1$

$$F^{\mu\nu} = \frac{\partial A^\mu}{\partial x_\mu} - \frac{\partial A^\nu}{\partial x_\nu}$$

$$= \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

Current 4-vector

$$J^\mu = (\rho, J_x, J_y, J_z)$$

Can relate:

$$\partial_\nu F^{\mu\nu} \alpha J^\mu$$